



Eurocodes

Background and Applications

Dissemination of information for training

18-20 February 2008, Brussels

Eurocode 4

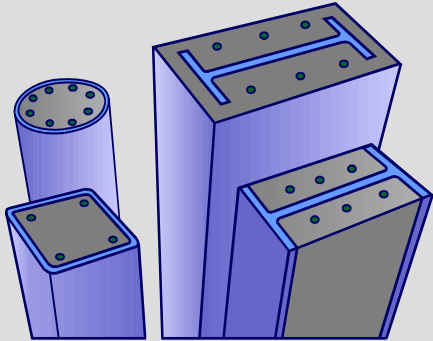
Composite Columns

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University of Wuppertal

Germany



Part 1: Introduction

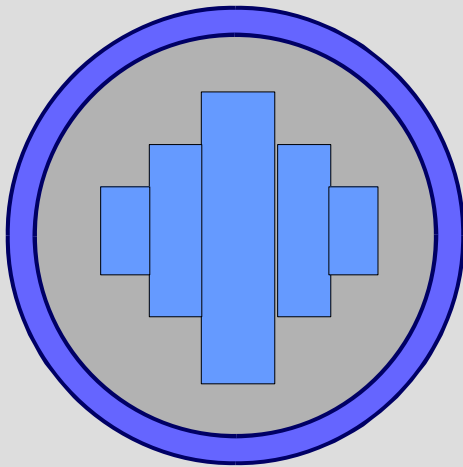
Part 2: General method of design

Part 3: Plastic resistance of cross-sections and interaction curve

Part 4: Simplified design method

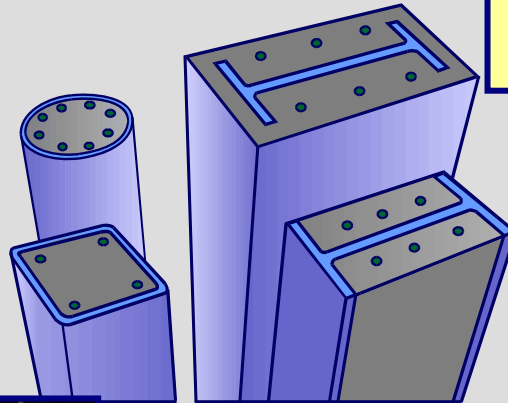
Part 5: Special aspects of columns with inner core profiles

Part 6: Load introduction and longitudinal shear

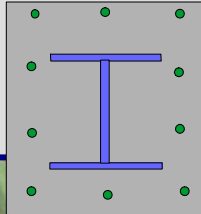


Part 1: Introduction

Composite columns



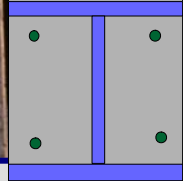
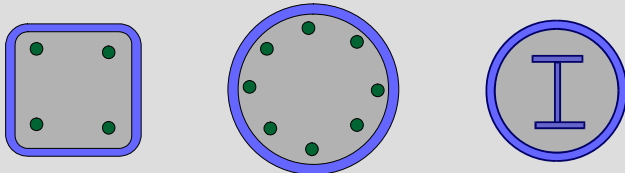
concrete encased sections

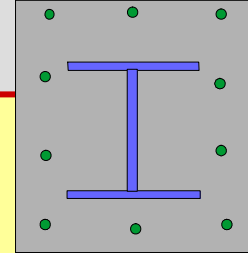


concrete filled hollow sections



partially concrete encased sections



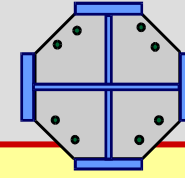
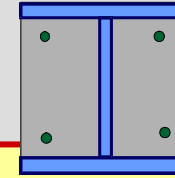
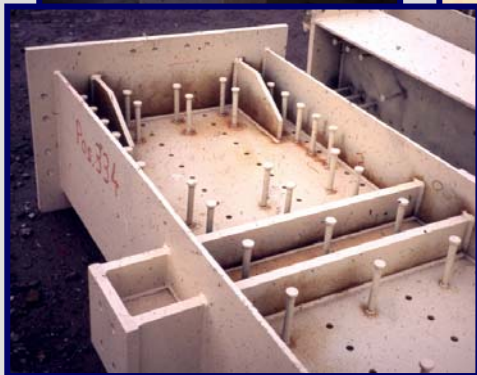
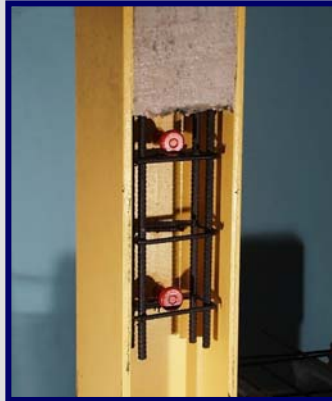


advantages:

- high bearing resistance
- high fire resistance
- economical solution with regard to material costs

disadvantages:

- high costs for formwork
- difficult solutions for connections with beams
- difficulties in case of later strengthening of the column
- in special case edge protection is necessary

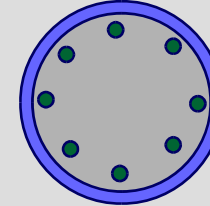
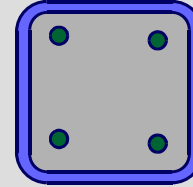


advantages:

- high bearing resistance, especially in case of welded steel sections
- no formwork
- simple solution for joints and load introduction
- easy solution for later strengthening and additional later joints
- no edge protection

disadvantages:

- lower fire resistance in comparison with concrete encased sections.



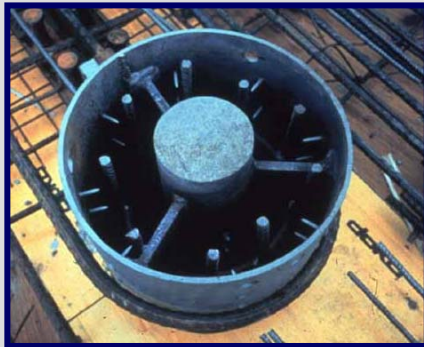
advantages:

- high resistance and slender columns
- advantages in case of biaxial bending
- no edge protection

disadvantages :

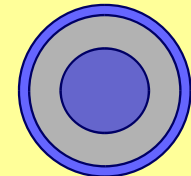
- high material costs for profiles
- difficult casting
- additional reinforcement is needed for fire resistance

Concrete filled hollow sections with additional inner profiles



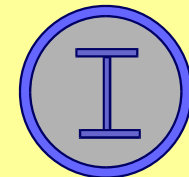
advantages:

- extreme high bearing resistance in combination with slender columns
- constant cross section for all stories is possible in high rise buildings
- high fire resistance and no additional reinforcement
- no edge protection

