



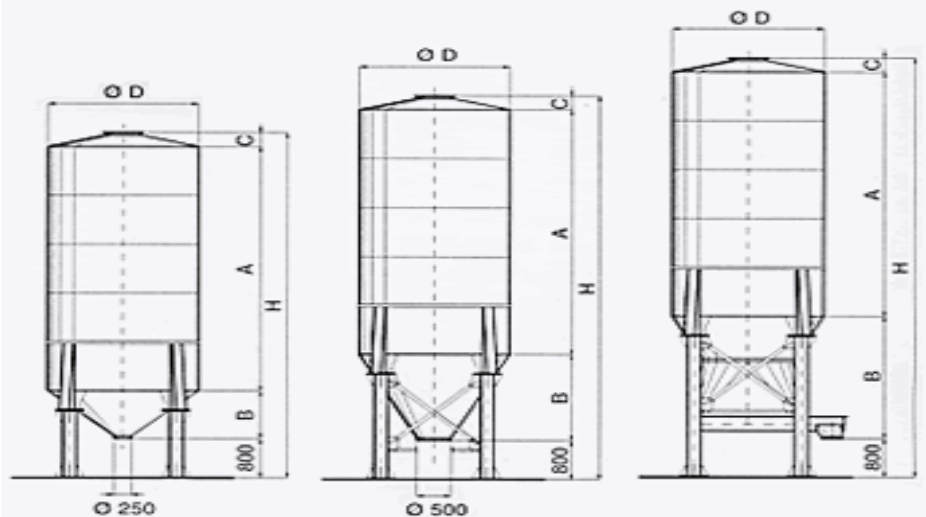
EN 1999 - Eurocode 9: Design of aluminium structures Part 1.5 - Shell structures

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Aluminium shells – applications



The EN1999-1-5 General part

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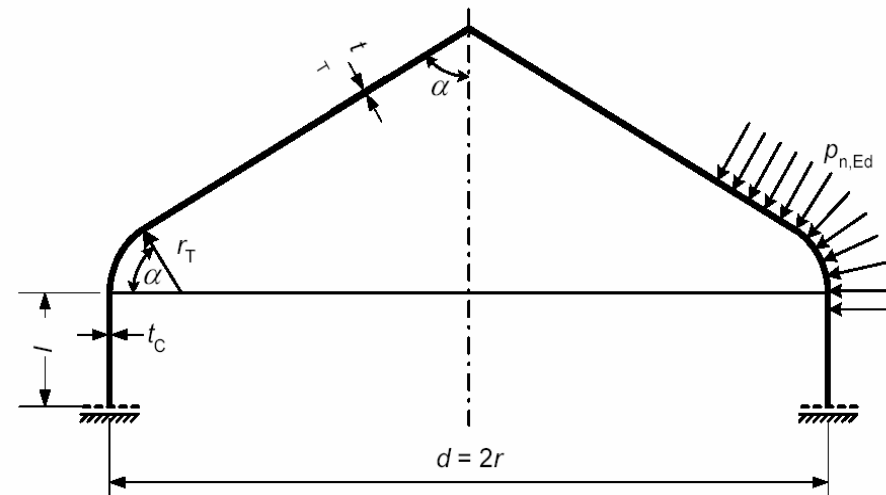
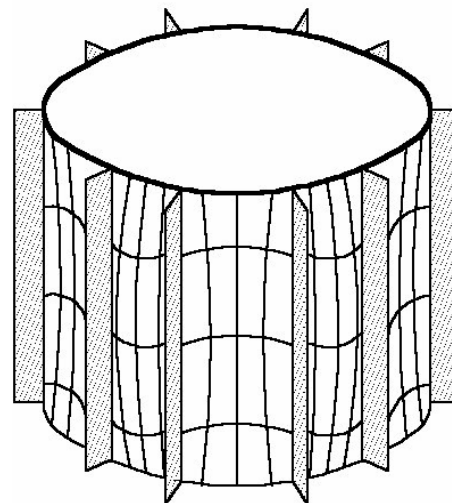
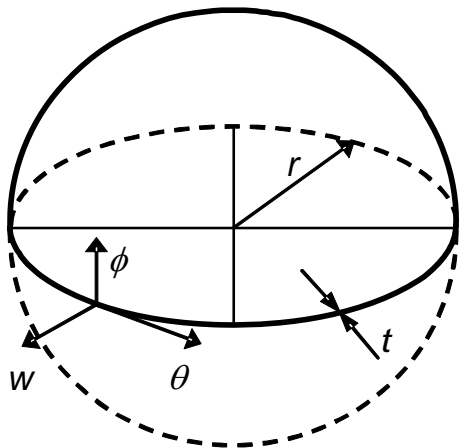
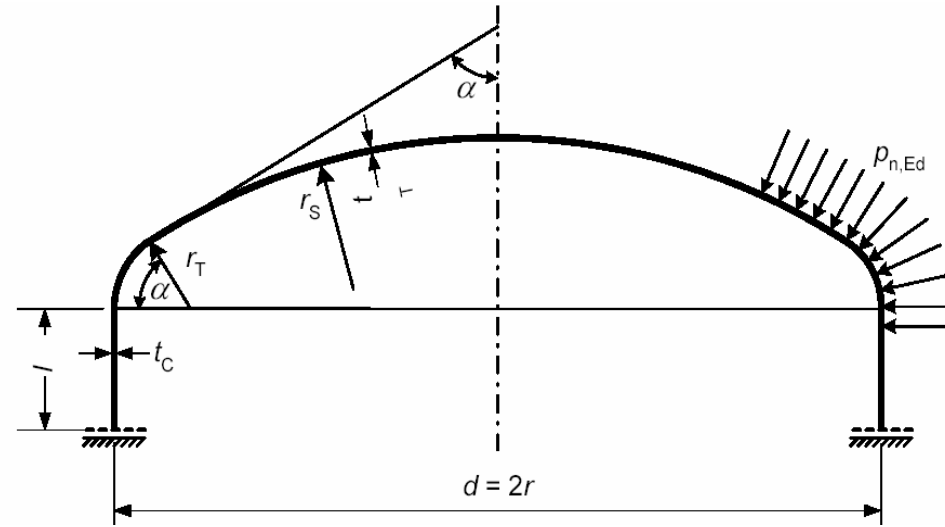
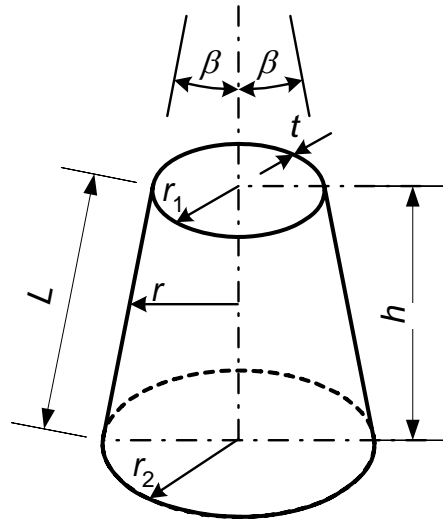
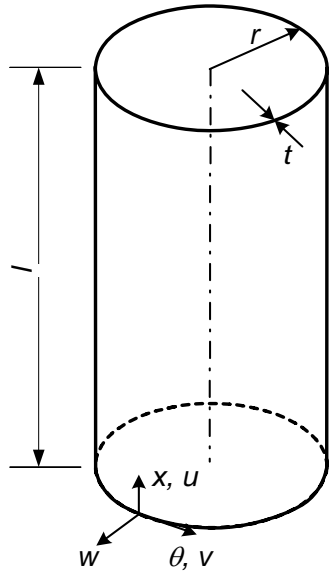


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Shell configurations allowed for in EN1999-1-5



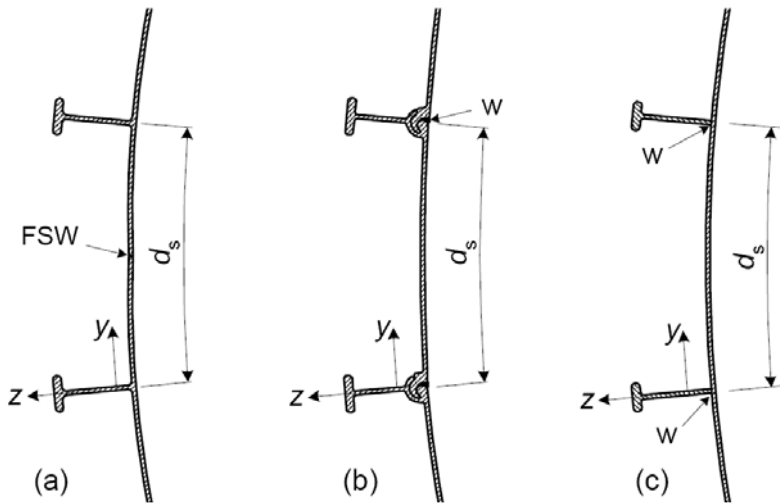


Types of shell analysis in EN1999-1-5

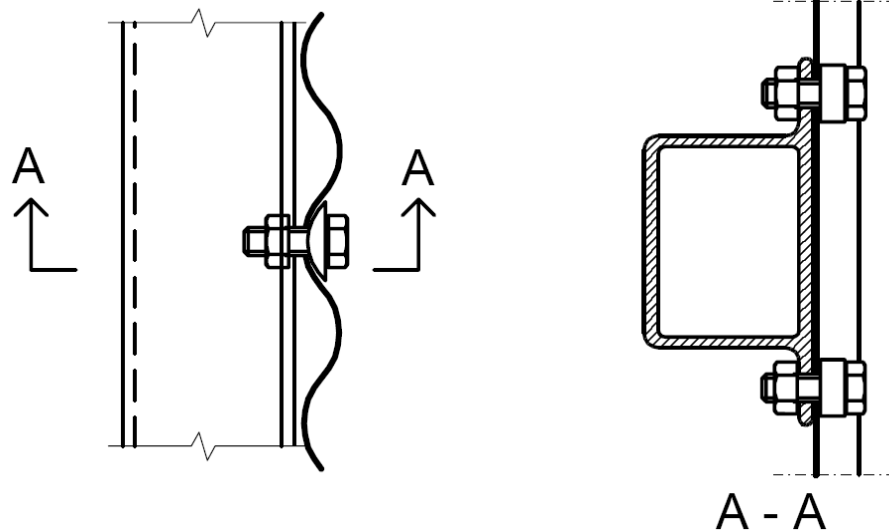
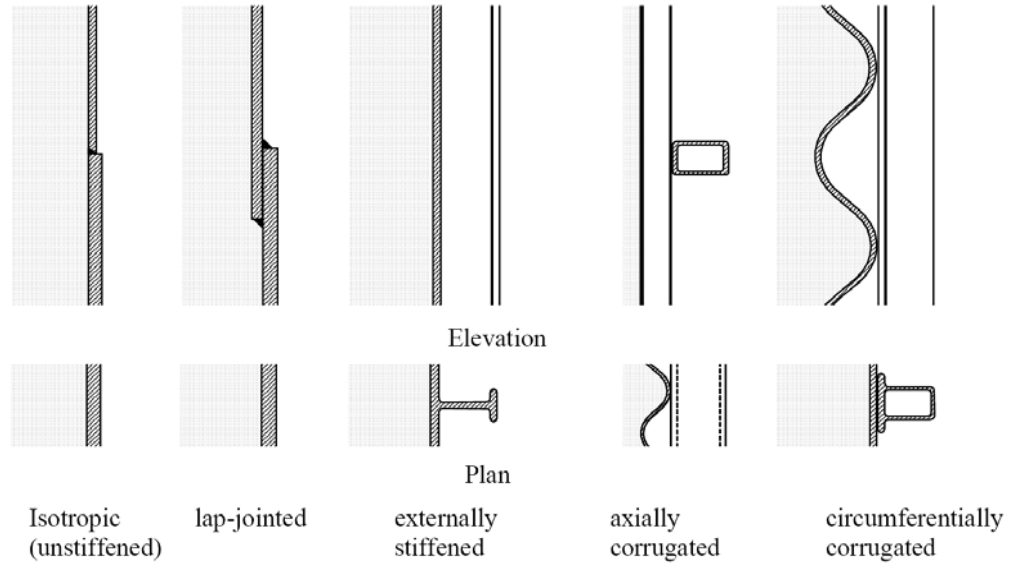
Membrane theory analysis (MTA)	An analysis of a shell structure under distributed loads assuming a set of membrane forces that satisfy equilibrium with the external loads.
Linear elastic analysis (LA)	An analysis on the basis of the small deflection linear elastic shell bending theory assuming perfect geometry.
Linear elastic bifurcation (eigenvalue) analysis (LBA)	An analysis that calculates the linear elastic bifurcation eigenvalue on the basis of small deflections using the linear elastic shell bending theory, assuming perfect geometry. Note that eigenvalue in this context does not refer to vibration modes.
Geometrically non-linear analysis (GNA)	An analysis on the basis of the shell bending theory assuming perfect geometry, considering non-linear large deflection theory and linear elastic material properties.
Materially non-linear analysis (MNA)	An analysis equal to (LA), however, considering non-linear material properties. For welded structure the material in the heat-affected zone should be modelled.
Geometrically and materially non-linear analysis (GMNA)	An analysis applying the shell bending theory assuming perfect geometry, considering non-linear large deflection theory and non-linear material properties. For welded structure the material in the heat-affected zone should be modelled.
Geometrically non-linear elastic analysis with imperfections included (GNIA) ¹⁾	An analysis equal to (GNA), however, considering an imperfect geometry.
Geometrically and materially non-linear analysis with imperfections included (GMNIA)	An analysis equal to (GMNA), however, considering an imperfect geometry.

1) This type of analyses is not covered in this standard, however, listed here for the purpose of having a complete presentation of types of shell analysis.

Specific issues for aluminium alloy shells in EN1999-1-5



Key: w = weld, FSW = friction stir welding



Background activity - Main investigated aspects

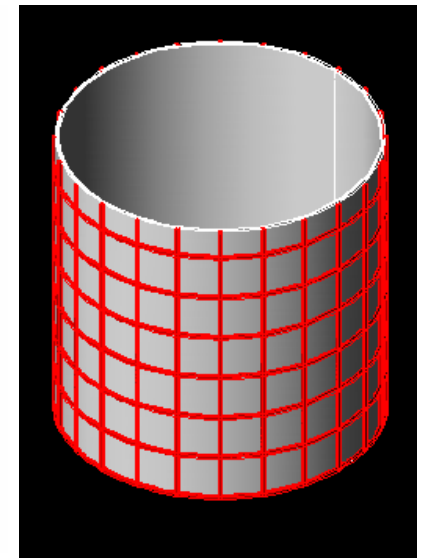
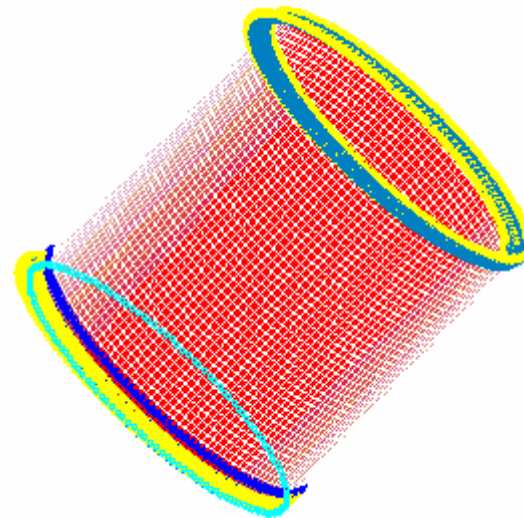
- **shell plastic buckling**
 - **imperfection sensitivity analysis of aluminium cylinders;**
 - set-up of buckling curves for aluminium shells;
 - definition of imperfection classes for plastic buckling;
 - interaction between load cases;
 - introduction of additional shell configurations;
- **stiffened shells**
 - imperfection sensitivity analysis of stiffened cylinders;
 - validation of EN1993-1-6 procedures and harmonization with EN1999 rules;
- **effect of welding effect (HAZ zones)**
 - imperfection sensitivity analysis of welded cylinders;
 - definition of simplified design procedures;

Parametric analysis: Shell geometrical data and material features

R/t	R [mm]	t [mm]	L [mm]	L/R
Cylinders under axial compression				
200	1000	5	2000	2
100	1000	10	2000	2
50	1000	20	2000	2
25	1000	40	2000	2
12.5	1000	80	2000	2
Cylinders under external pressure				
200	1000	5	4000	4
100	1000	10	4000	4
50	1000	20	4000	4
200	1000	5	2000	2
100	1000	10	2000	2
50	1000	20	2000	2
200	1000	5	1000	1
100	1000	10	1000	1
50	1000	20	1000	1
Cylinders under torsion				
200	1000	5	4000	4
100	1000	10	4000	4
50	1000	20	4000	4
200	1000	5	2000	2
100	1000	10	2000	2
50	1000	20	2000	2

	$f_{0.2}$ [MPa]	$n_{R.0}$
Strong hardening alloys	100	10
Weak hardening alloys (Heat-treated alloys)	200	20
	300	30

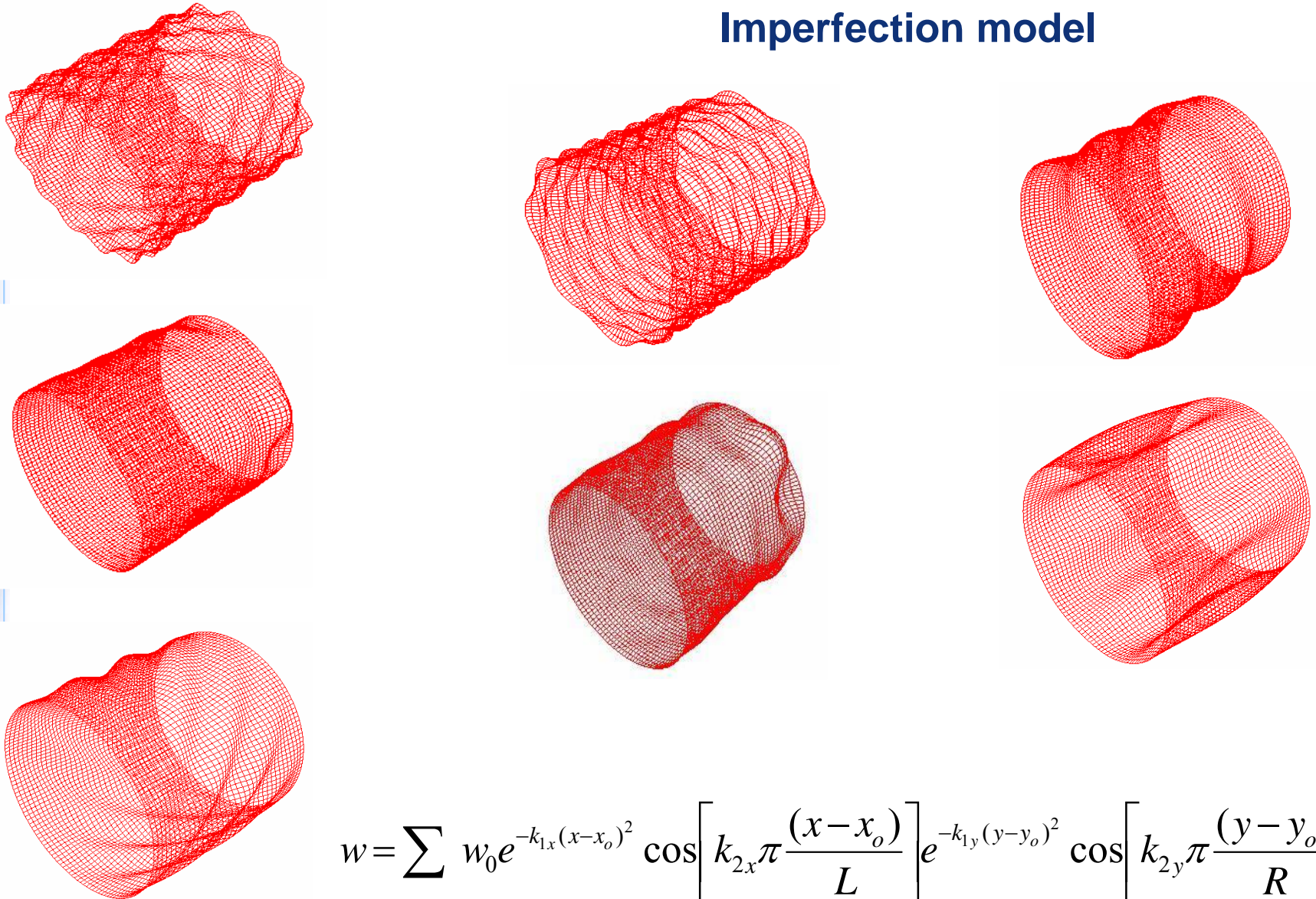
Mechanical features of alloys under consideration.