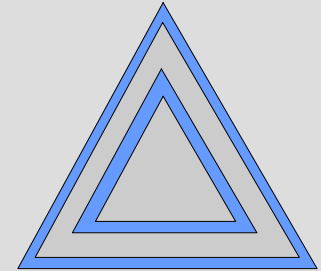
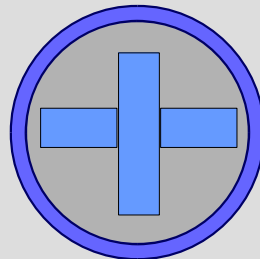
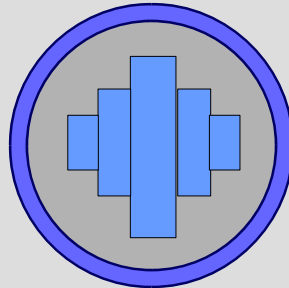
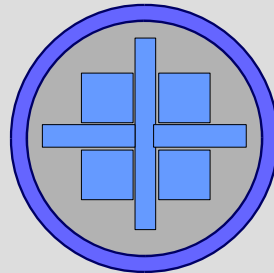
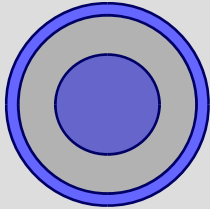


disadvantages:

- high material costs
- difficult casting



Composite columns with hollow sections and additional inner core-profiles



Commerzbank
Frankfurt

Verifications for composite columns



Resistance of the member for
structural stability

General method

Simplified method

Resistance to local Buckling

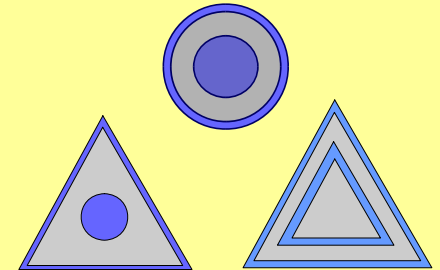
Introduction of loads

Longitudinal shear outside the areas of load
introduction

Methods of verification

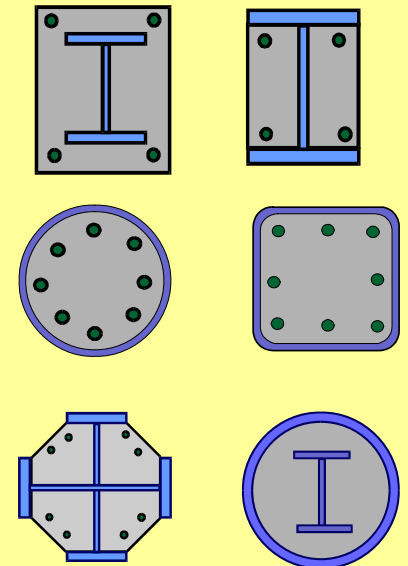
general method:

- any type of cross-section and any combination of materials



simplified method:

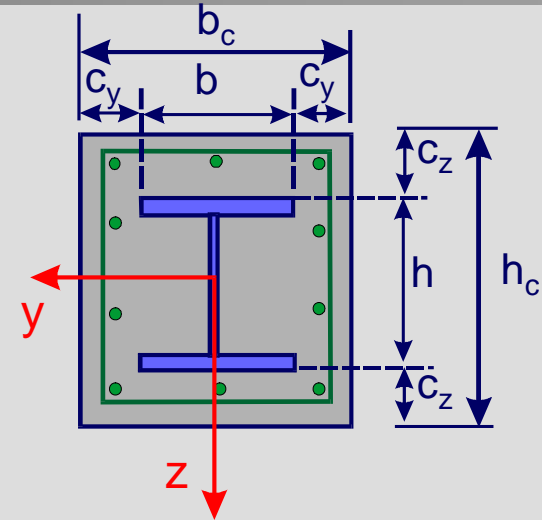
- double-symmetric cross-section
- uniform cross-section over the member length
- limited steel contribution factor δ
- related Slenderness smaller than 2,0
- limited reinforcement ratio
- limitation of b/t-values



concrete encased cross-sections

Verification is not necessary where

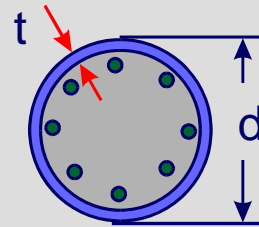
$$c_z \geq \begin{cases} 40 \text{ mm} \\ b/6 \end{cases}$$



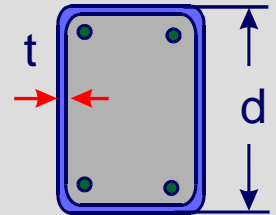
concrete filled hollow section



$$\max \left(\frac{d}{t} \right) = 90 \varepsilon^2$$



$$\max \left(\frac{d}{t} \right) = 52 \varepsilon$$



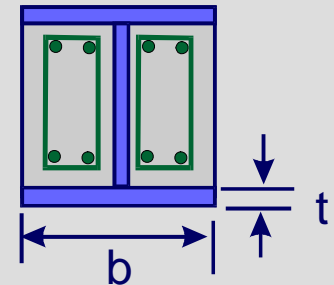
partially encased I sections

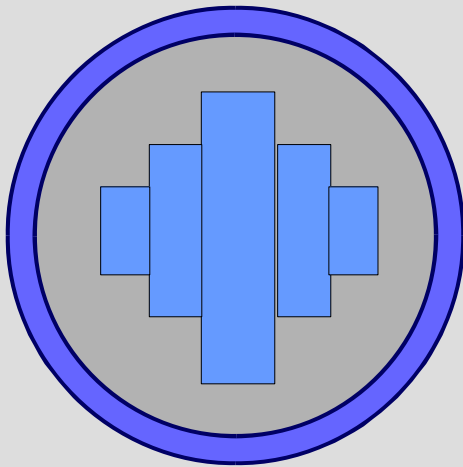
$$\varepsilon = \sqrt{\frac{f_{yk,o}}{f_{yk}}}$$

$$f_{yk,o} = 235 \text{ N/mm}^2$$



$$\max \left(\frac{d}{t} \right) = 44 \varepsilon$$



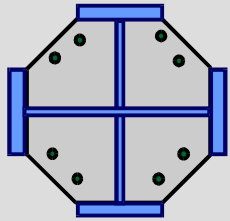


Part 2: General design method

Design for structural stability shall take account of

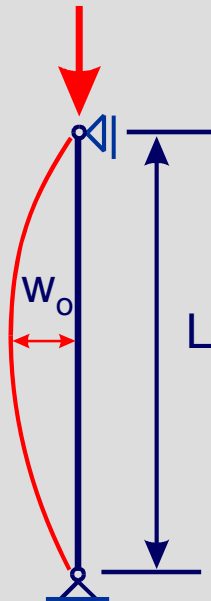
- second-order effects including residual stresses,
- geometrical imperfections,
- local instability,
- cracking of concrete,
- creep and shrinkage of concrete
- yielding of structural steel and of reinforcement.

The design shall ensure that instability does not occur for the most unfavourable combination of actions at the ultimate limit state and that the resistance of individual cross-sections subjected to bending, longitudinal force and shear is not exceeded. Second-order effects shall be considered in any direction in which failure might occur, if they affect the structural stability significantly. Internal forces shall be determined by elasto-plastic analysis. Plane sections may be assumed to remain plane. Full composite action up to failure may be assumed between the steel and concrete components of the member. The tensile strength of concrete shall be neglected. The influence of tension stiffening of concrete between cracks on the flexural stiffness may be taken into account.

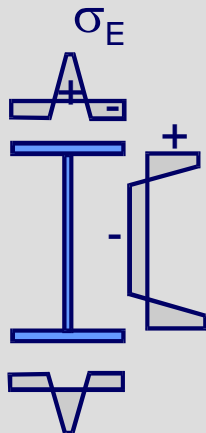


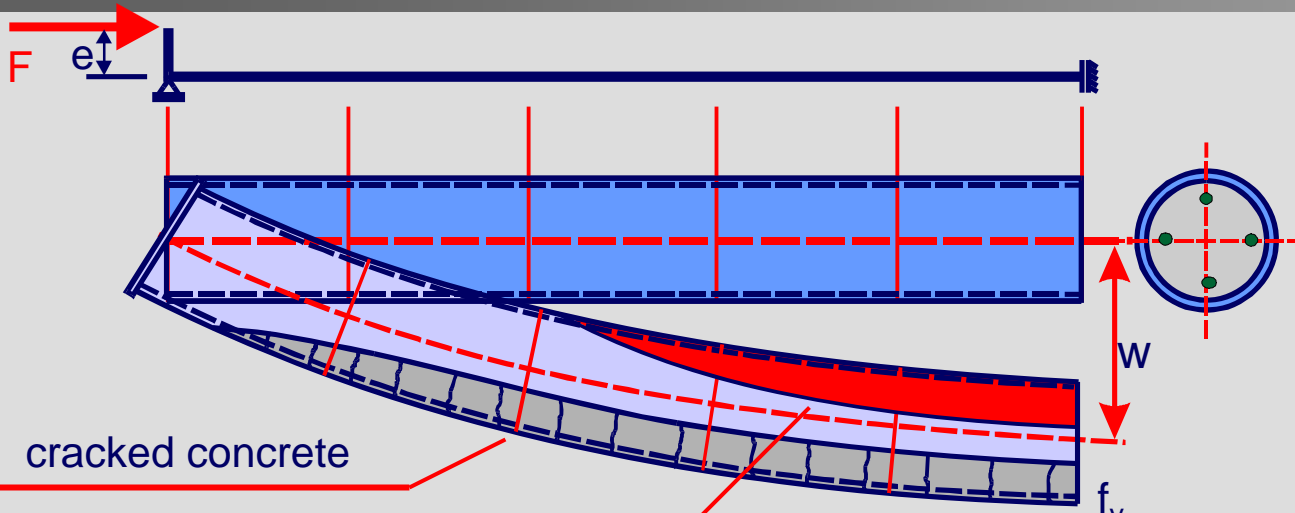
geometrical
imperfection

$$w_o = \frac{L}{1000}$$



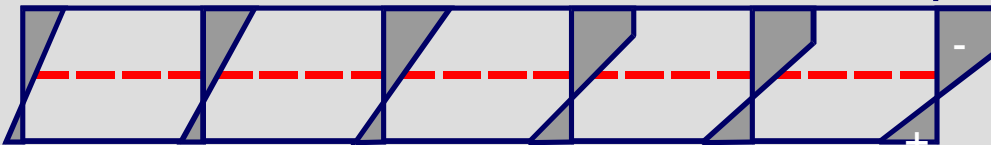
residual
stresses due
to rolling or
welding