Geometric data of analysed cylinders (R mean radius, t wall thickness, L overall length)

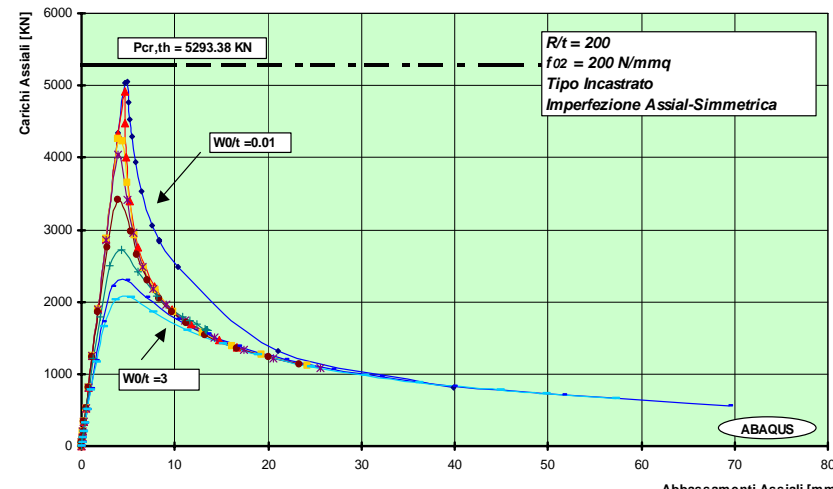
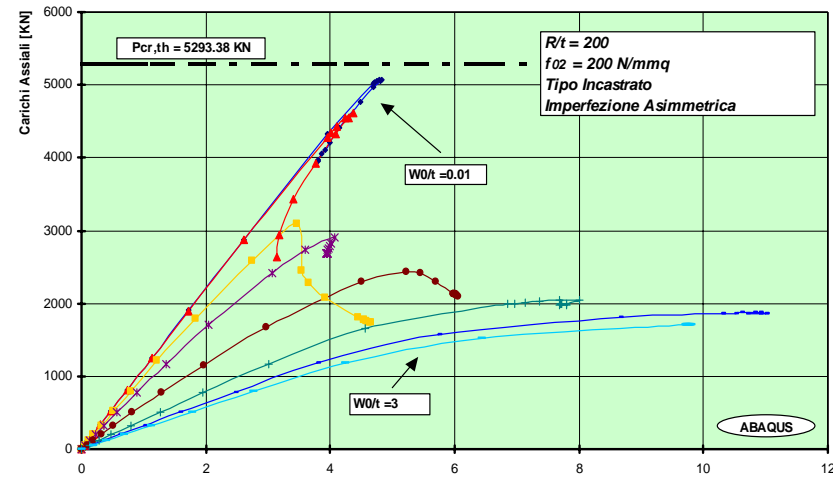
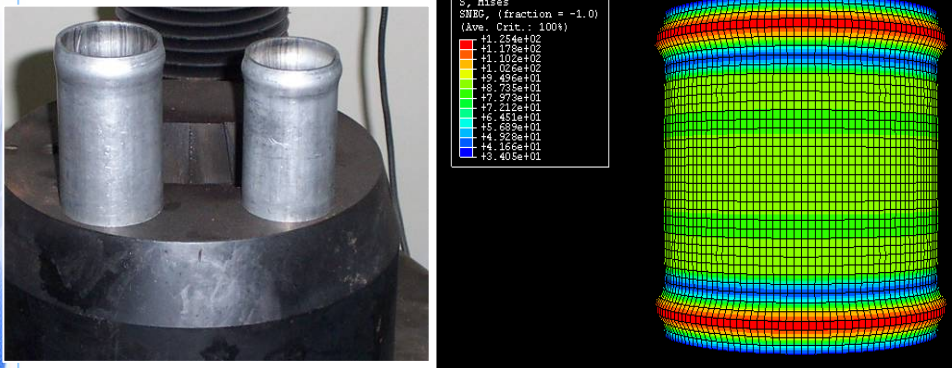
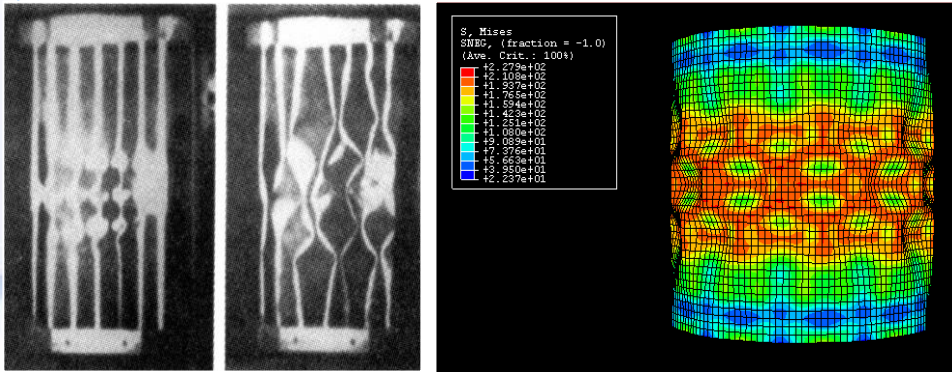
The ABAQUS model

Imperfection model



$$w = \sum w_0 e^{-k_{1x}(x-x_0)^2} \cos \left[k_{2x} \pi \frac{(x-x_0)}{L} \right] e^{-k_{1y}(y-y_0)^2} \cos \left[k_{2y} \pi \frac{(y-y_0)}{R} \right]$$

Buckling response of axially loaded cylinders





Deflected shapes at buckling (cylinders under uniform external pressure)

Diagramma Carichi-Spostamenti Radiali

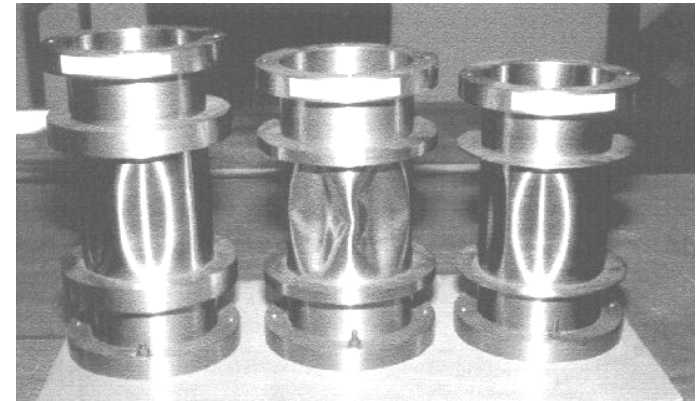
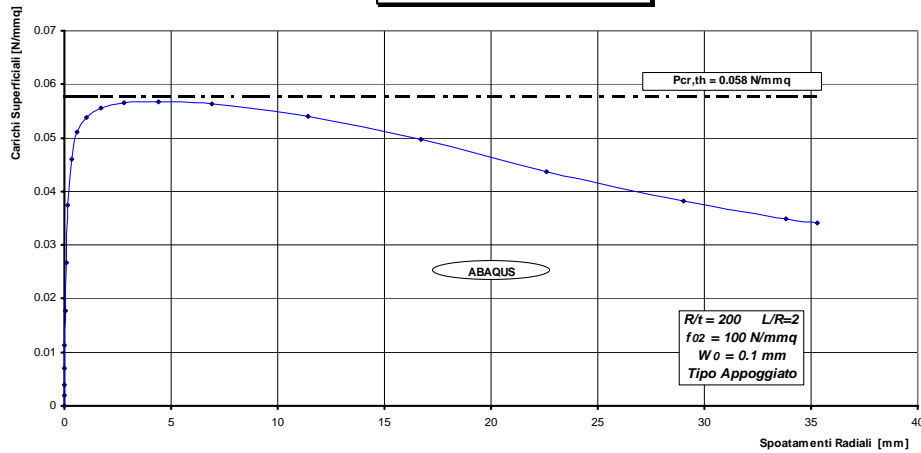
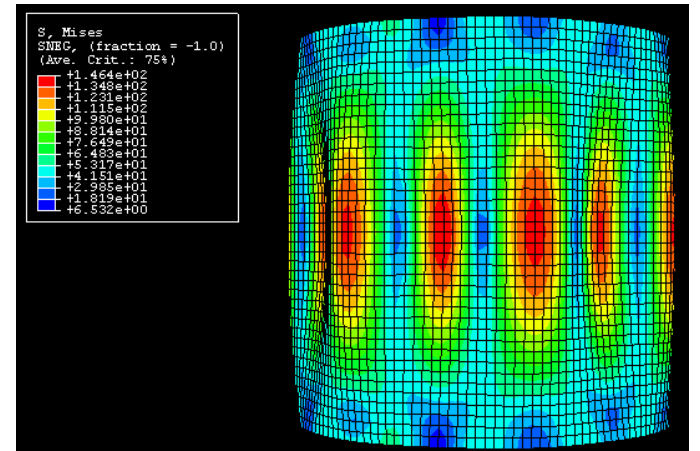
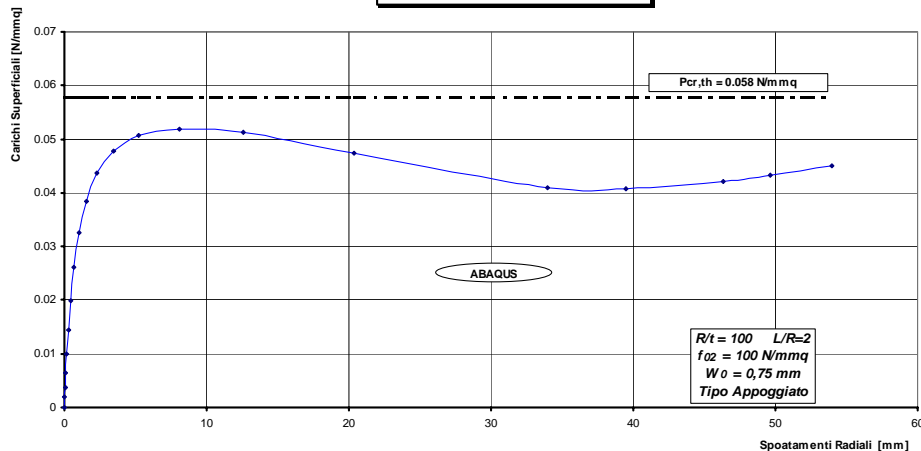
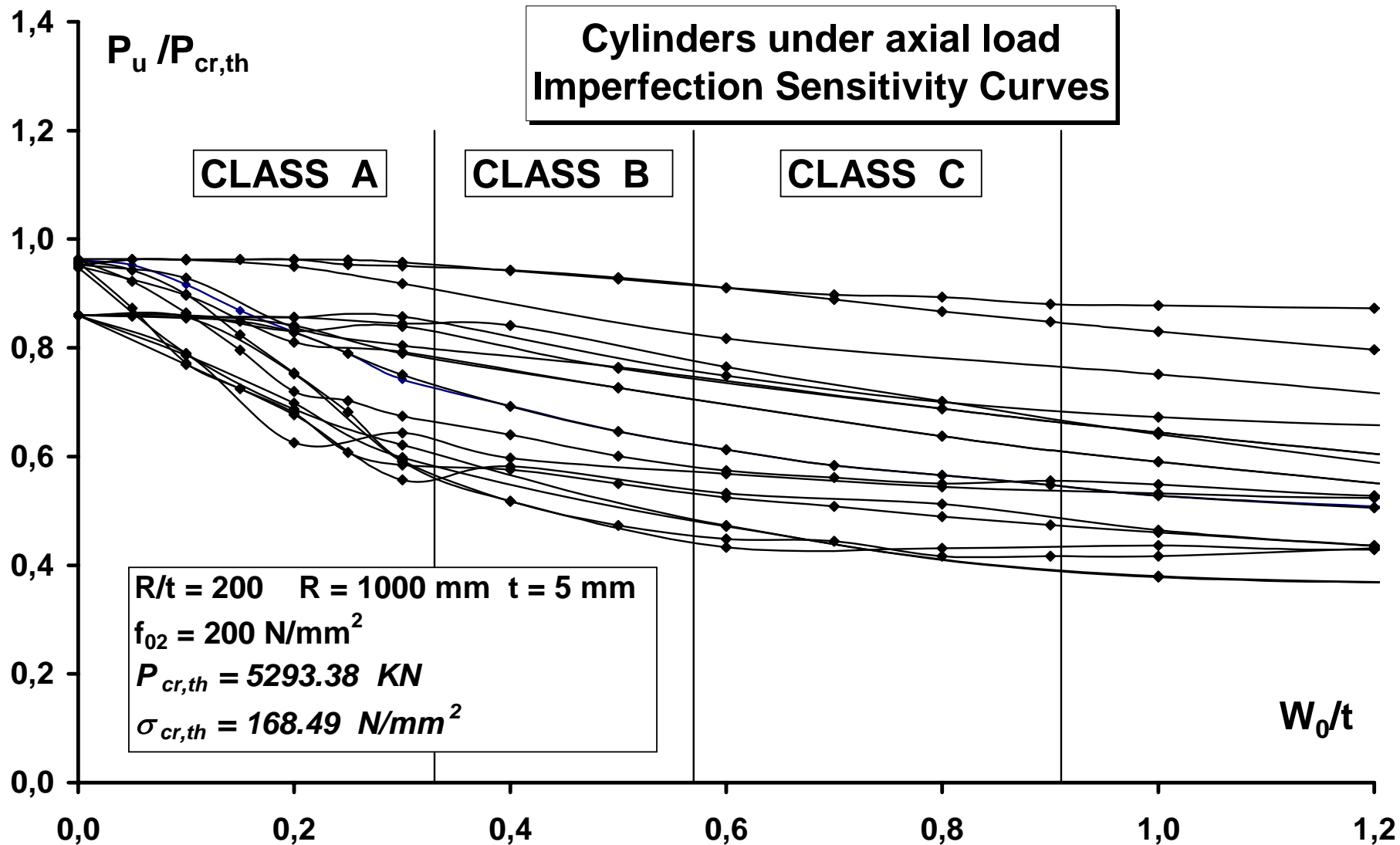


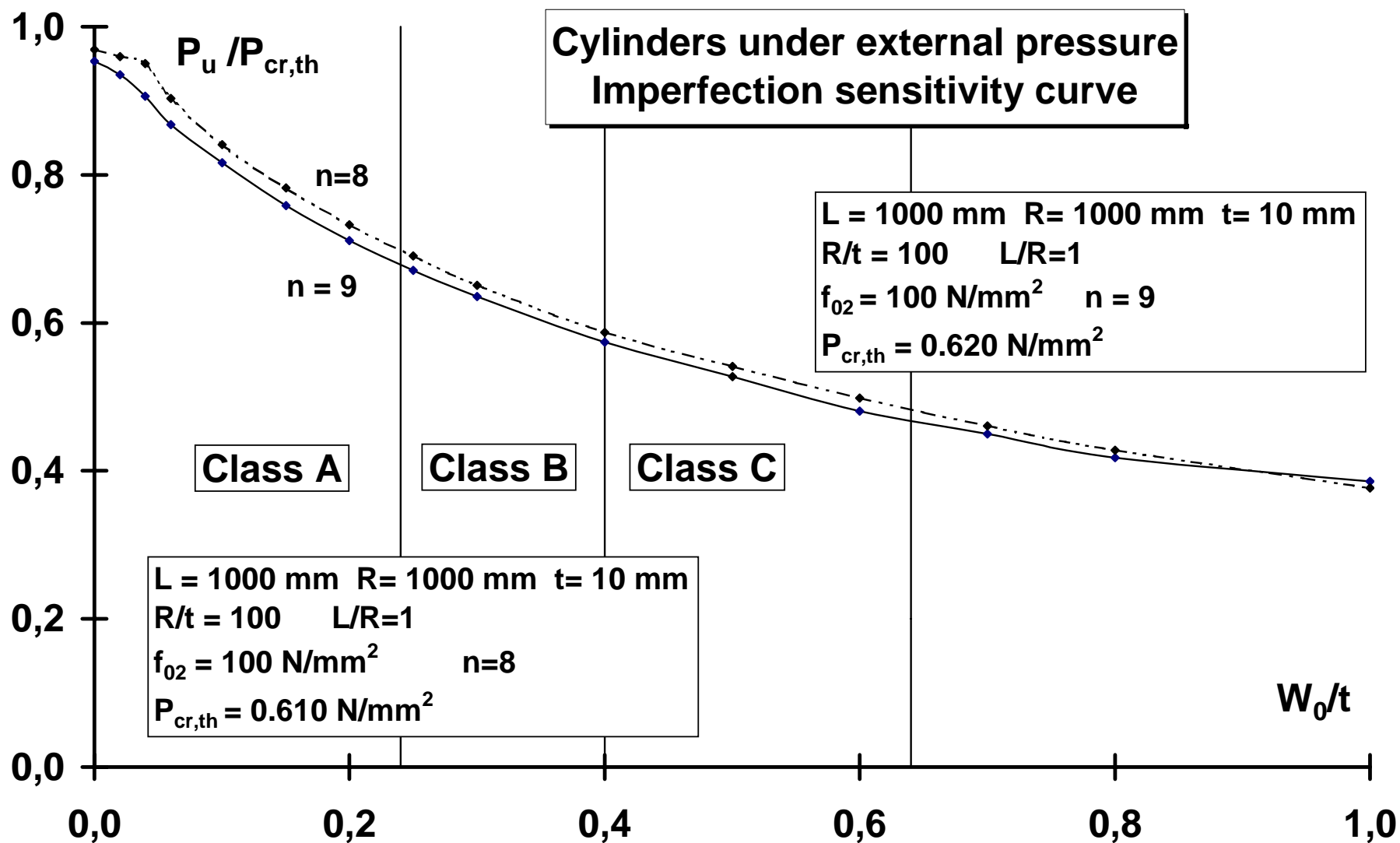
Diagramma Carichi-Spostamenti Radiali



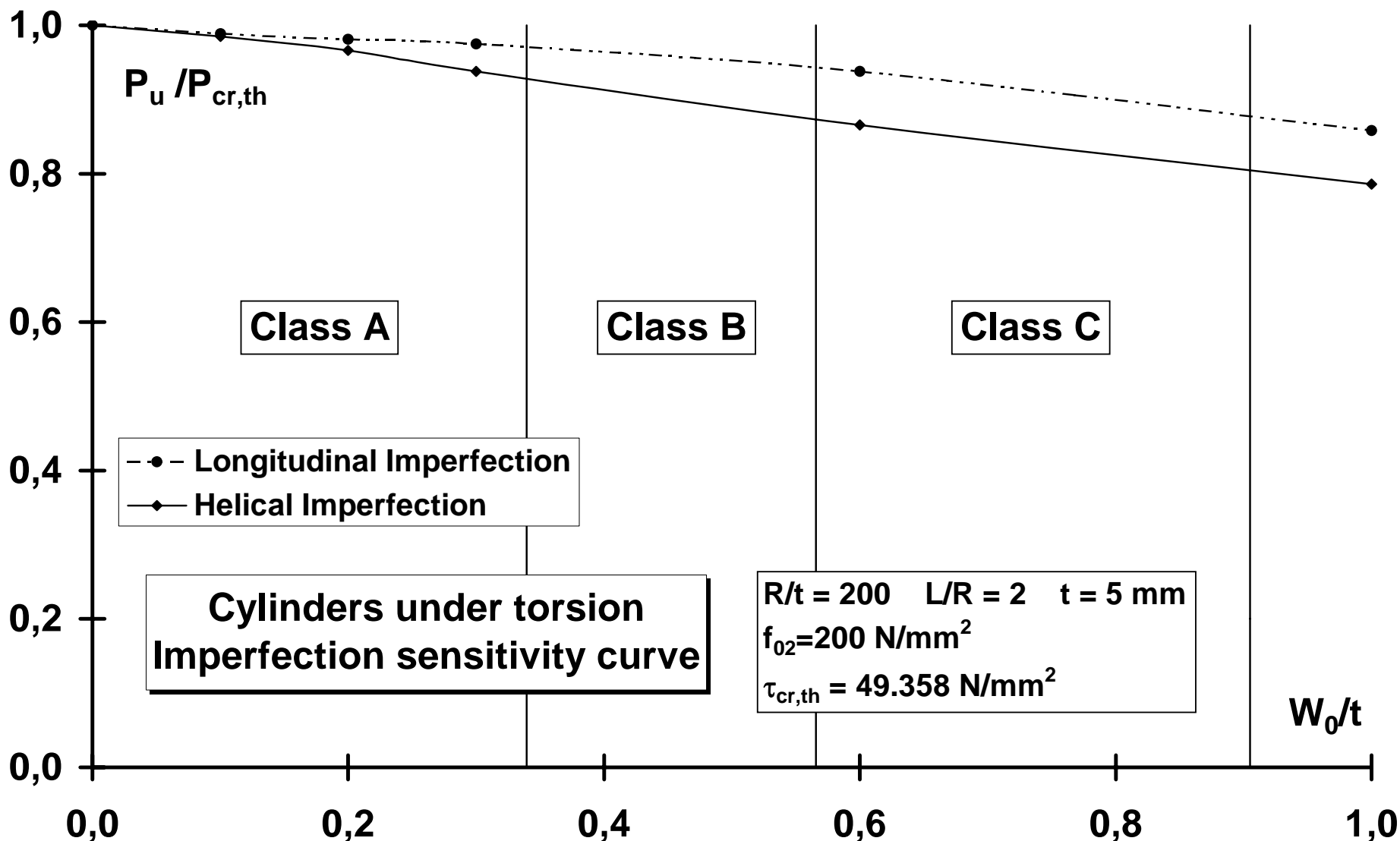
Imperfection sensitivity curves (axially loaded cylinders)