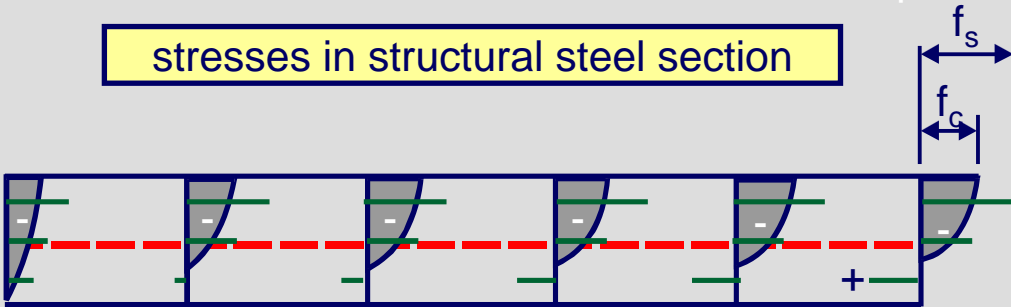


cracked concrete

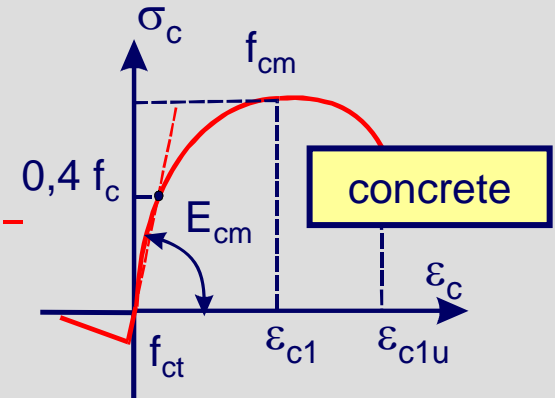
plastic zones in structural steel



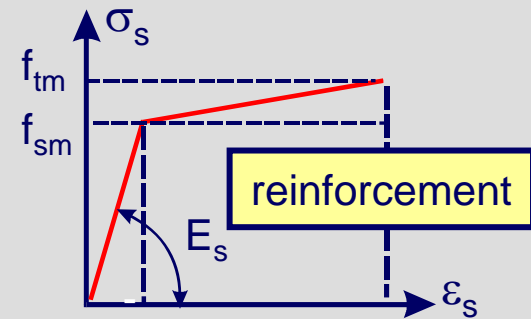
stresses in structural steel section



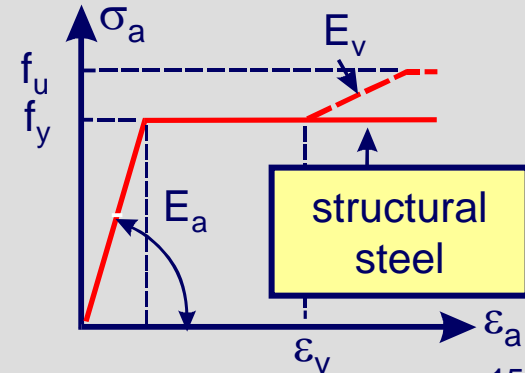
stresses in concrete and reinforcement



concrete

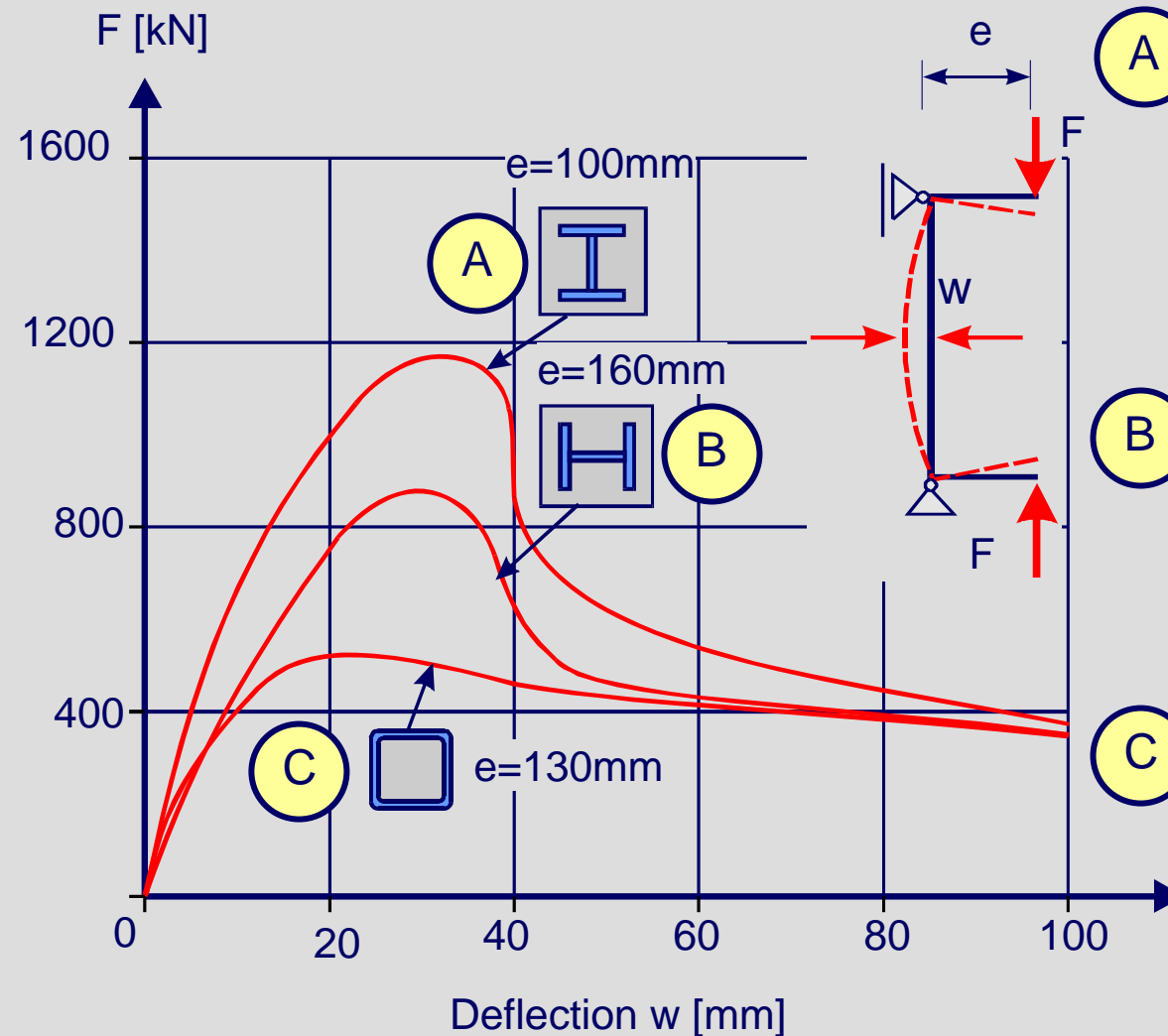


reinforcement



structural steel

Typical load-deformation behaviour of composite columns in tests

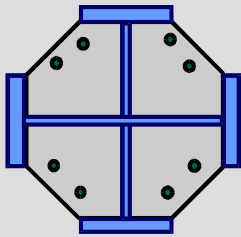
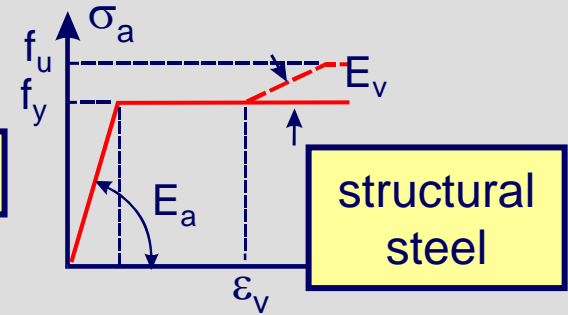
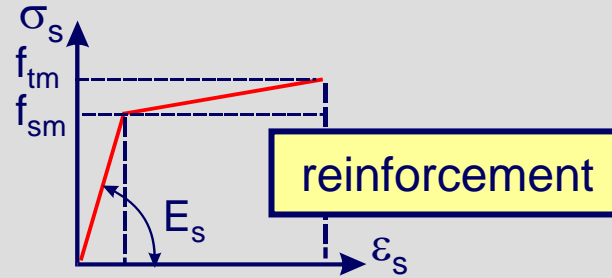
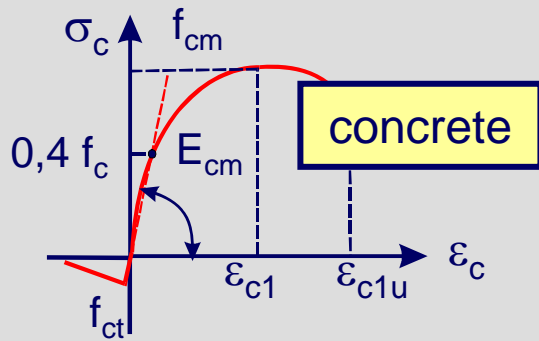


(A) concrete encased section and bending about the strong axis:
 Failure due to exceeding the ultimate strain in concrete, buckling of longitudinal reinforcement and spalling of concrete.

(B) concrete encased section and bending about the weak axis :
 Failure due to exceeding the ultimate strain in concrete.

(C) concrete filled hollow section:
 cross-section with high ductility and rotation capacity. Fracture of the steel profile in the tension zone at high deformations and local buckling in the compression zone of the structural steel section.