Imperfection sensitivity curves (cylinders under external pressure)

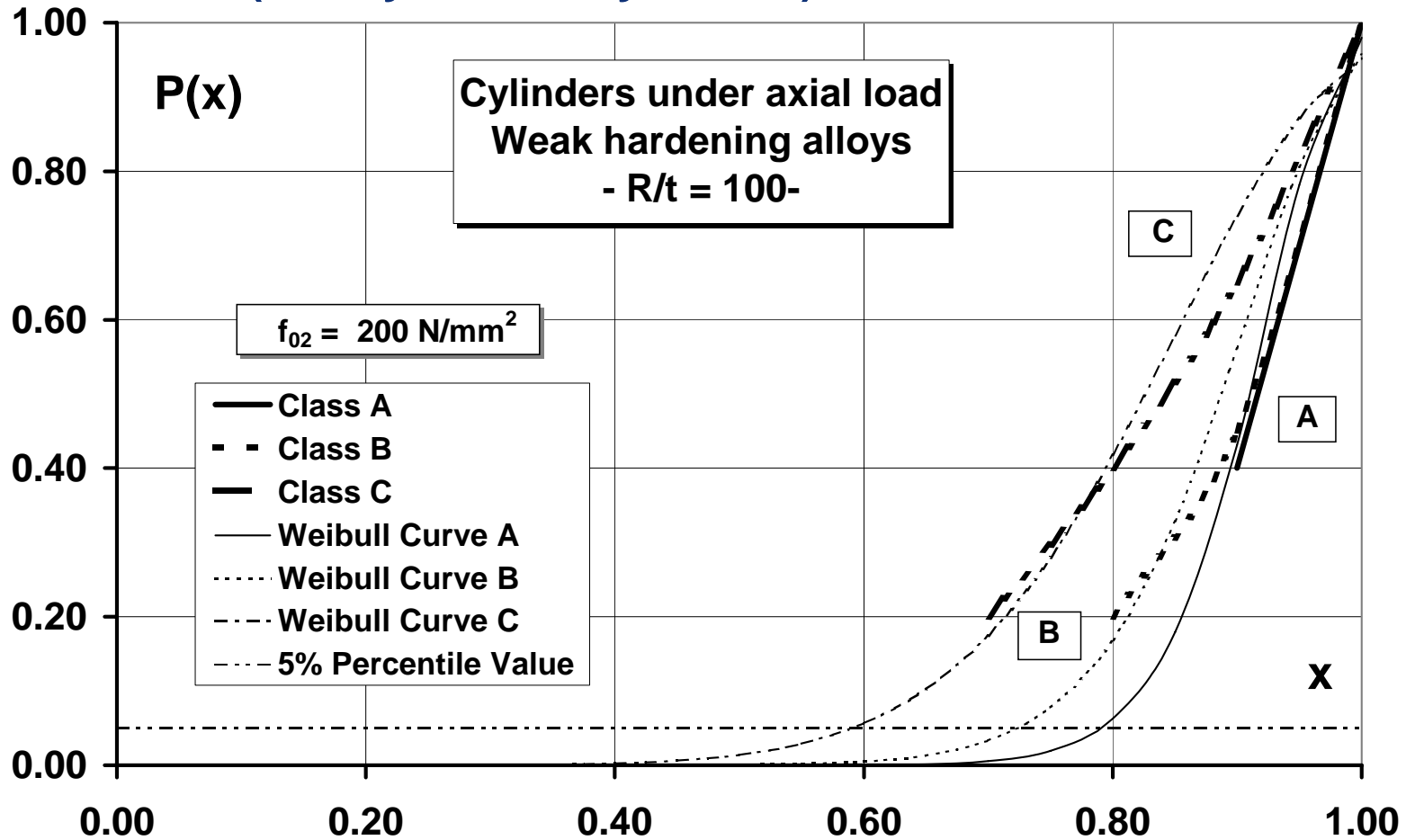


Imperfection sensitivity curves (cylinders under torsion)



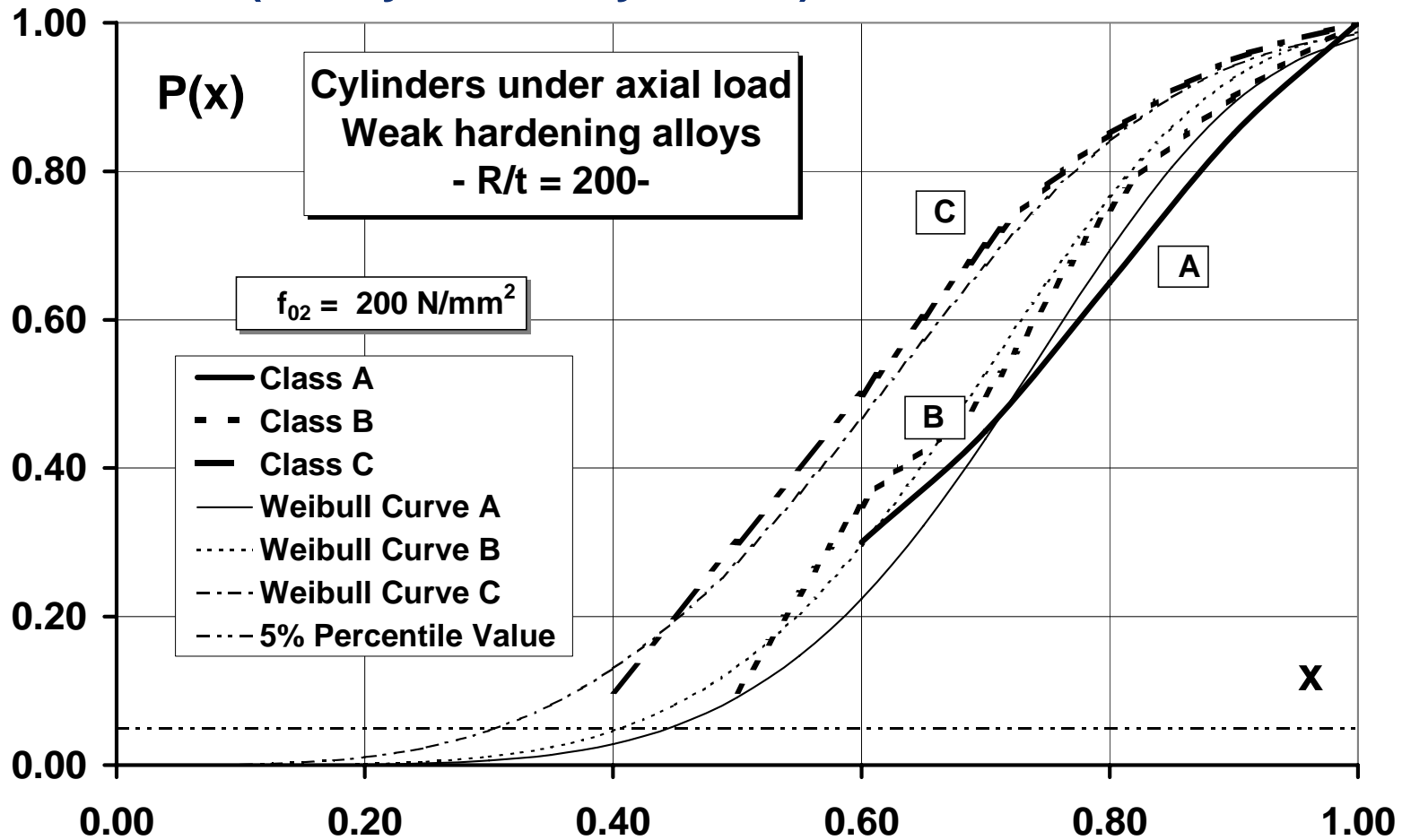


Semi-probabilistic interpretation of buckling data (axially loaded cylinders) - Weibull's law



$$p(x) = \frac{dP(x)}{dx} = \frac{1}{\alpha\beta^{1/\alpha}} x^{(1/\alpha - 1)} e^{-(x/\beta)^{1/\alpha}}$$

Semi-probabilistic interpretation of buckling data (axially loaded cylinders) - Weibull's law



$$p(x) = \frac{dP(x)}{dx} = \frac{1}{\alpha\beta^{1/\alpha}} x^{(1/\alpha - 1)} e^{-(x/\beta)^{1/\alpha}}$$

Background activity - Main investigated aspects

- **shell plastic buckling**
 - imperfection sensitivity analysis of aluminium cylinders;
 - **set-up of buckling curves for aluminium shells;**
 - definition of imperfection classes for plastic buckling;
 - interaction between load cases;
 - introduction of additional shell configurations;
- **stiffened shells**
 - imperfection sensitivity analysis of stiffened cylinders;
 - validation of EN1993-1-6 procedures and harmonization with EN1999 rules;
- **effect of welding effect (HAZ zones)**
 - imperfection sensitivity analysis of welded cylinders;
 - definition of simplified design procedures;



Shell buckling – EC3 formulation

$$\chi = 1 \Leftrightarrow \lambda \leq \lambda_0$$

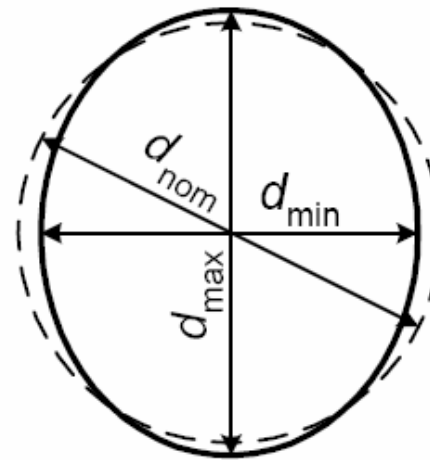
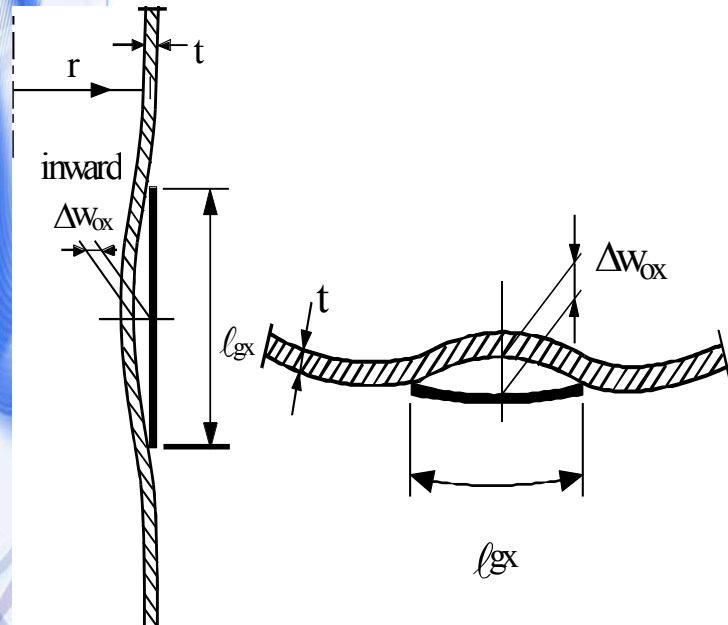
$$\chi = 1 - \beta \left(\frac{\lambda - \lambda_0}{\lambda_p - \lambda_0} \right)^\eta \Leftrightarrow \lambda_0 < \lambda \leq \lambda_p$$

$$\chi = \frac{\alpha}{\lambda^2} \Leftrightarrow \lambda_p \leq \lambda$$

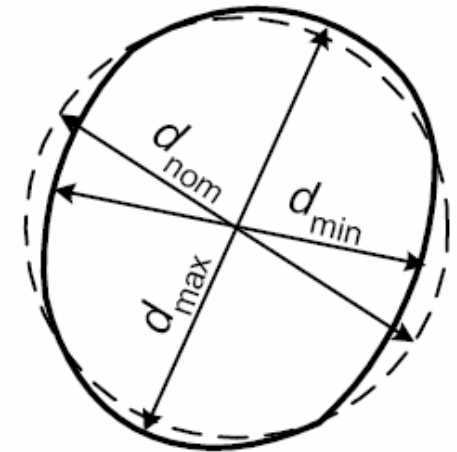
$$\lambda = \sqrt{\frac{f_{yk}}{\sigma_{xRc}}}$$

$$\lambda_p = \sqrt{\frac{\alpha}{1 - \beta}}$$

Shell buckling – fabrication tolerance classes in EC3



a) flattening



b) unsymmetrical shape

$$U_r = \frac{d_{\max} - d_{\min}}{d_{\text{nom}}} \quad \frac{w_0}{l_{gx}} \leq U_{0 \max} \Leftrightarrow \frac{w_0}{t} \leq \frac{U_{0 \max} l_{gx}}{t}$$

$U_{0 \max}$ Dimple tolerance parameter

$$l_{gx} = 4 \sqrt{R t} \quad \text{Gauge length}$$

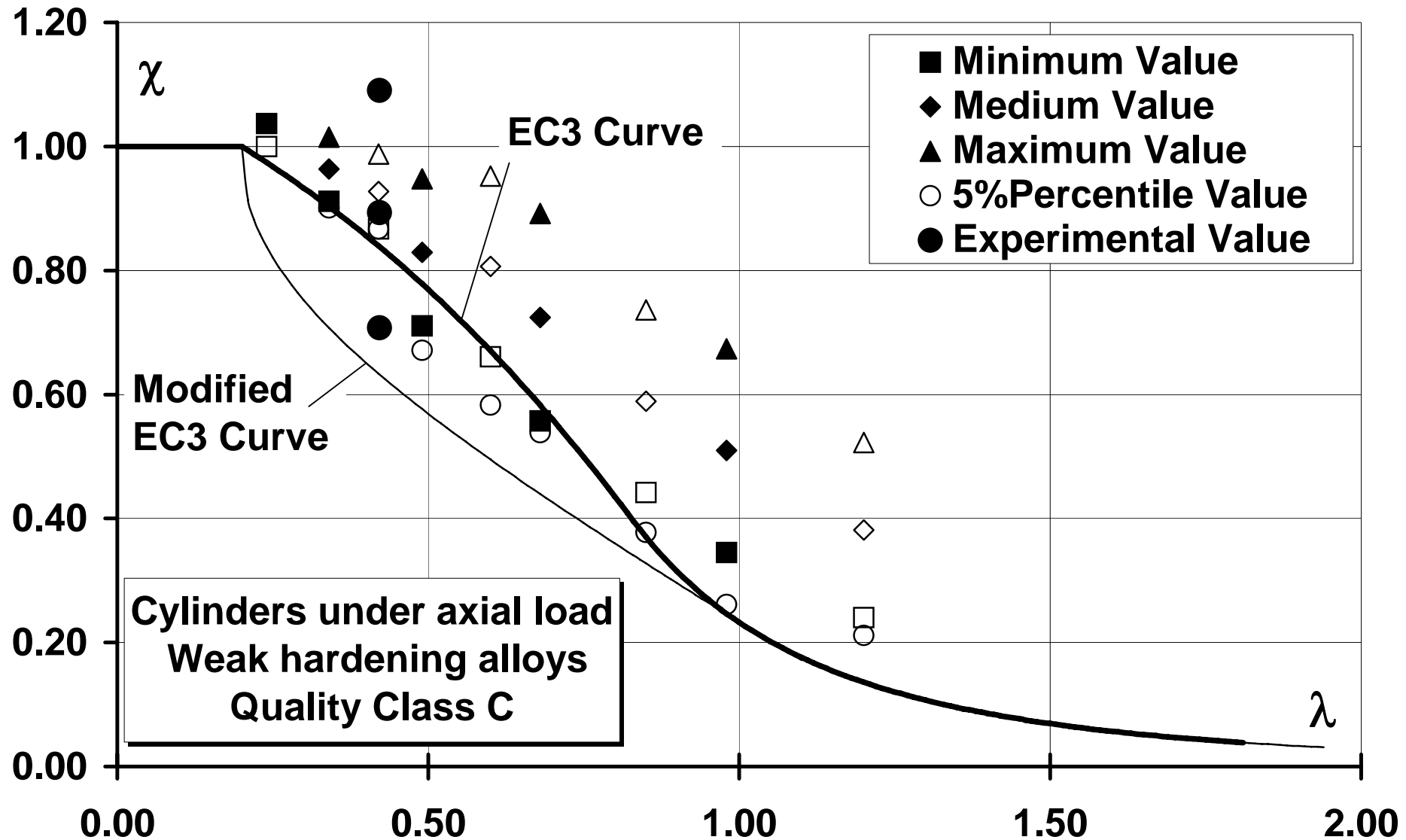
Fabrication tolerance quality class	Description
Class A	Excellent
Class B	High
Class C	Normal

Expressions of buckling factors according to EC3

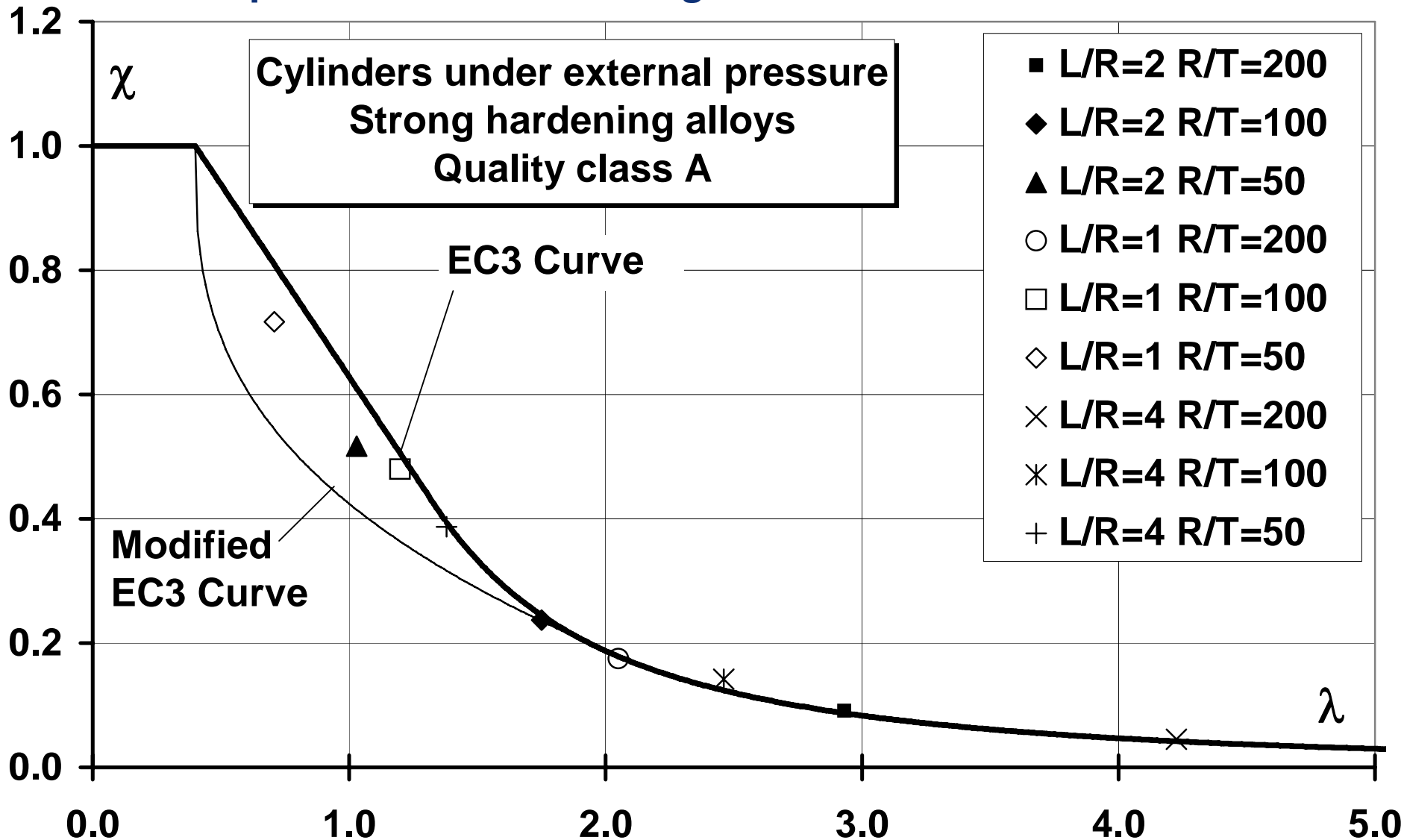
	Axial (meridional) load	External pressure and torsion (shear)
λ_0	0.20	0.40
β	0.60	0.60
η	1.00	1.00

Fabrication tolerance quality class	Description	Axial (meridional) load		External pressure (α_θ) and torsion (shear) (α_τ)
		Q	α_x	α_θ or α_τ
Class A	Excellent	40	$\alpha_x = \frac{0.62}{1 + 1.91 \left(1 / Q \sqrt{r/t} \right)^{1.44}}$	0,75
Class B	High	25		0,65
Class C	Normal	16		0,50

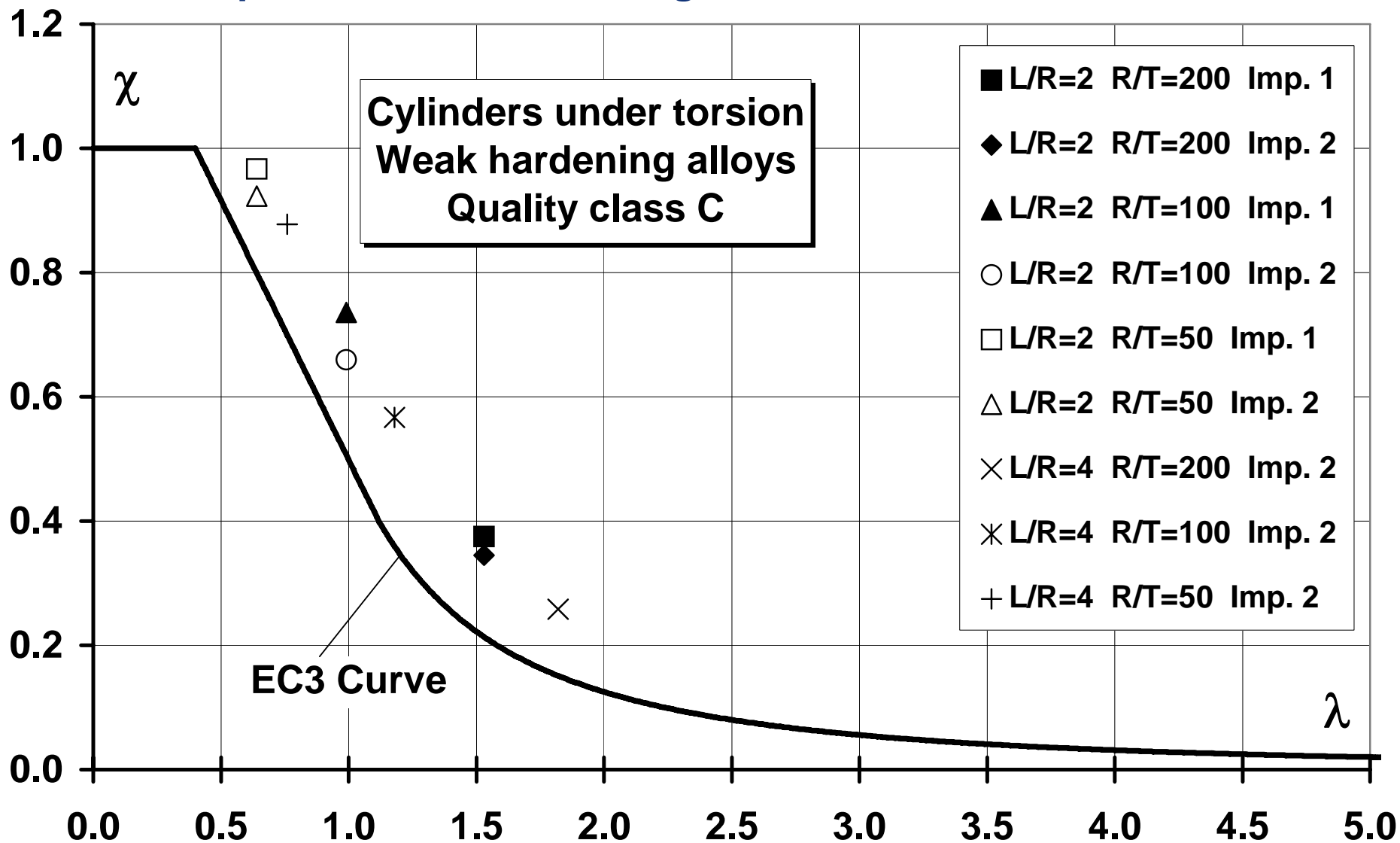
Comparison of EC3 buckling curves with simulation data



Comparison of EC3 buckling curves with simulation data



Comparison of EC3 buckling curves with simulation data



Shell buckling - proposal for pr1999-1-5

$$\chi_x = \alpha \chi_{perf}$$

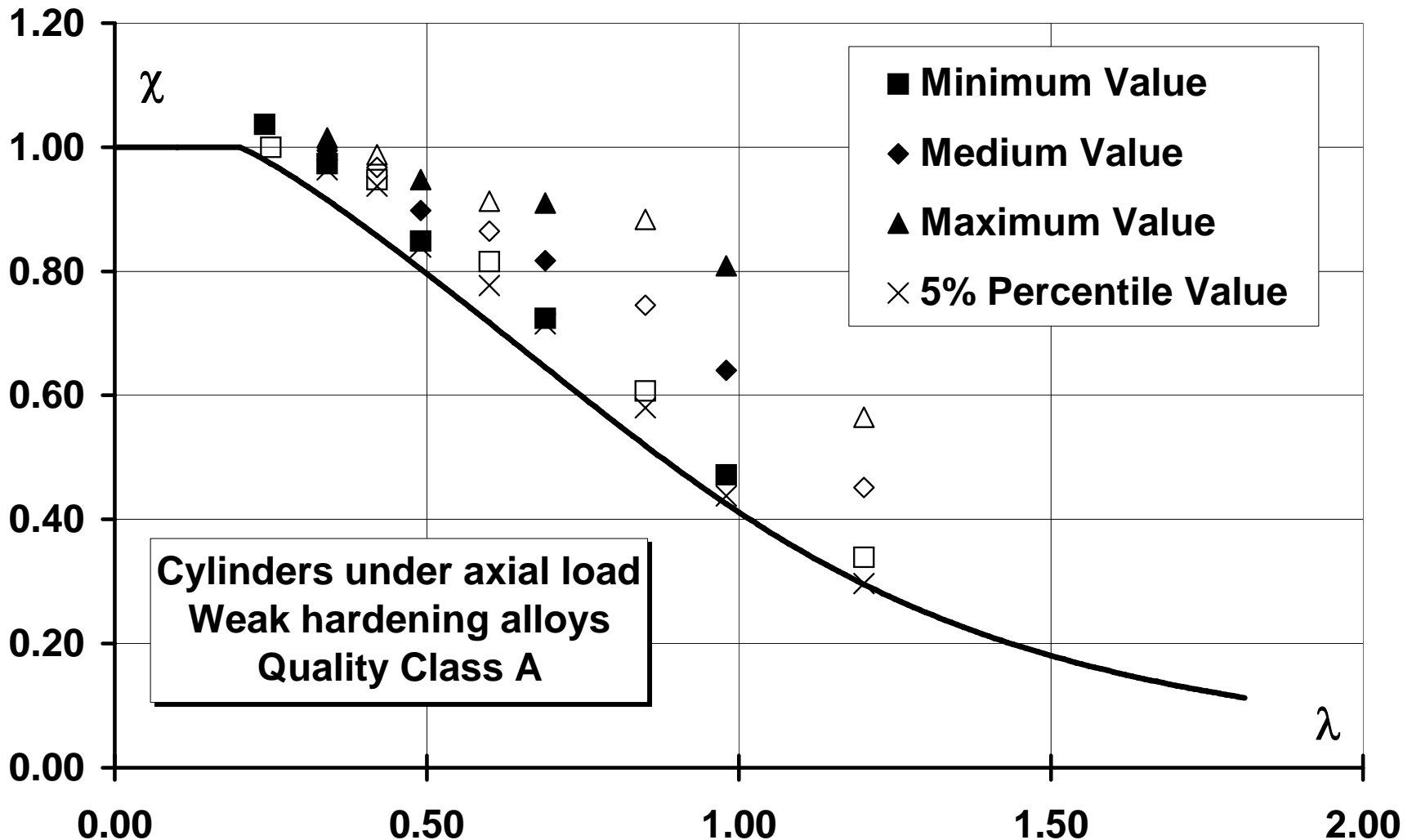
$$\chi_{perf} = \frac{1}{\phi + \sqrt{\phi^2 - \lambda^2}} \quad \lambda = \sqrt{\frac{f_{yk}}{\sigma_{xRc}}}$$

$$\phi = 0.5 \left[1 + \alpha_0 (\lambda - \lambda_0) + \lambda^2 \right]$$

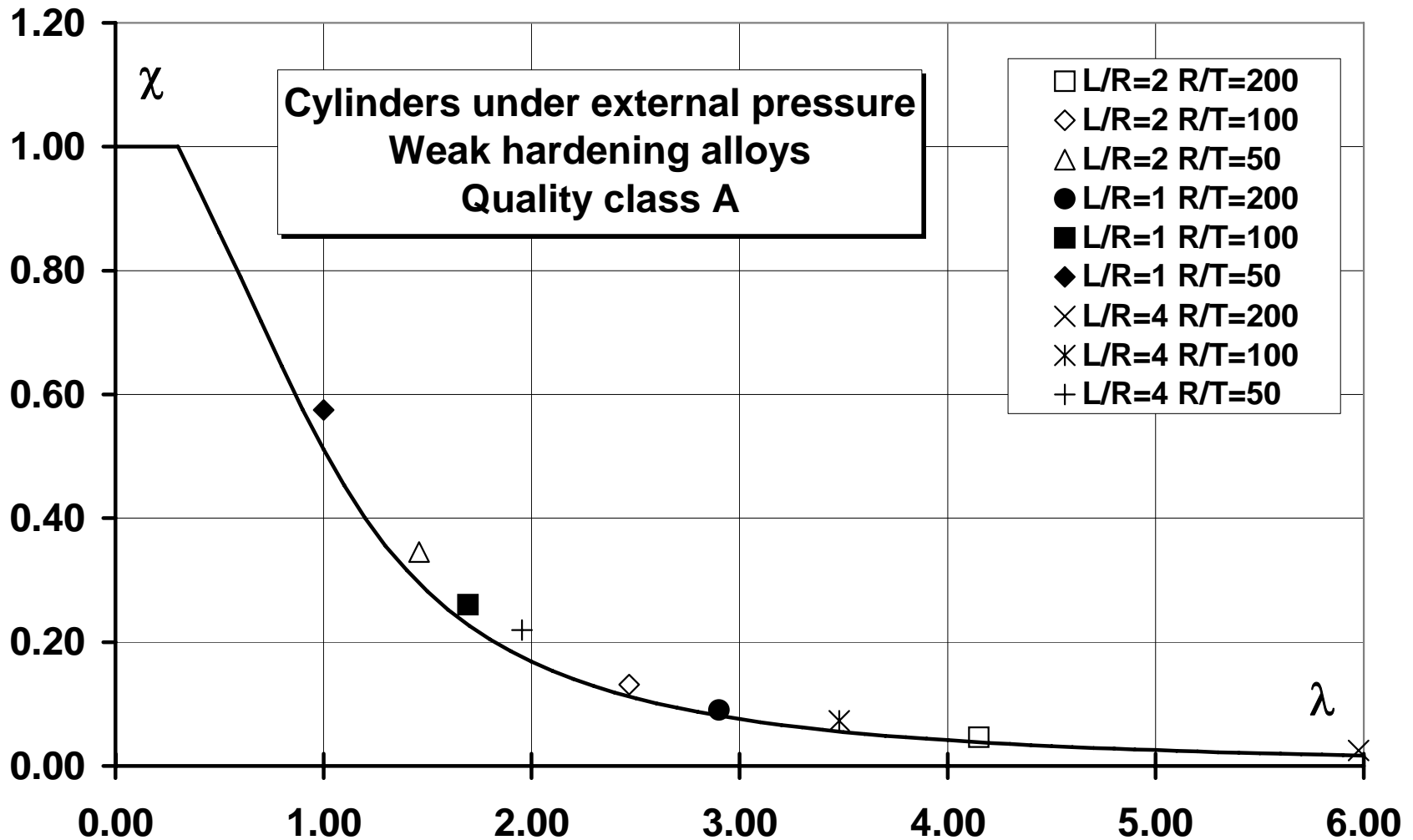
Alloy	Axial (meridional) load		External pressure		Shear (torsion)	
	λ_0	α_0	λ_0	α_0	λ_0	α_0
Weak hardening alloys	0.2	0.35	0.3	0.55	0.5	0.3
Strong hardening alloys	0.1	0.2	0.2	0.7	0.4	0.4

Fabrication tolerance quality class	Description	Axial (meridional) load		External pressure (α_θ) and torsion (α_τ)	
		Q	α_x	α_{ref}	α_θ or α_τ
Class A	Excellent	40	$\alpha_x = \frac{0.62}{1 + 1.91 \left(1 / Q \sqrt{r/t} \right)^{1.44}}$	0,75	$\alpha_{\theta,\tau} = \frac{1}{1 + 0,2 \left(1 - \alpha_{ref} \right) \left(\bar{\lambda} - \bar{\lambda}_0 \right) / \alpha_{ref}^2}$
Class B	High	25		0,65	
Class C	Normal	16		0,50	