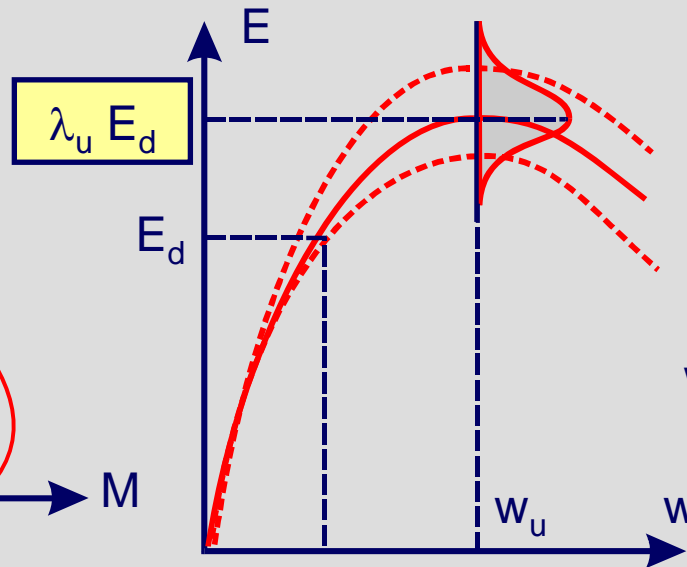
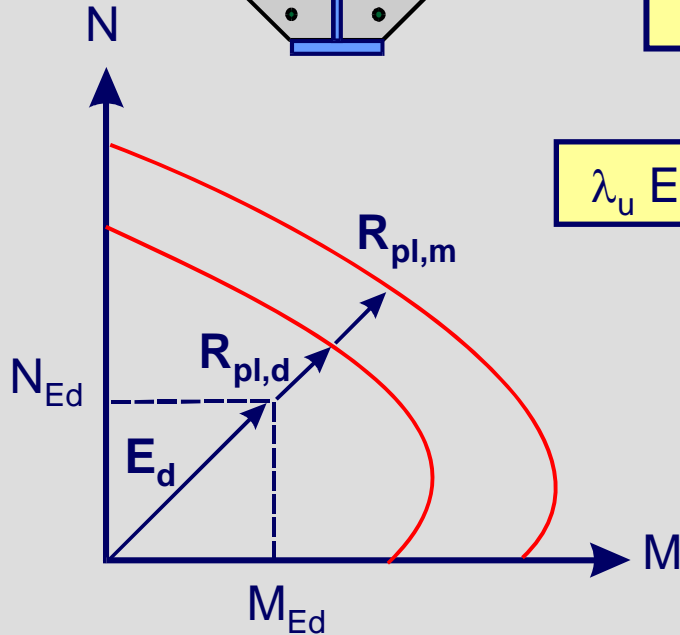
General Method – Safety concept based on DIN 18800-5 (2004) and German national Annex for EN 1994-1-1



λ_u : amplification factor for ultimate system capacity

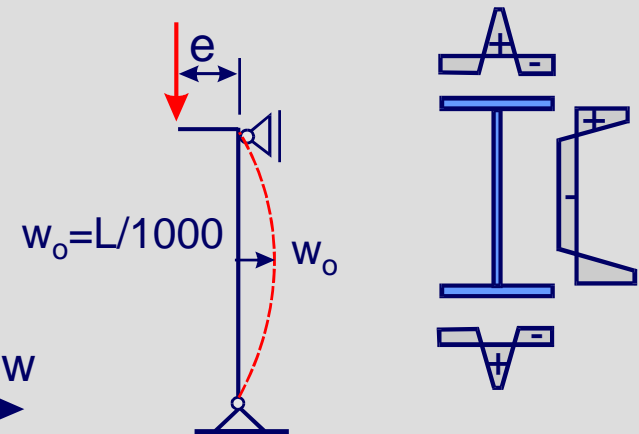
$$\gamma_R = \frac{R_{pl,m}}{R_{pl,d}}$$

Verification $\lambda_u \geq \gamma_R$

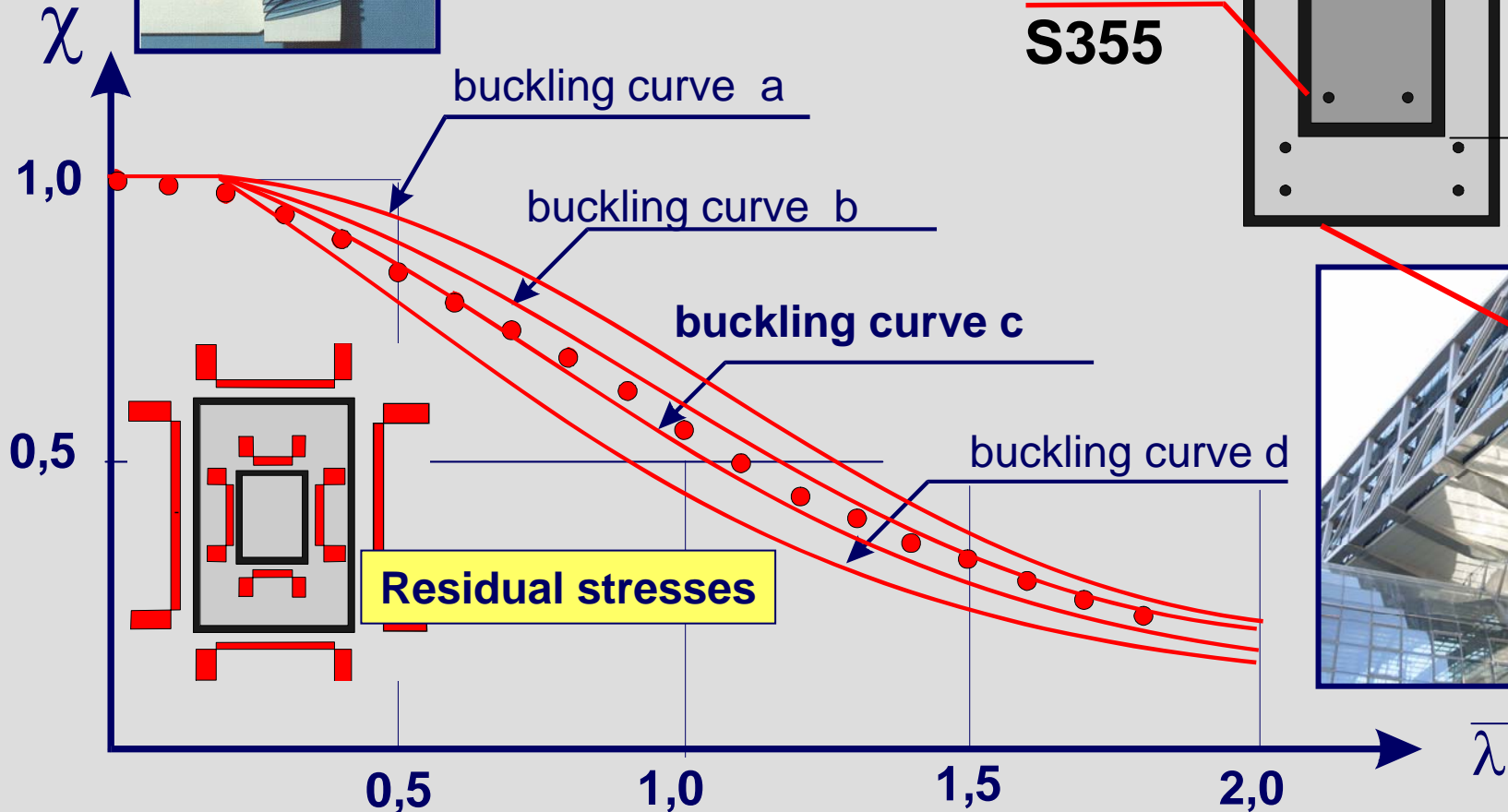
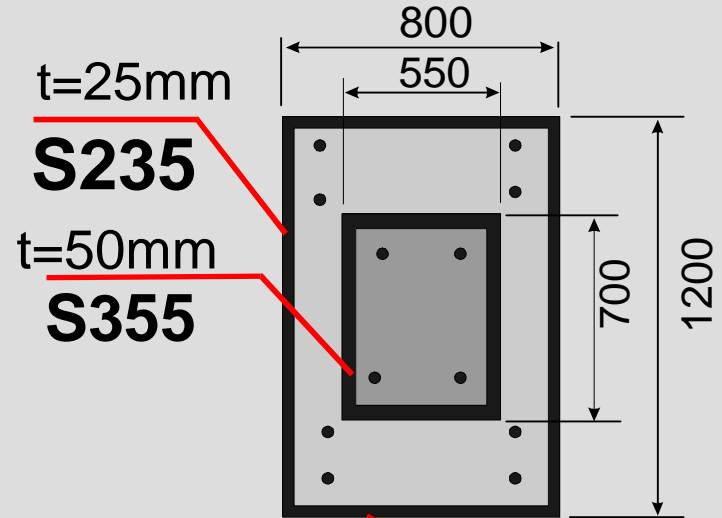
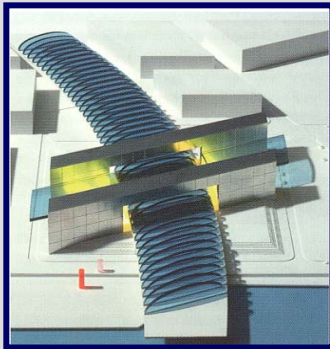