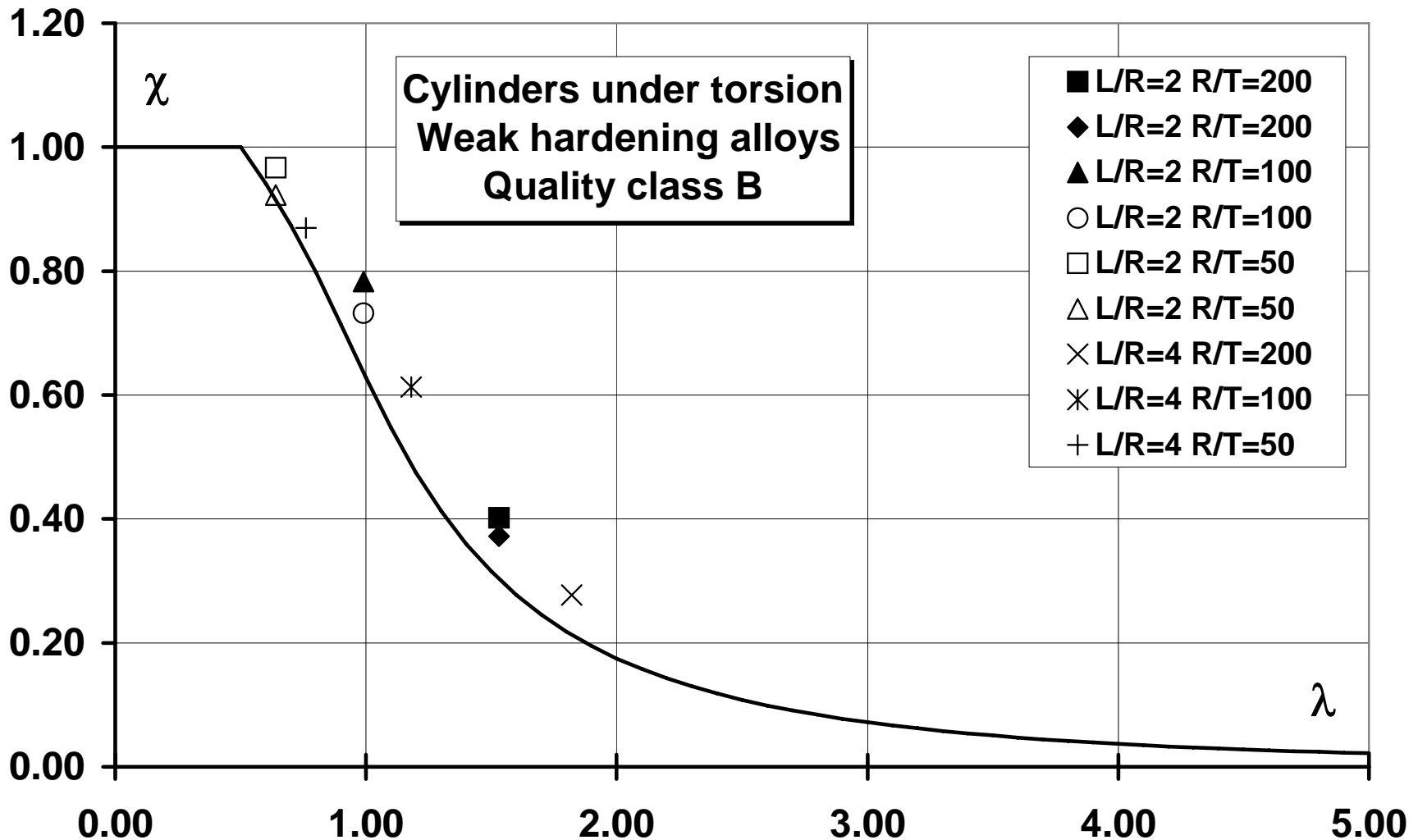
Buckling curves - proposal for EC9



Buckling curves - proposal for EC9



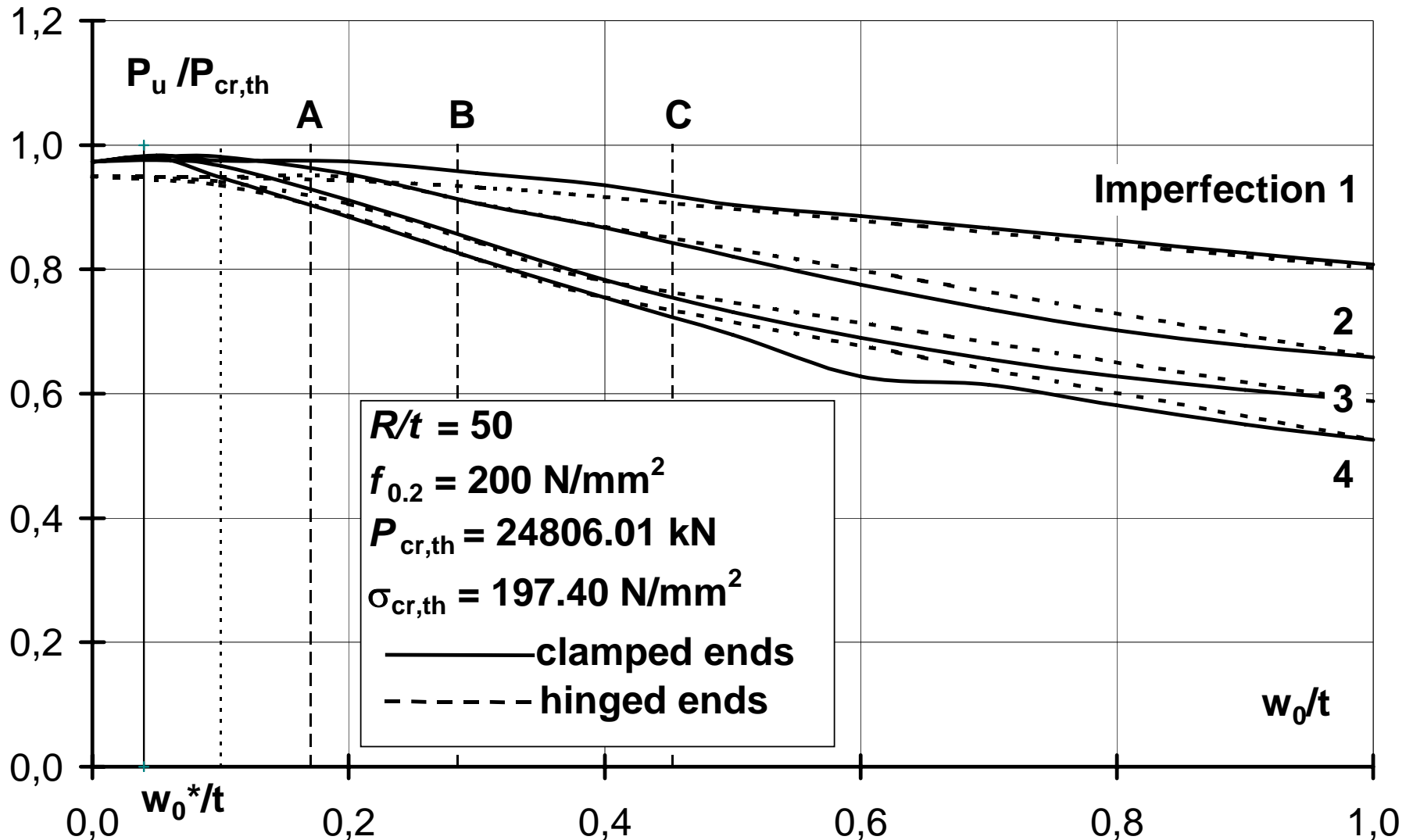
Buckling curves - proposal for EC9



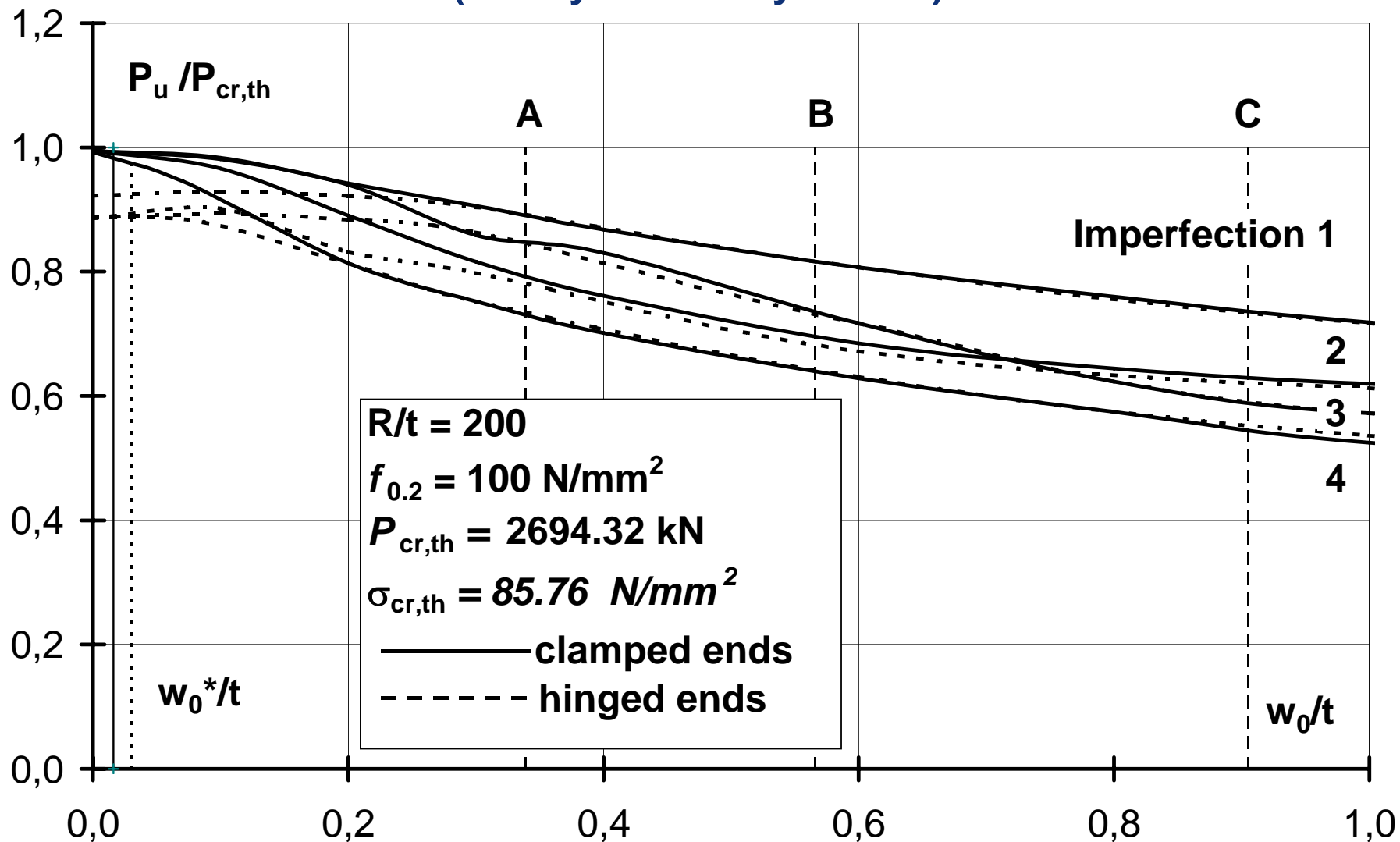
Background activity - Main investigated aspects

- **shell plastic buckling**
 - imperfection sensitivity analysis of aluminium cylinders;
 - set-up of buckling curves for aluminium shells;
 - **definition of imperfection classes for plastic buckling;**
 - interaction between load cases;
 - introduction of additional shell configurations;
- **stiffened shells**
 - imperfection sensitivity analysis of stiffened cylinders;
 - validation of EN1993-1-6 procedures and harmonization with EN1999 rules;
- **effect of welding effect (HAZ zones)**
 - imperfection sensitivity analysis of welded cylinders;
 - definition of simplified design procedures;

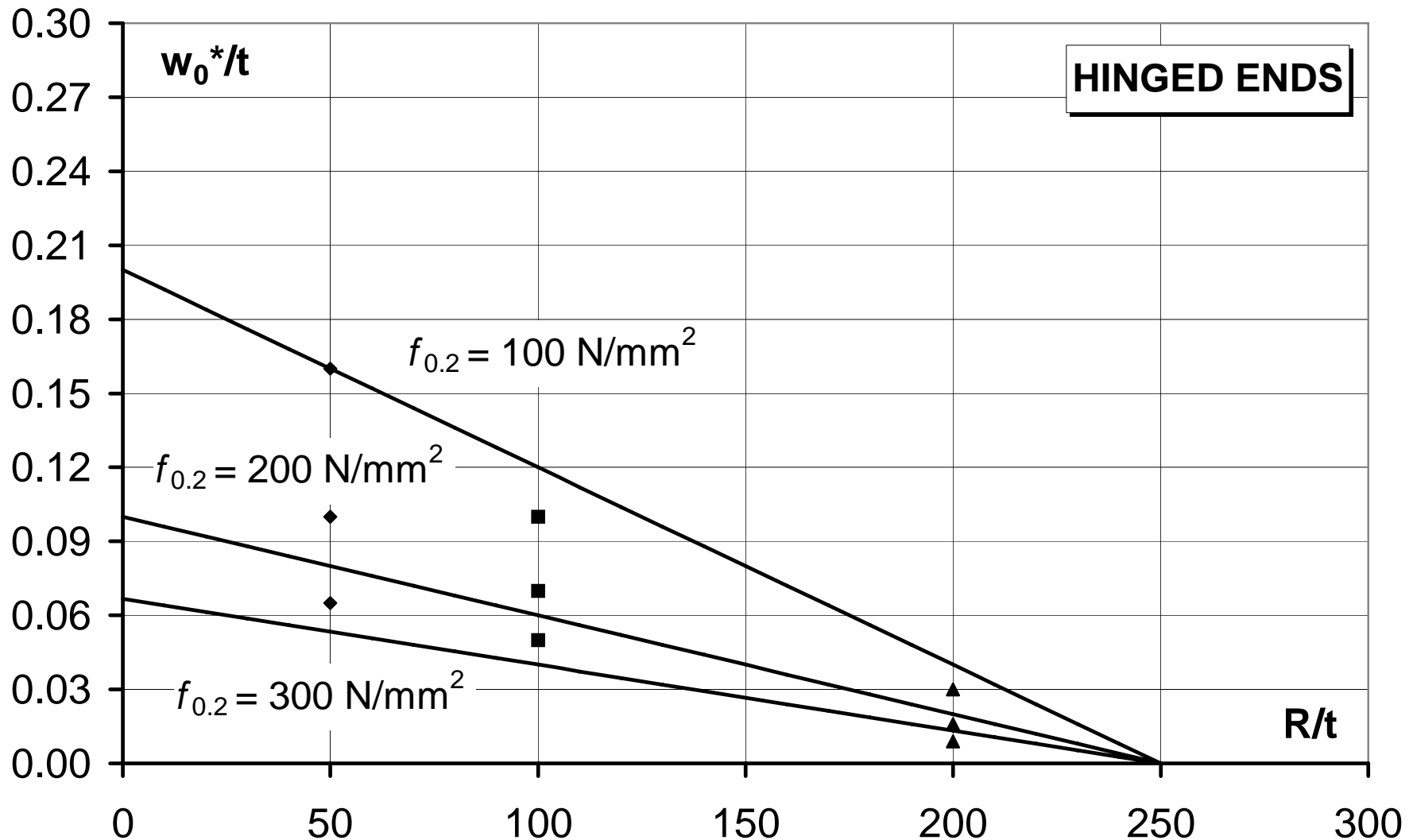
Exploitation of plastic buckling features (axially loaded cylinders)



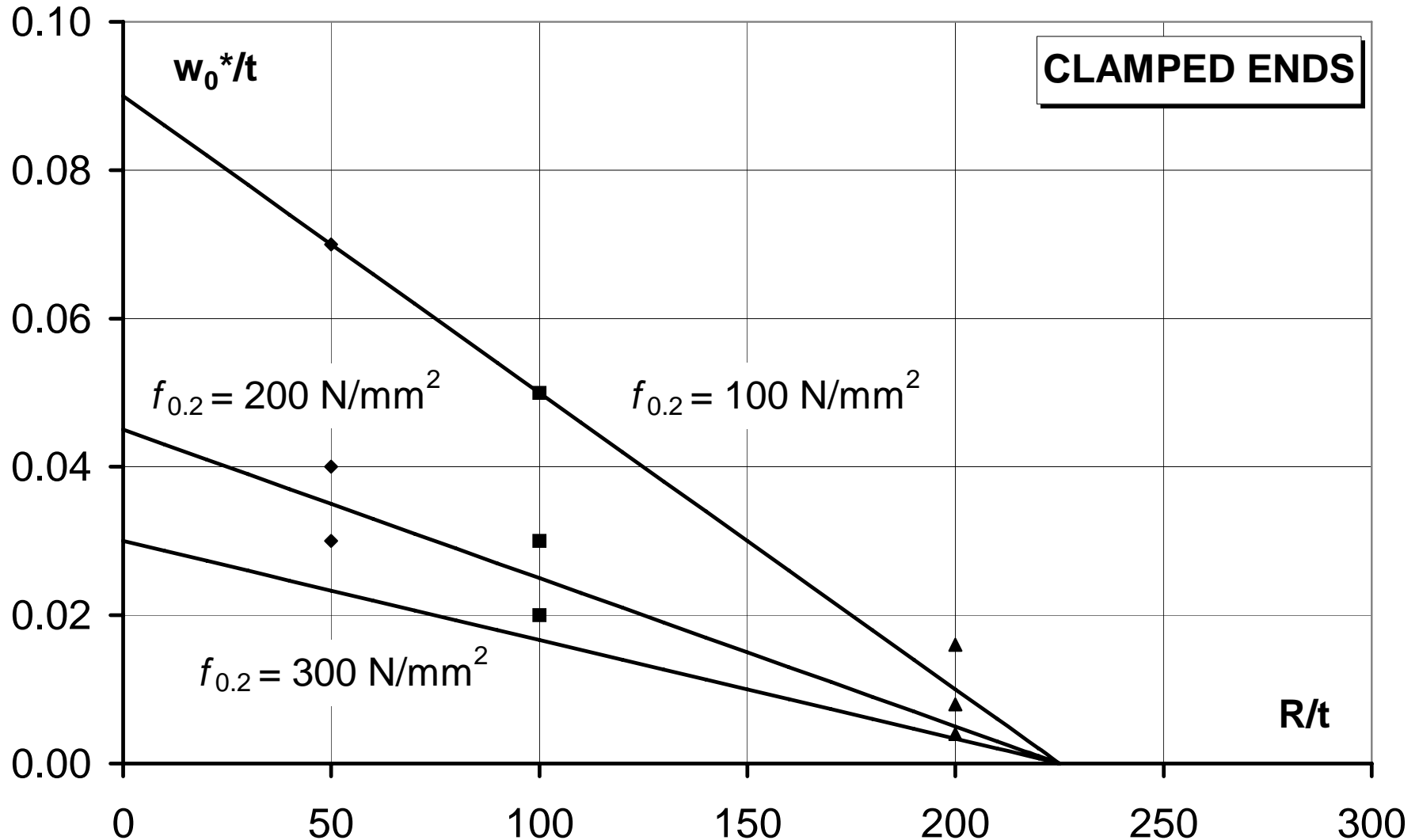
Exploitation of plastic buckling features (axially loaded cylinders)



Exploitation of plastic buckling features (axially loaded cylinders) – Imperfection limit w_0^*/t



Exploitation of plastic buckling features (axially loaded cylinders) – Imperfection limit w_0^*/t



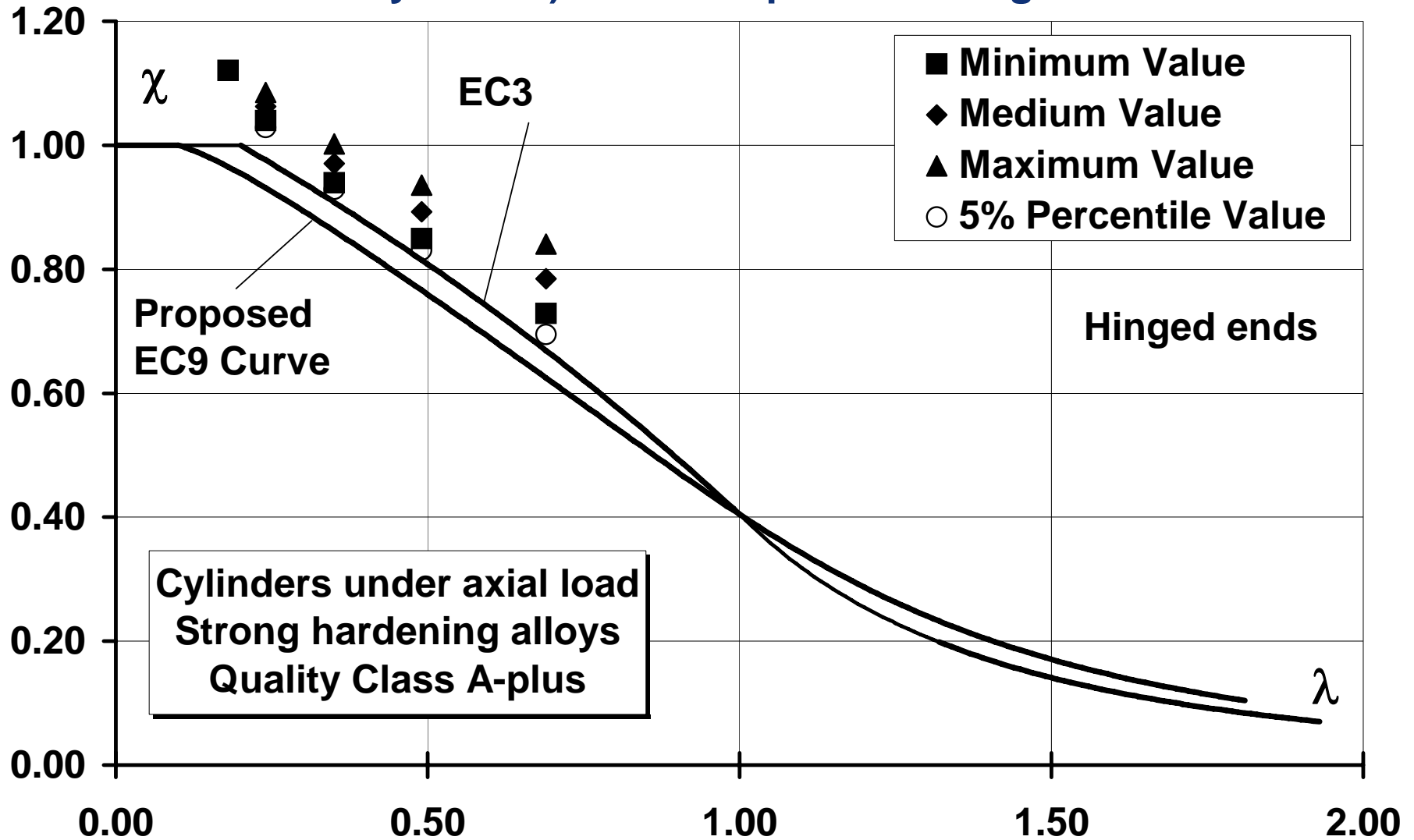
Exploitation of plastic buckling features (axially loaded cylinders)

Definition of quality Class A-plus in prEN-1999-1-5

Fabrication tolerance quality class	Description	Value of $U_{0,max}$ ($f_{0,2}$ in N/mm ²)	
		Clamped ends	Hinged ends
Class A-plus	Excellent	$\frac{1}{f_{0,2}} \left(2.25 \sqrt{\frac{t}{R}} + 0.01 \sqrt{\frac{R}{t}} \right)$	$\frac{1}{f_{0,2}} \left(5 \sqrt{\frac{t}{R}} + 0.02 \sqrt{\frac{R}{t}} \right)$
Class A	Very high	0,006	
Class B	High	0,01	
Class C	Normal	0,016	

Fabrication tolerance quality class	Description	Q		α_x
		Clamped ends	Hinged ends	
Class A-plus	Excellent	60	50	$\alpha_x = \frac{1}{1 + 2.60 \left(\frac{1}{Q} \sqrt{\frac{0.6E}{f_{0,2}}} (\lambda_x - \lambda_{x,0}) \right)^{1.44}}$
Class A	Very high	40		
Class B	High	25		
Class C	Normal	16		

Exploitation of plastic buckling features (axially loaded cylinders) - Class A-plus buckling curves





Shell buckling – summary of EC9 formulation

Load cases

- axial compression
- external pressure
- torsion

Unstiffened shells

$$\sigma_{x,Rd} = \alpha_x \rho_{x,w} \chi_{x,perf} \frac{f_o}{\gamma_{M1}}$$

$$\sigma_{\theta,Rd} = \alpha_{\theta} \rho_{\theta,w} \chi_{\theta,perf} \frac{f_o}{\gamma_{M1}}$$

$$\tau_{Rd} = \alpha_{\tau} \rho_{\tau,w} \chi_{\tau,perf} \frac{f_o}{\sqrt{3} \gamma_{M1}}$$

Stiffened shells

$$n_{x,Rd} = \alpha_{n,x} \chi_{x,perf} \frac{n_{x,Rk}}{\gamma_{M1}}$$

$$p_{n,Rd} = \alpha_{p,\theta} \chi_{\theta,perf} \frac{p_{n,Rk}}{\gamma_{M1}}$$

Shell buckling – summary of EC9 formulation

$$\chi_{i,\text{perf}} = \frac{1}{\phi_i + \sqrt{\phi_i^2 - \bar{\lambda}_i^2}} \quad \text{but} \quad \chi_{i,\text{perf}} \leq 1,00$$

with:

$$\phi_i = 0,5 \left(1 + \mu_i (\bar{\lambda}_i - \bar{\lambda}_{i,0}) + \bar{\lambda}_i^2 \right)$$

$$\bar{\lambda}_x = \sqrt{\frac{f_o}{\sigma_{x,\text{cr}}}}$$

$$\bar{\lambda}_\theta = \sqrt{\frac{f_o}{\sigma_{\theta,\text{cr}}}}$$

$$\bar{\lambda}_\tau = \sqrt{\frac{f_o}{\sqrt{3}\tau_{\text{cr}}}}$$

Material buckling class	Axial (meridional) load		External pressure		Shear (torsion)	
	$\lambda_{x,0}$	μ_x	$\lambda_{\theta,0}$	μ_θ	$\lambda_{\tau,0}$	μ_τ
A (Weak hardening alloys)	0.2	0.35	0.3	0.55	0.5	0.3
B (Strong hardening alloys)	0.1	0.2	0.2	0.7	0.4	0.4

Shell buckling – fabrication tolerance classes according to EC9

Fabrication tolerance quality class	Diameter range		
	$d \leq 0,5 \text{ m}$	$0,5 \text{ m} < d < 1,25 \text{ m}$	$1,25 \text{ m} \geq d$
Class 4	0,010	$0,005 + 0,0067(1,25 - d)$	0,005
Class 3	0,014	$0,007 + 0,0090(1,25 - d)$	0,007
Class 2	0,020	$0,010 + 0,0133(1,25 - d)$	0,010
Class 1	0,030	$0,015 + 0,0200(1,25 - d)$	0,015

Fabrication tolerance quality class	Axial (meridional) load		External pressure (α_θ) and torsion (α_τ)	
	Q	α_x	α_{ref}	α_θ OR α_τ
Class 1	16	$\alpha_x = \frac{0.62}{1 + 1.91(1/Q\sqrt{r/t})^{1.44}}$	0,50	$\alpha_{\theta,\tau} = \frac{1}{1 + 0,2(1 - \alpha_{ref})(\bar{\lambda} - \bar{\lambda}_0) / \alpha_{ref}^2}$
Class 2	25		0,65	
Class 3	40		0,75	
Class 4	50-60		-	

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 - definition of simplified design procedures;



Interaction between load cases

- **interaction between load cases**
 - axial compression – external pressure
 - axial compression – torsion
 - external pressure – torsion
- validation of EN1993-1-6 procedures
- proposal for an alternative formulation

Interaction between load cases ENV1993-1-6 formulation

$$\left(\frac{\sigma_{xE d}}{\sigma_{xR d}}\right)^{k_x} + \left(\frac{\sigma_{\theta E d}}{\sigma_{\theta R d}}\right)^{k_{\theta}} + \left(\frac{\tau_{E d}}{\tau_{R d}}\right)^{k_{\tau}} \leq 1 \quad \text{if } \sigma_{xE d} \text{ and } \sigma_{\theta E d} \text{ are compressive,}$$

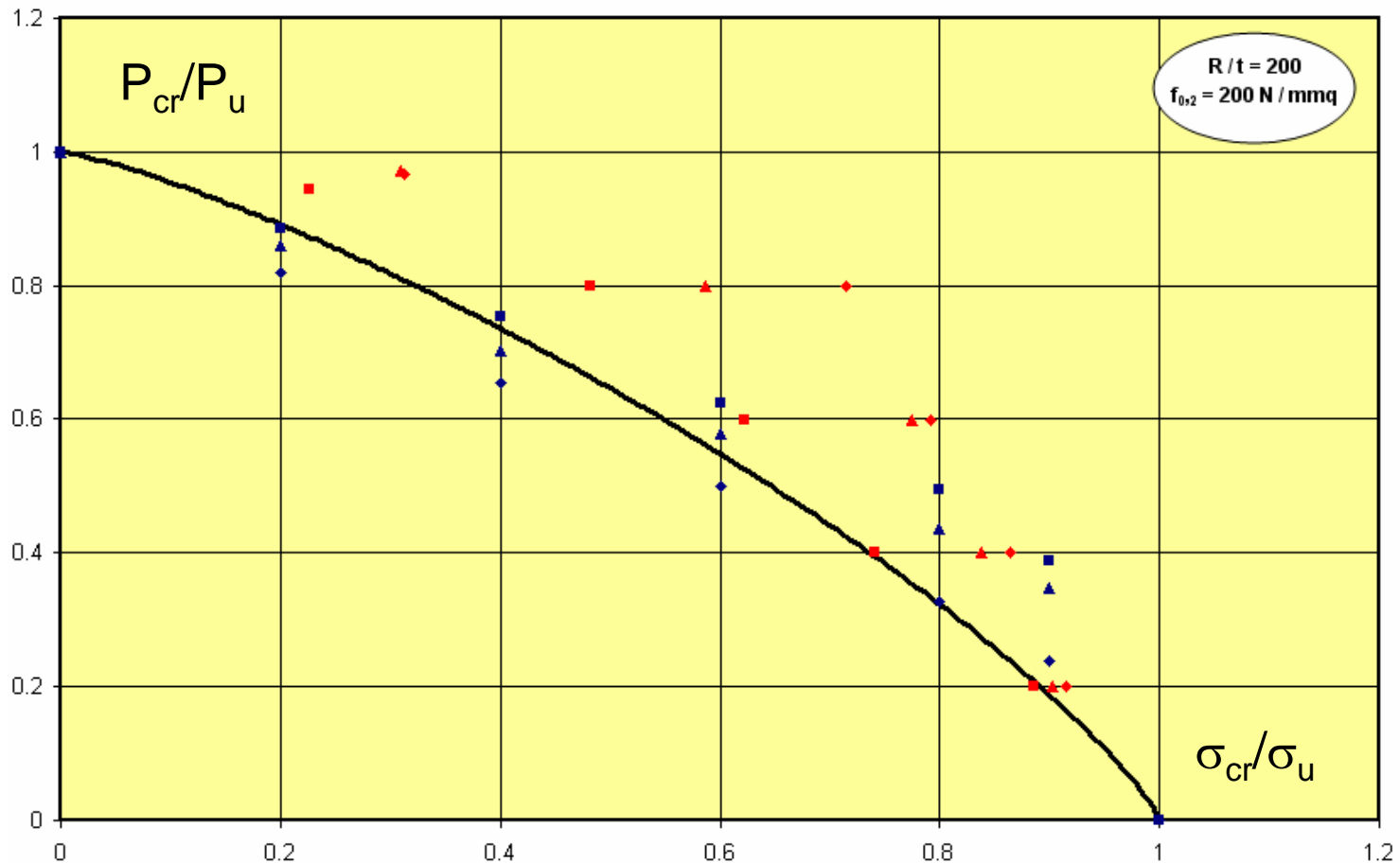
$$\left(\frac{\sigma_{xE d}}{\sigma_{xR d}}\right)^{k_x} + \left(\frac{\tau_{E d}}{\tau_{R d}}\right)^{k_{\tau}} \leq 1 \quad \text{if } \sigma_{\theta E d} \text{ is tensile or zero,}$$

$$\left(\frac{\sigma_{\theta E d}}{\sigma_{\theta R d}}\right)^{k_{\theta}} + \left(\frac{\tau_{E d}}{\tau_{R d}}\right)^{k_{\tau}} \leq 1 \quad \text{if } \sigma_{xE d} \text{ is tensile or zero.}$$

$$k_x = k_{\theta} = \boxed{1,25} \quad \text{and} \quad k_{\tau} = \boxed{2,0}$$

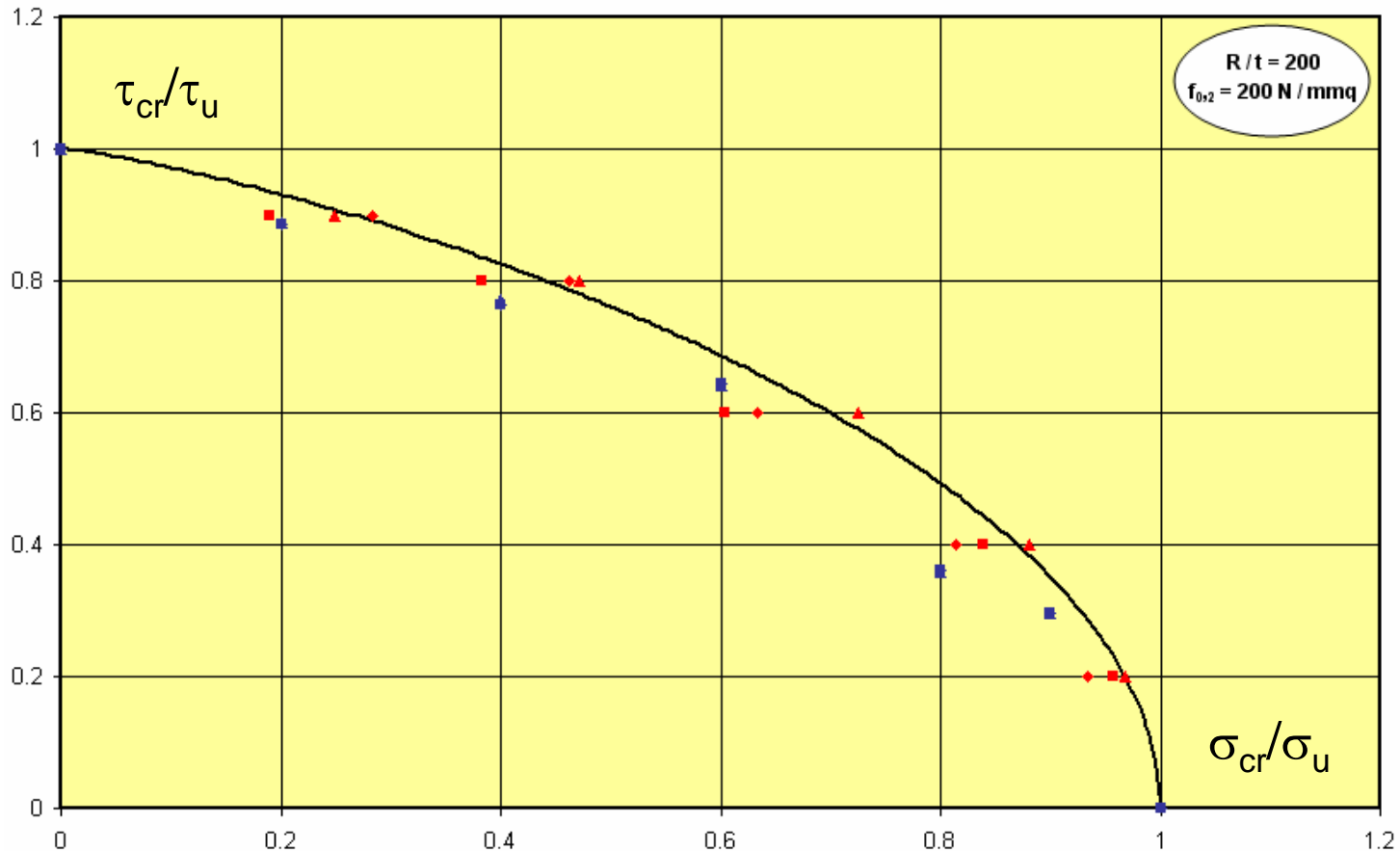
Interaction between load case ENV 1993-1-6 – Interaction domains

Axial compression and External pressure



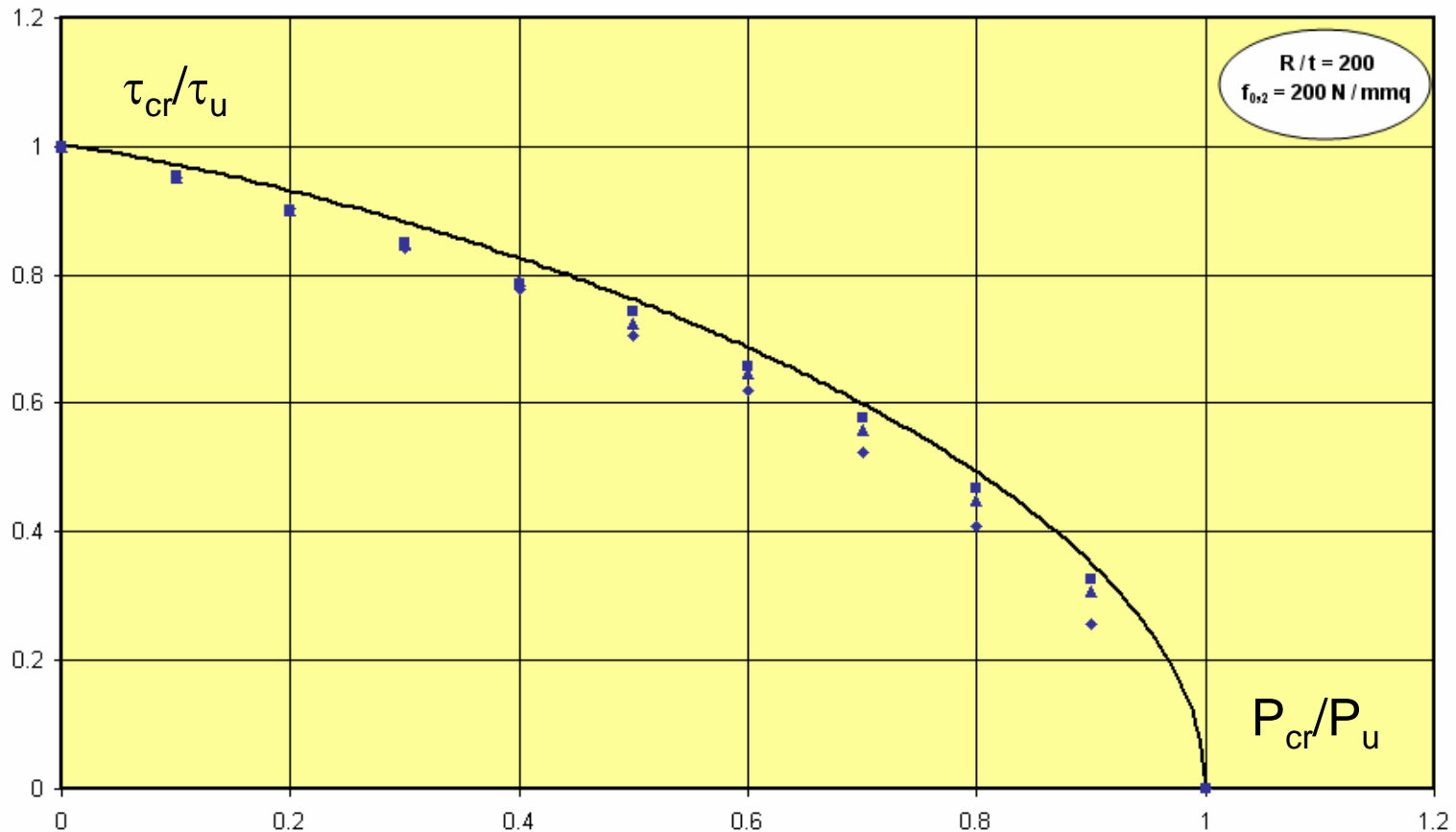
Interaction between load case ENV 1993-1-6 – Interaction domains

Axial compression and Torsion



Interaction between load case ENV 1993-1-6 – Interaction domains

External pressure and Torsion





Interaction between load cases

prEN1993-1-6 formulation and proposal for prEN1999-1-5

$$\left(\frac{\sigma_{xE d}}{\sigma_{xR d}} \right)^{k_x} + \left(\frac{\sigma_{\theta E d}}{\sigma_{\theta R d}} \right)^{k_{\theta}} - k_i \left(\frac{\sigma_{xE d}}{\sigma_{xR d}} \right) \left(\frac{\sigma_{\theta E d}}{\sigma_{\theta R d}} \right) + \left(\frac{\tau_{x\theta E d}}{\tau_{x\theta R d}} \right)^{k_{\tau}} \leq 1$$

$$k_x = 1,0 + \chi_x^2$$

$$k_x = 1,0 + \chi_x^2$$

$$k_{\theta} = 1,0 + \chi_{\theta}^2$$

$$k_{\theta} = 1,0 + \chi_{\theta}^2$$

$$k_{\tau} = 1,5 + 0,5 \chi_{\tau}^2$$

$$k_{\tau} = 1,0 + \chi_{\tau}^2$$

$$k_i = (\chi_x \chi_{\theta})^2$$

$$k_i = (\chi_x \chi_{\theta})^2$$

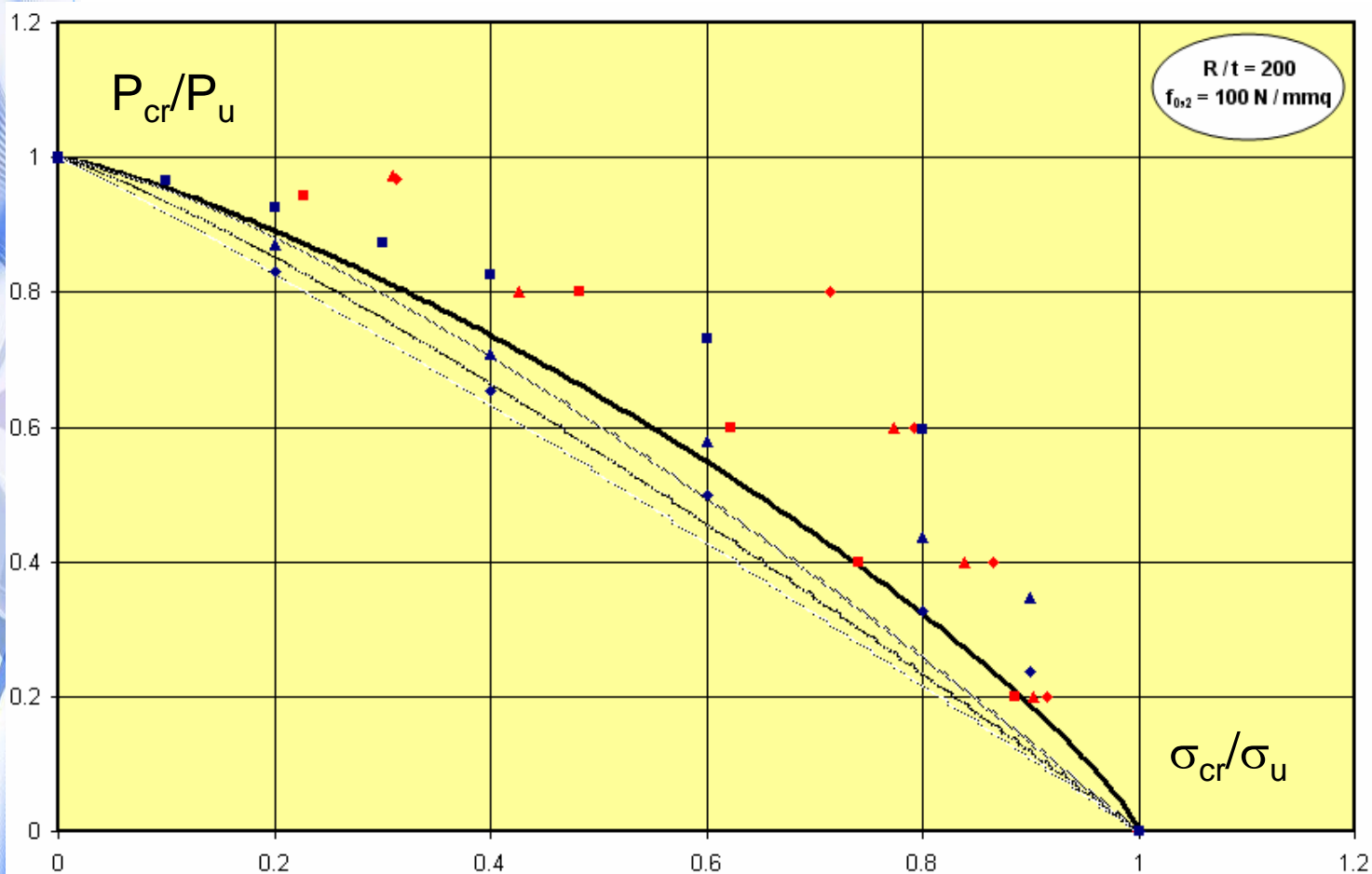
prEN1993-1-6

prEN1999-1-5

Interaction between load cases

EN 1999-1-5 – Interaction domains

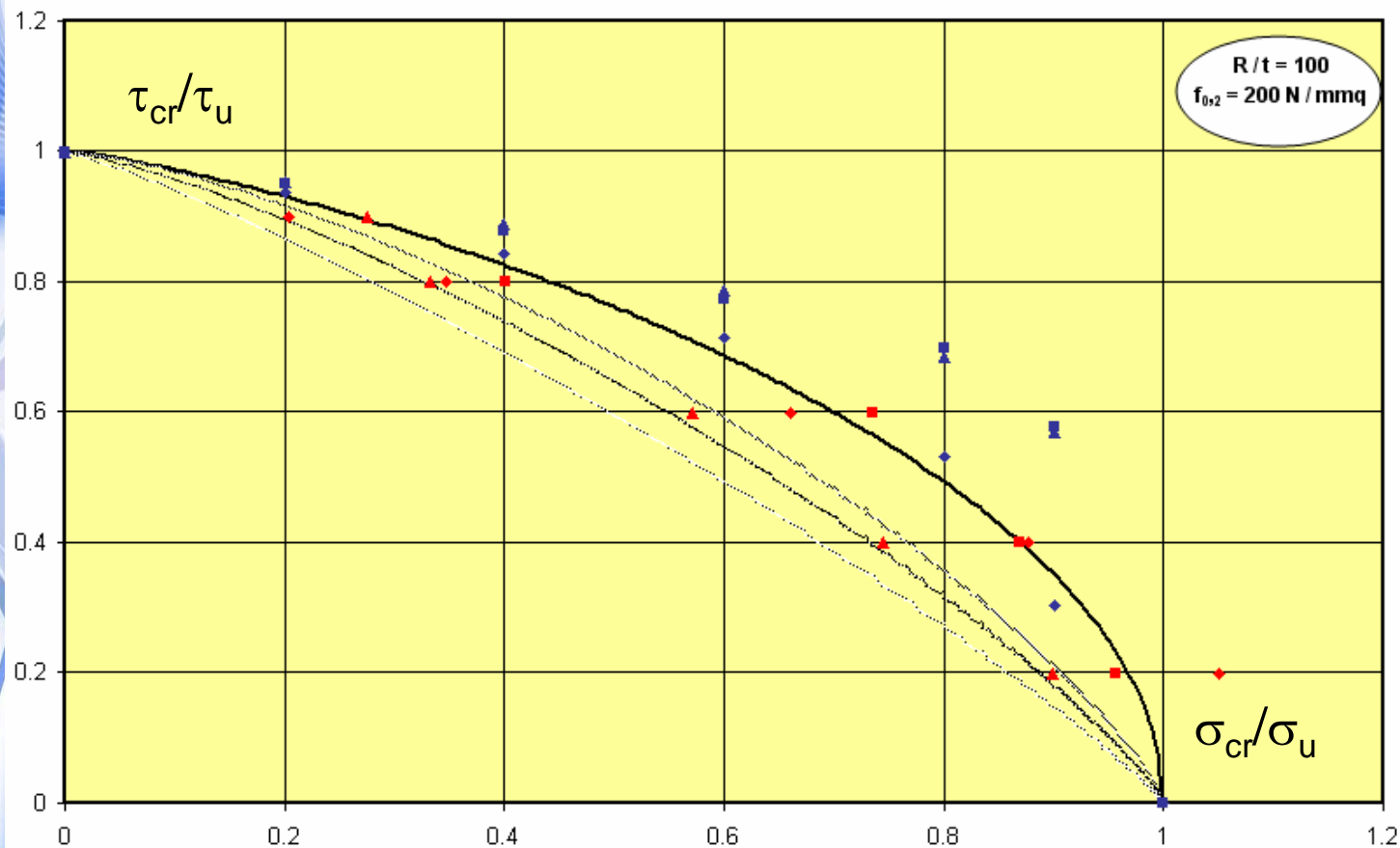
Axial compression and External pressure



- EC3
- Class A
- ▲ Class B
- ◆ Class C
- Class A
- ▲ Class B
- ◆ Class C
- Proposal Class A
- Proposal Class B
- Proposal Class C