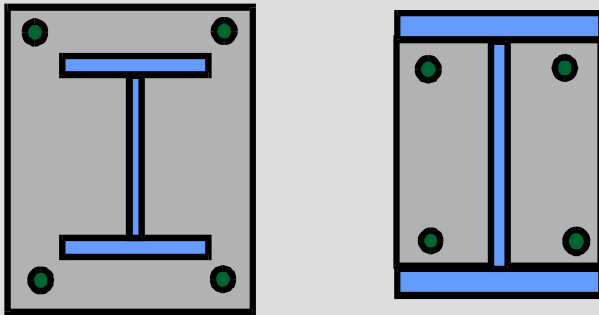
geometrical Imperfection

Residual stresses



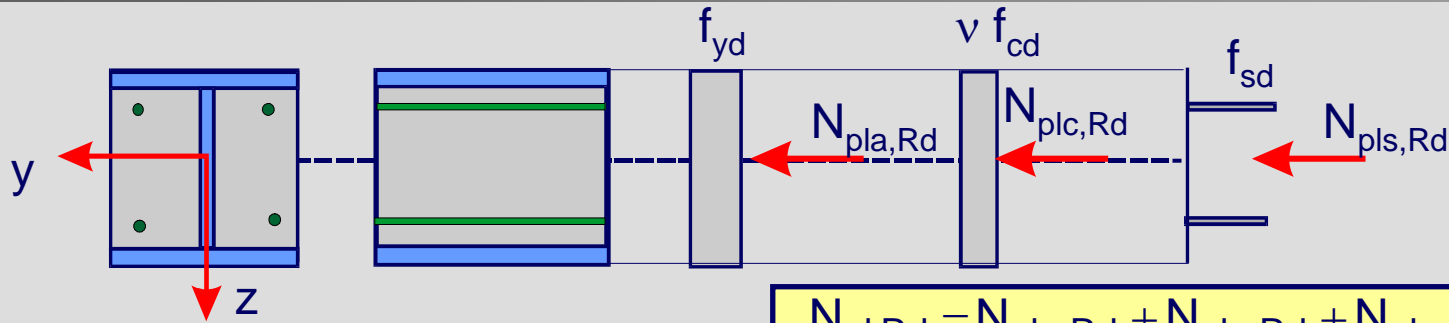
Composite columns for the central station in Berlin





Part IV-3:

Plastic resistance of cross-sections and interaction curve



Design value of the plastic resistance to compressive forces:

$$N_{pl,Rd} = N_{pla,Rd} + N_{plc,Rd} + N_{pls,Rd}$$

$$N_{pl,Rd} = A_a f_{yd} + v A_c f_{cd} + A_s f_{sd}$$

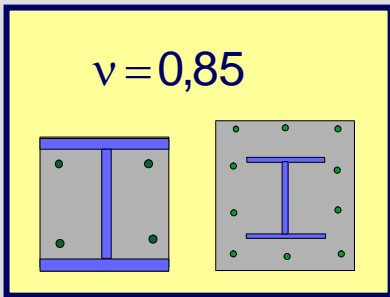
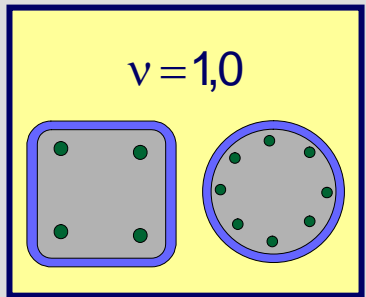
Characteristic value of the plastic resistance to compressive forces:

$$N_{pl,Rk} = A_a f_{yk} + A_s f_{sk} + v A_c f_{ck}$$

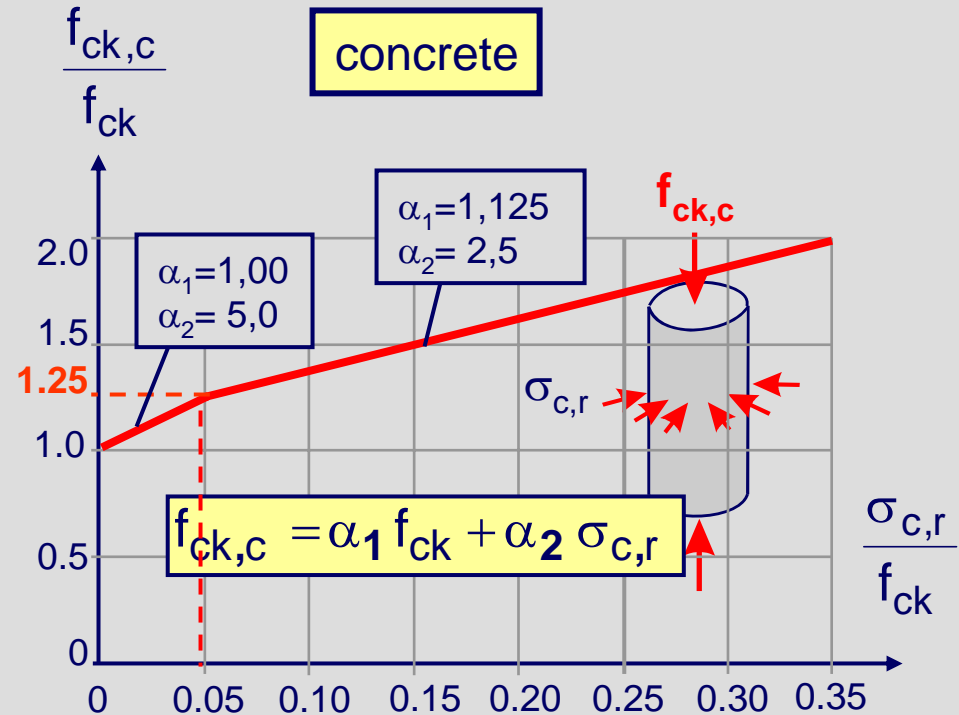
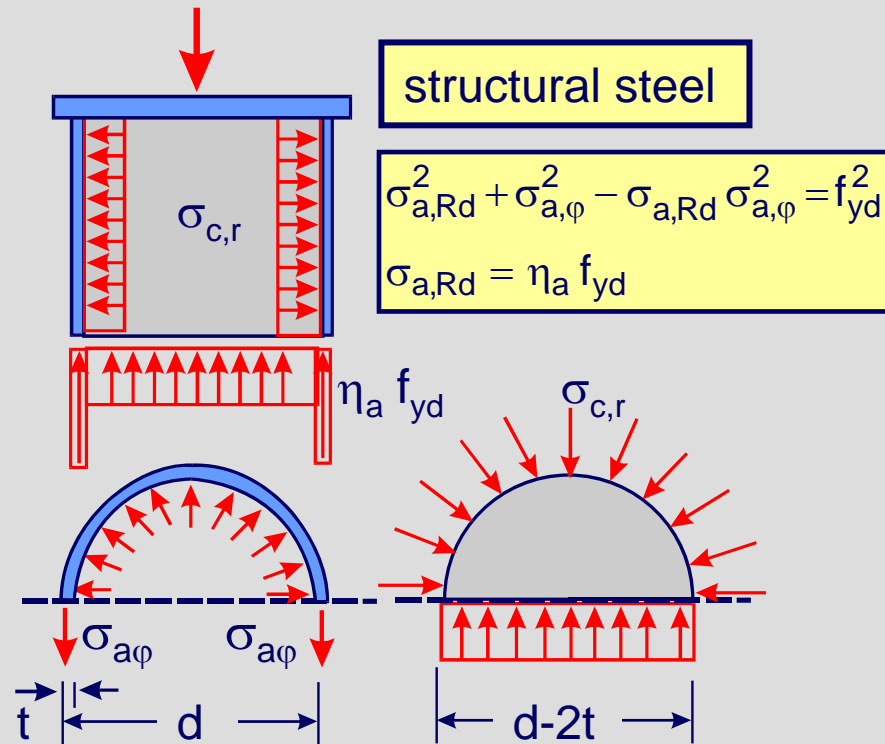
Design strength:

$$f_{yd} = \frac{f_{yk}}{\gamma_a} \quad f_{sd} = \frac{f_{sk}}{\gamma_s} \quad f_{cd} = \frac{f_{ck}}{\gamma_c}$$

Increase of concrete strength due to better curing conditions in case of concrete filled hollow sections:



Confinement effects in case of concrete filled tubes



For concrete stresses $\sigma_c > 0,8 f_{ck}$ the Poisson's ratio of concrete is higher than the Poisson's ratio of structural steel. The confinement of the circular tube causes radial compressive stresses $\sigma_{c,r}$. This leads to an increased strength and higher ultimate strains of the concrete. In addition the radial stresses cause friction in the interface between the steel tube and the concrete and therefore to an increase of the longitudinal shear resistance.

Design value of the plastic resistance to compressive forces taking into account the confinement effect:

$$N_{pl,Rd} = \eta_a f_{yd} A_a + A_c f_{cd} \left(1 + \eta_c \frac{t}{d} \frac{f_{yk}}{f_{ck}} \right)$$

Basic values η for stocky columns
 centrically loaded:

$$\eta_{ao} = 0,25 \quad \eta_{co} = 4,9$$

influence of
 slenderness for
 $\bar{\lambda} \leq 0,5$

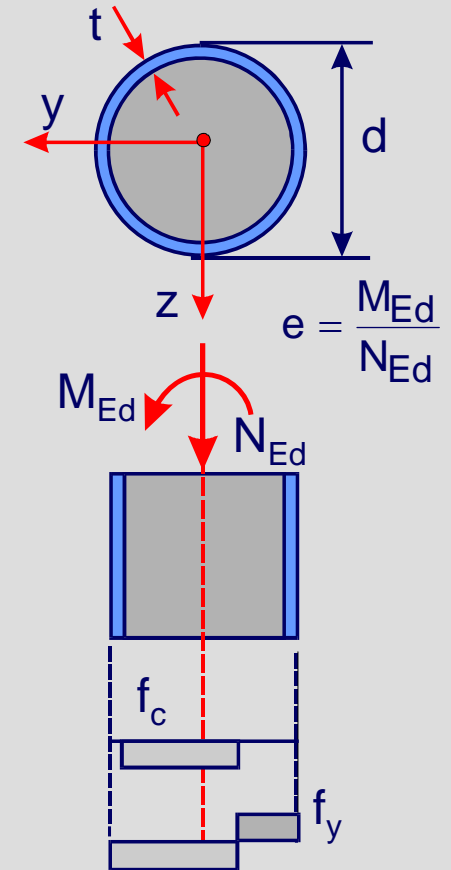
$$\eta_{a,\lambda} = \eta_{ao} + 0,5 \bar{\lambda}_K \leq 1,0$$

$$\eta_{c,\lambda} = \eta_{co} - 18,5 \bar{\lambda}_K (1 - 0,92 \bar{\lambda}_K) \geq 0$$

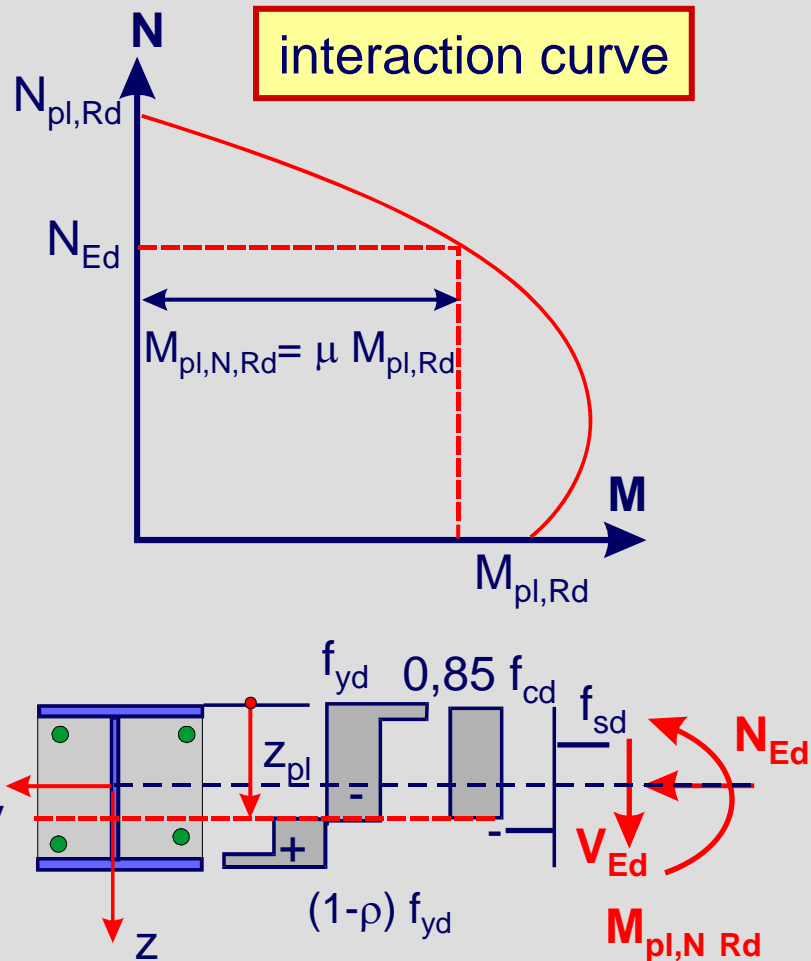
influence of load
 eccentricity :