Interaction between load cases EN 1999-1-5 – Interaction domains

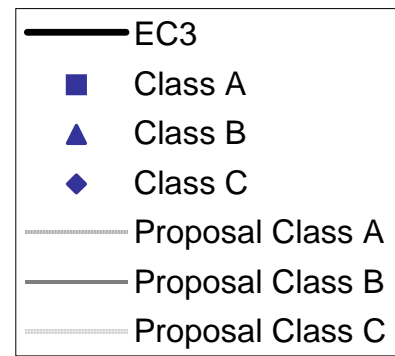
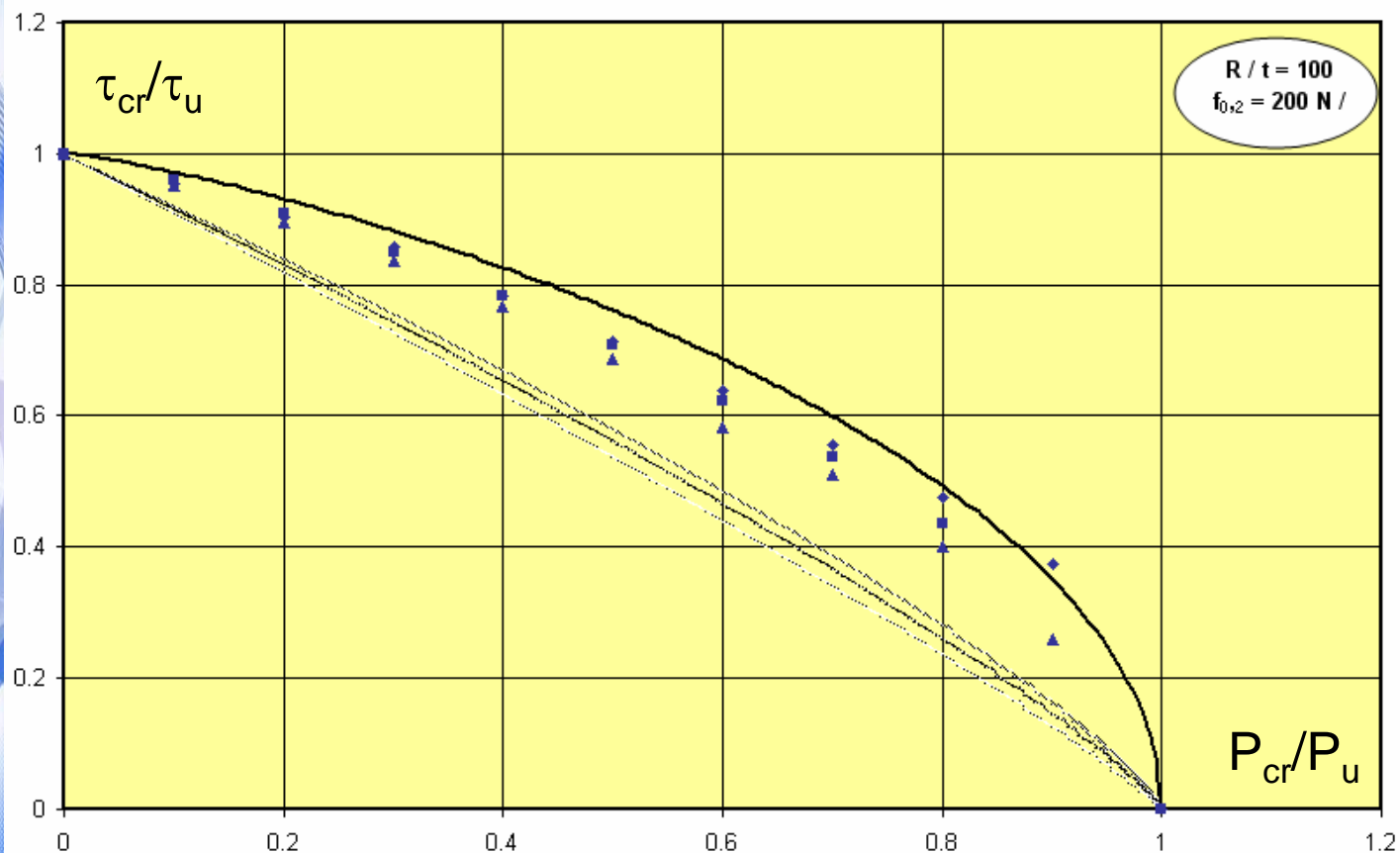
Axial compression and Torsion



- EC3
- Class A
- ▲ Class B
- ◆ Class C
- Class A
- ▲ Class B
- ◆ Class C
- Proposal Class A
- Proposal Class B
- Proposal Class C

Interaction between load cases EN 1999-1-5 – Interaction domains

External pressure and Torsion





Interaction buckling check according to EC9

$$\left(\frac{\sigma_{xE d}}{\sigma_{xR d}} \right)^{k_x} + \left(\frac{\sigma_{\theta E d}}{\sigma_{\theta R d}} \right)^{k_{\theta}} - k_i \left(\frac{\sigma_{xE d}}{\sigma_{xR d}} \right) \left(\frac{\sigma_{\theta E d}}{\sigma_{\theta R d}} \right) + \left(\frac{\tau_{x\theta E d}}{\tau_{x\theta R d}} \right)^{k_{\tau}} \leq 1$$

$$k_x = 1,0 + \chi_x^2$$

$$k_{\theta} = 1,0 + \chi_{\theta}^2$$

$$k_{\tau} = 1,5 + 0,5 \chi_{\tau}^2$$

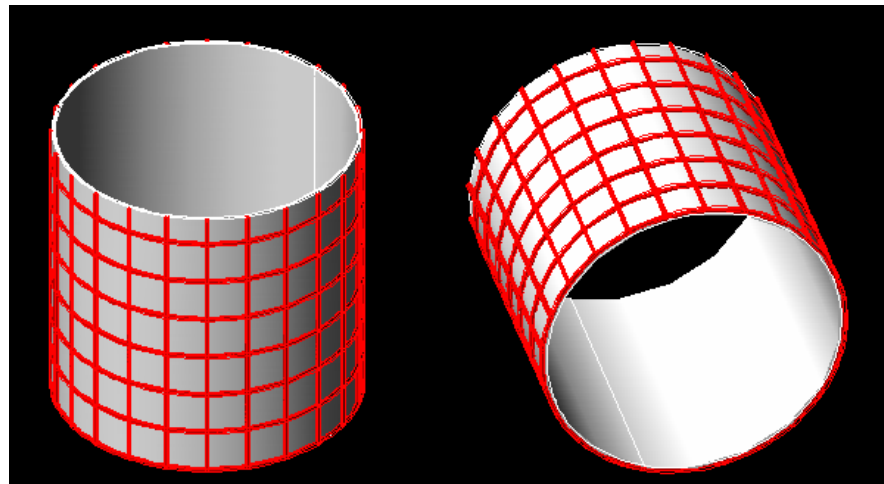
$$k_i = (\chi_x \chi_{\theta})^2$$

Background activity - Main investigated aspects

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 - imperfection sensitivity analysis of aluminium cylinders;
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 - interaction between load cases;
 - introduction of additional shell configurations;
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 - imperfection sensitivity analysis of stiffened cylinders;
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 - imperfection sensitivity analysis of welded cylinders;
 - definition of simplified design procedures;

Parametric analysis – Stiffened shells

Stiffener section	Shell geometry	Stiffener size	[mm]	[mm]	[mm]	[mm]
Circular	R/t=50	radius	5	10	25	50
Square		side	5	10	25	50
Rectangular	R/t=100	sides	5x20	10x20	25x20	50x20
	R/t=200		5x10	10x10	25x10	50x10
			5x5	10x5	25x5	50x5



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Stiffened shells – Proposal for EN19991-5

Axial load

$$n_{xRc} = \alpha_x \frac{1.2}{1 + \frac{5EA}{C_\phi d_s}} \frac{1}{\omega^2} \left(1 + \frac{EA_s}{C_\theta d_s} \right) \left(A_1 + \frac{A_2}{A_3} \right)$$

with $\alpha_x = 0.80$

External pressure

$$p_{nRc} = \alpha_\theta \frac{1}{rj^2} \left(A_1 + \frac{A_2}{A_3} \right)$$

with $\alpha_\theta = 0.50$

$$A_1 = j^4 \left[\omega^4 C_{44} + 2\omega^2 (C_{45} + C_{66}) + C_{55} \right] + C_{22} + 2j^2 C_{25}$$

$$A_2 = 2\omega^2 (C_{12} + C_{33}) (C_{22} + j^2 C_{25}) (C_{12} + j^2 \omega^2 C_{14})$$

$$- (\omega^2 C_{11} + C_{33}) (C_{22} + j^2 C_{25})^2 - \omega^2 (C_{22} + C_{25} + \omega^2 C_{33}) (C_{12} + j^2 \omega^2 C_{14})^2$$

$$A_3 = (\omega^2 C_{11} + C_{33}) (C_{22} + C_{25} + \omega^2 C_{33}) - \omega^2 (C_{12} + C_{33})^2$$

where

$$C_{11} = C_\varphi + EA_s / d_s$$

$$C_{22} = C_\theta + EA_r / d_r$$

$$C_{12} = \nu \sqrt{C_\varphi C_\theta}$$

$$C_{33} = C_{\varphi\theta}$$

$$C_{14} = e_s EA_s / (rd_s)$$

$$C_{25} = -e_r EA_r / (rd_r)$$

$$C_{44} = [D_\varphi + EI_s d_s] / r^2$$

$$C_{55} = [D_\theta + EI_r d_r] / r^2$$

$$C_{45} = \nu \sqrt{D_\varphi D_\theta} / r^2$$

$$C_{66} = [D_{\varphi\theta} + 0,5(GI_{ts}/d_s + GI_{tr}/d_r)] / r^2$$

$$\omega = \frac{\pi r}{jl_i}$$



Stiffened shells – Proposal for EN1999-1-5

Equivalent orthotropic properties of corrugated sheeting (from prEN1999-1-6)

$$C_{\varphi} = E \cdot t_x = E \cdot \frac{2t^3}{3d^2}$$

$$C_{\theta} = E \cdot t_y = E \cdot t \left(1 + \frac{\pi^2 d^2}{4l^2} \right)$$

$$C_{\theta\varphi} = G \cdot t_{xy} = \frac{G \cdot t}{\left(1 + \frac{\pi^2 d^2}{4l^2} \right)}$$

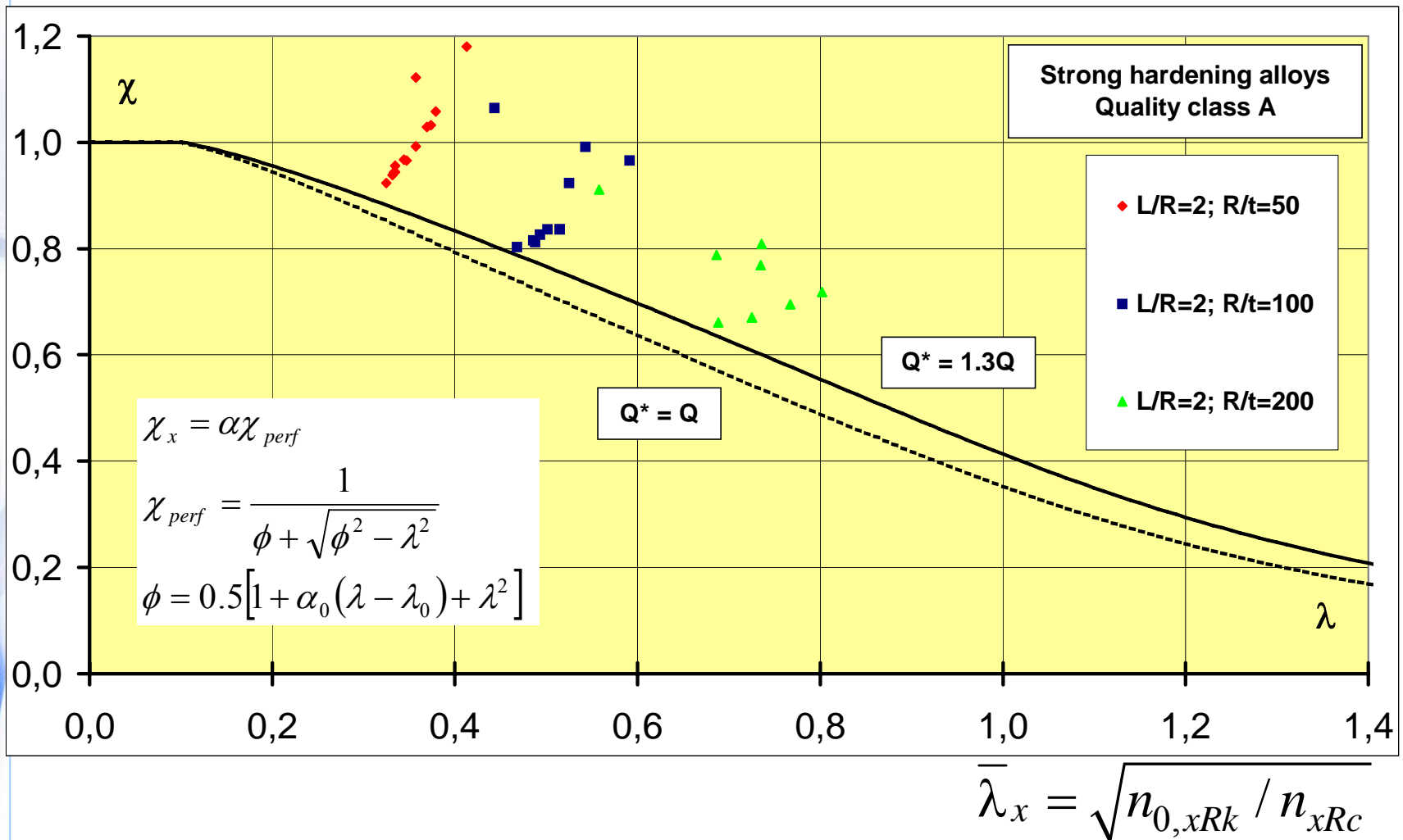
$$D_{\varphi} = E \cdot I_x = \frac{Et^3}{12(1-\nu^2)} \frac{1}{\left(1 + \frac{\pi^2 d^2}{4l^2} \right)}$$

$$D_{\varphi} = E \cdot I_y = 0,13Etd^2$$

$$D_{\theta\varphi} = G \cdot I_{xy} = \frac{G \cdot t^3}{12} \left(1 + \frac{\pi^2 d^2}{4l^2} \right)$$

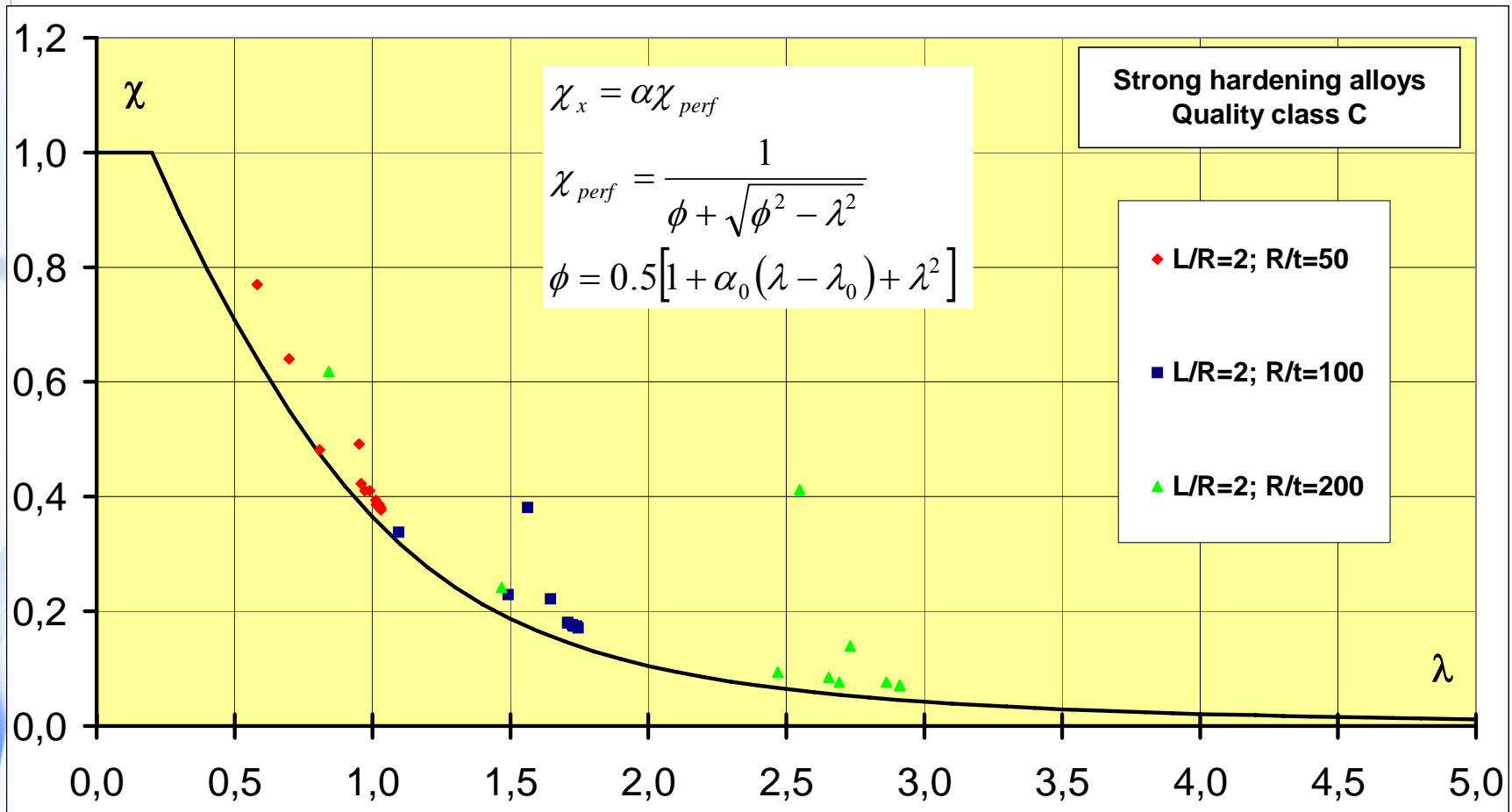
Stiffened shells – Proposal for prEN19991-5

Axial load



Stiffened shells – Proposal for prEN19991-5

External pressure



$$\chi_x = \alpha \chi_{perf}$$

$$\chi_{perf} = \frac{1}{\phi + \sqrt{\phi^2 - \lambda^2}}$$

$$\phi = 0.5 [1 + \alpha_0 (\lambda - \lambda_0) + \lambda^2]$$

Strong hardening alloys
Quality class C

- ◆ L/R=2; R/t=50
- L/R=2; R/t=100
- ▲ L/R=2; R/t=200

$$\bar{\lambda}_\theta = \sqrt{P_{0,nRk} / P_{nRc}}$$

Stiffened shells – EC9 formulation

General buckling curve formulation

$$\chi_x = \alpha \chi_{perf}$$

$$\chi_{perf} = \frac{1}{\phi + \sqrt{\phi^2 - \lambda^2}}$$

$$\phi = 0.5 \left[1 + \alpha_0 (\lambda - \lambda_0) + \lambda^2 \right]$$

$$\bar{\lambda}_x = \sqrt{f_{0k} / \sigma_{xRc}}$$

$$\bar{\lambda}_\theta = \sqrt{f_{0k} / \sigma_{\theta Rc}}$$

$$\bar{\lambda}_\tau = \sqrt{(f_{0k} / \sqrt{3}) / \tau_{Rc}}$$

$$\bar{\lambda}_x = \sqrt{n_{0,xRk} / n_{xRc}}$$

$$\bar{\lambda}_\theta = \sqrt{p_{0,nRk} / p_{nRc}}$$

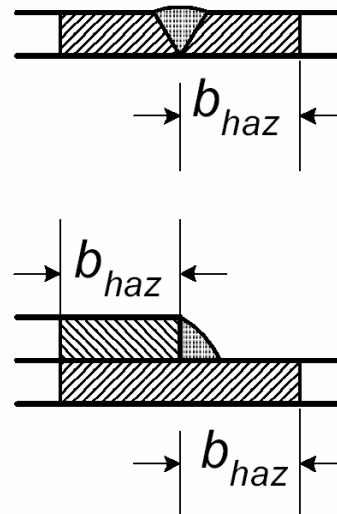
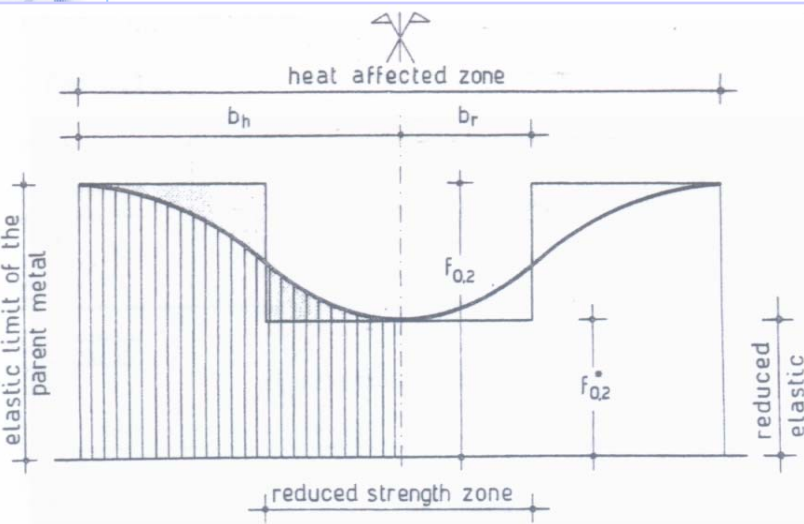
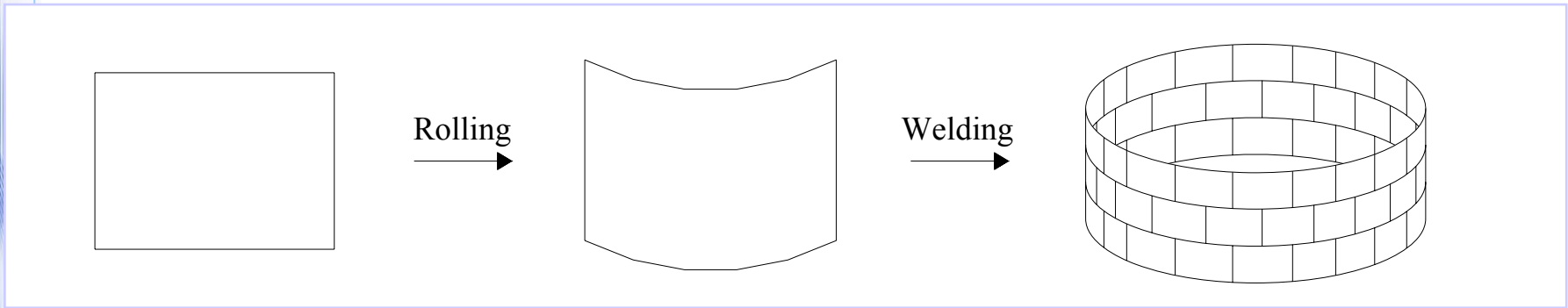
$$\bar{\lambda}_\tau = \sqrt{(f_{0k} / \sqrt{3}) / \tau_{Rc}}$$

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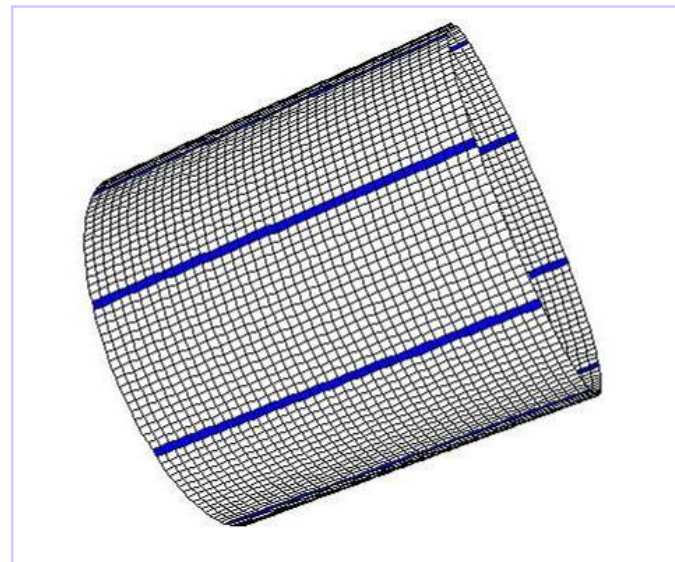
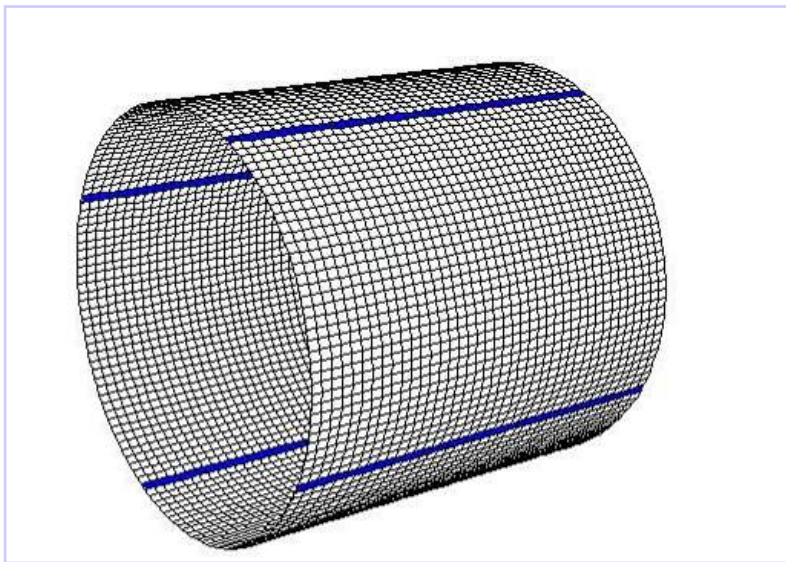
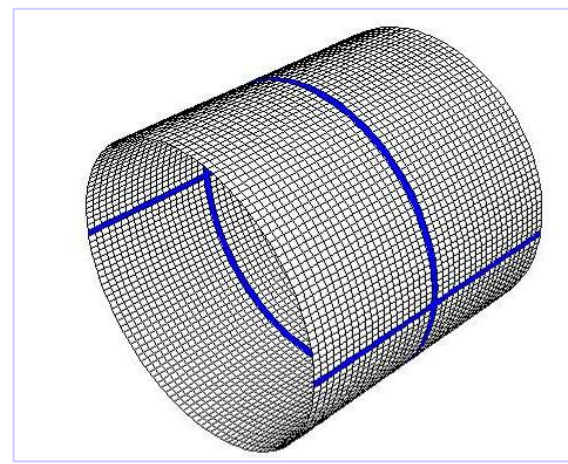
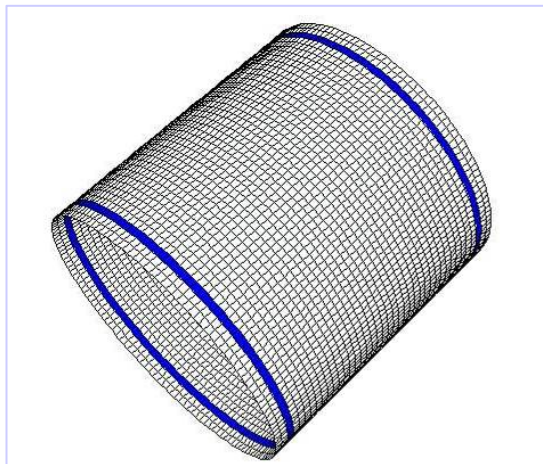
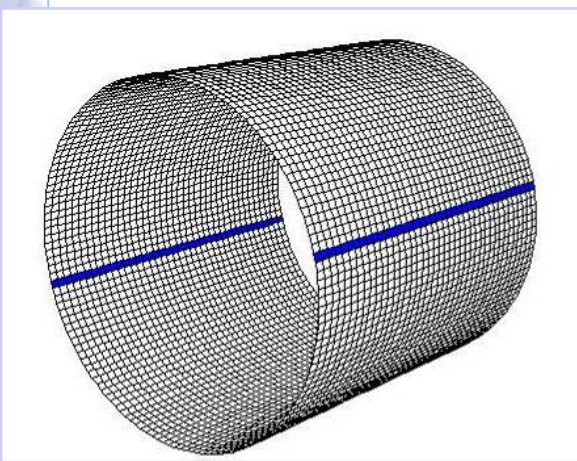


Effect of welding (HAZ zones): definition of simplified design procedures

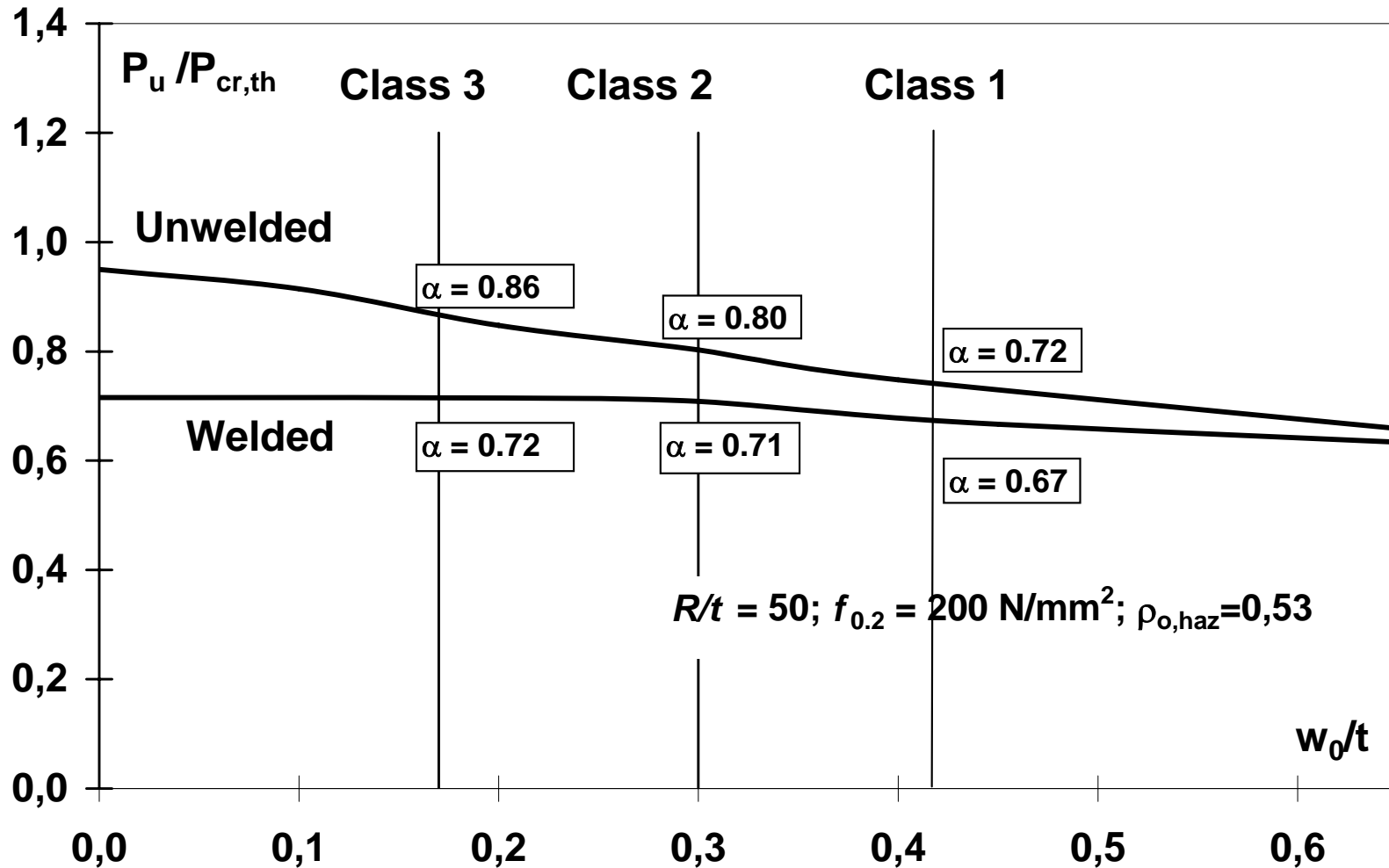


MIG	
$0 < t \leq 6\text{mm}$	$b_{haz} = 20 \text{ mm}$
$6 < t \leq 12\text{mm}$	$b_{haz} = 30 \text{ mm}$
$12 < t \leq 25\text{mm}$	$b_{haz} = 35 \text{ mm}$
$t > 25\text{mm}$	$b_{haz} = 40 \text{ mm}$
TIG	
$0 < t \leq 6\text{mm}$	$b_{haz} = 30 \text{ mm}$

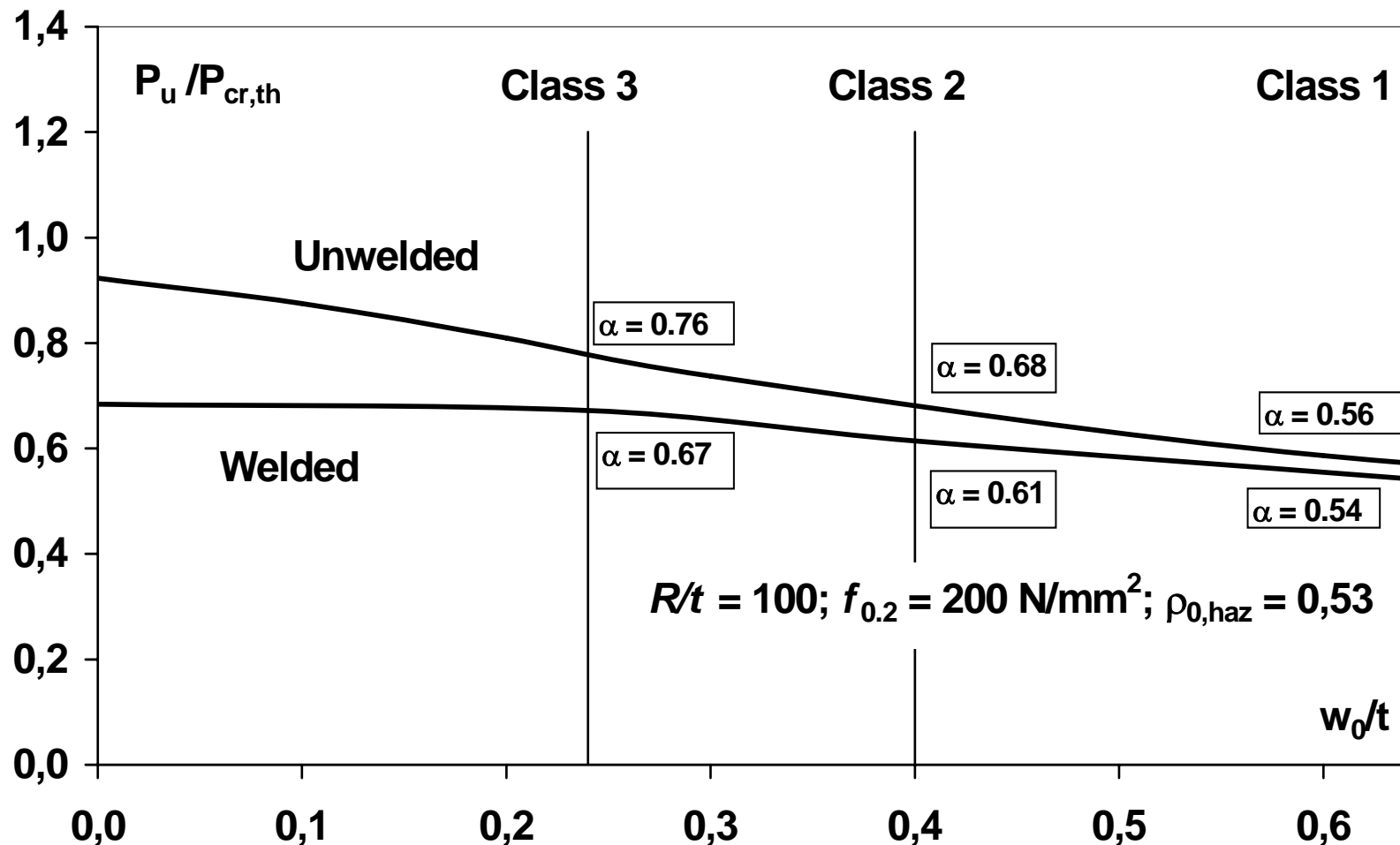
Effect of welding – Parametric analysis



Effect of welding – Imperfection sensitivity curves, axial compression



Effect of welding – Imperfection sensitivity curves, axial compression



Background activity - Main investigated aspects

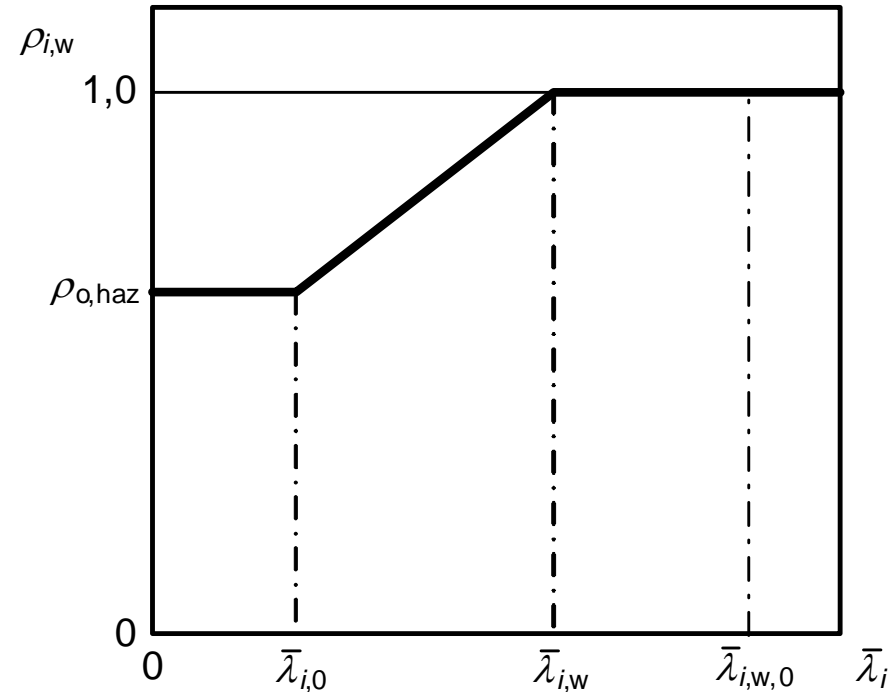
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 - **definition of simplified design procedures;**

Effect of welding according to EC9

$$\rho_{i,w} = \omega_0 + (1 - \omega_0) \frac{\bar{\lambda}_i - \bar{\lambda}_{i,0}}{\bar{\lambda}_{i,w} - \bar{\lambda}_{i,0}}$$

$$\omega_0 = \omega(\rho_{0,haz})$$

$$\lambda_{i,0} = \lambda_{i,0}(\lambda_{i,w,0})$$



Execution tolerance class	Axial compression $\bar{\lambda}_{x,w,0}$		Circumferential compression $\bar{\lambda}_{\theta,w,0}$		Torsion and shear $\bar{\lambda}_{\tau,w,0}$	
	Class A material	Class B material	Class A material	Class B material	Class A material	Class B material
Class 1	0,8	0,7	1,2	1,1	1,4	1,3
Class 2	1,0	0,9	1,3	1,2	1,5	1,4
Class 3	1,2	1,1	1,4	1,3	1,6	1,5
Class 4	1,3	1,2	-	-	-	-