$$\eta_a = \eta_{a,\lambda} + 10 (1 - \eta_{ao}) \frac{e}{d} \quad \eta_c = \eta_{c,\lambda} \left(1 - 10 \frac{e}{d} \right)$$

$$e/d \geq 0,1 : \eta_a = 1,0 \text{ and } \eta_c = 0$$



Plastic resistance to combined bending and compression



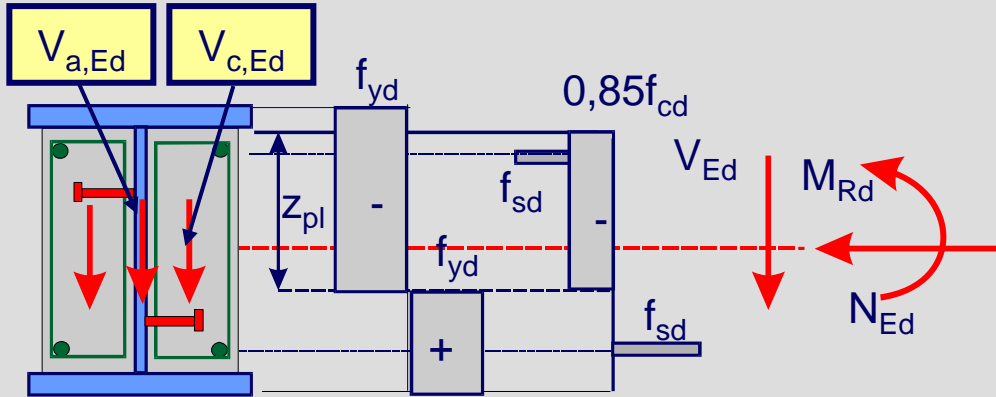
The resistance of a cross-section to combined compression and bending and the corresponding interaction curve may be calculated assuming rectangular stress blocks.

The tensile strength of the concrete should be neglected.

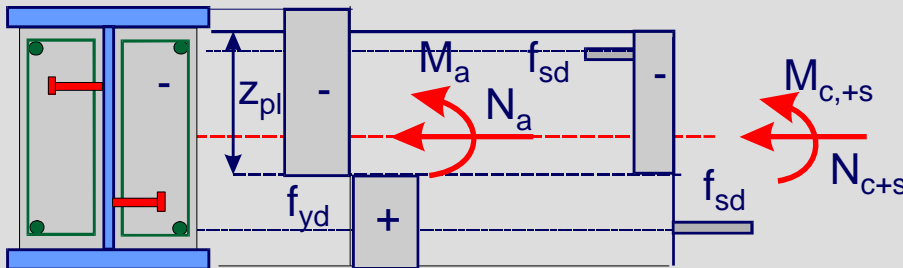
The influence of transverse shear forces on the resistance to bending and normal force should be considered when determining the interaction curve, if the shear force $V_{a,Ed}$ on the steel section exceeds 50% of the design shear resistance $V_{pl,a,Rd}$ of the steel section. The influence of the transverse shear on the resistance in combined bending and compression should be taken into account by a reduced design steel strength $(1 - \rho) f_{yd}$ in the shear area A_v .

$$V_{a,Ed} \leq 0,5 V_{pla,Rd} \Rightarrow \rho = 0$$

$$V_{a,Ed} > 0,5 V_{pla,Rd} \Rightarrow \rho = \left[\frac{2V_{a,Ed}}{V_{pla,Rd}} - 1 \right]^2$$



$$M_{Rd} = M_a + M_{c+s} \quad N_{Ed} = N_a + N_{c+s}$$



Verification for vertical shear:

$$V_{a,Ed} \leq V_{pla,Rd} \quad V_{c,Ed} \leq V_{c,Rd}$$

The shear force $V_{a,Ed}$ should not exceed the resistance to shear of the steel section. The resistance to shear $V_{c,Ed}$ of the reinforced concrete part should be verified in accordance with EN 1992-1-1, 6.2.

Unless a more accurate analysis is used, V_{Ed} may be distributed into $V_{a,Ed}$ acting on the structural steel and $V_{c,Ed}$ acting on the reinforced concrete section by :

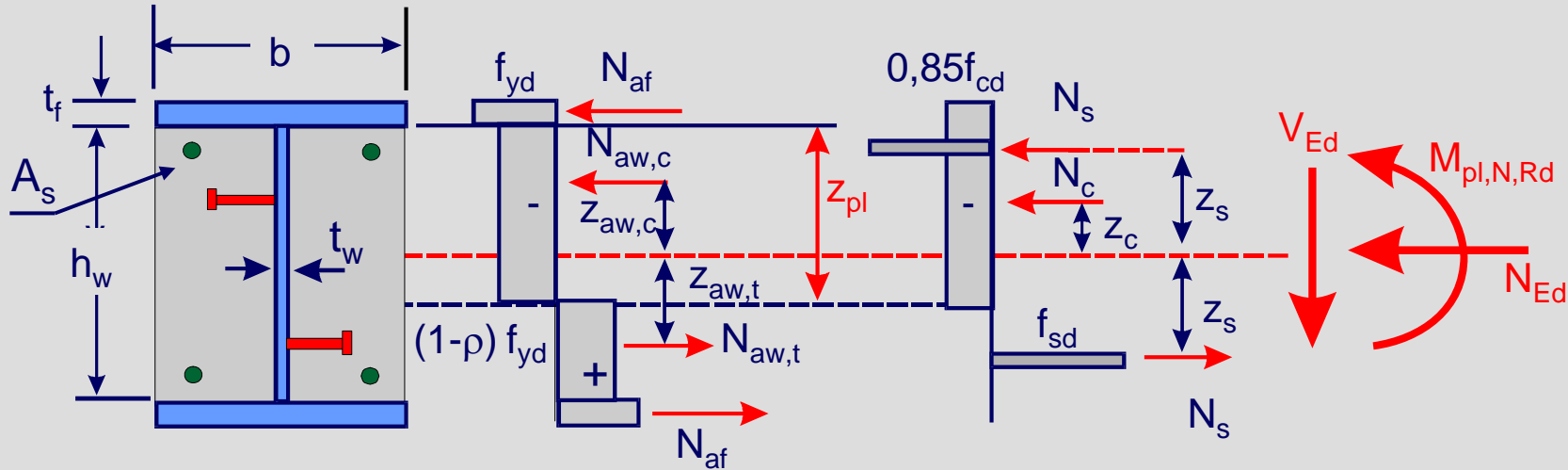
$$V_{a,Ed} = V_{Ed} \frac{M_a}{M_{Rd}} \approx \frac{M_{pla,Rd}}{M_{pl,Rd}}$$

$$V_{c,Ed} = V_{Ed} - V_{a,Ed}$$

$M_{pl,a,Rd}$ is the plastic resistance moment of the steel section.

$M_{pl,Rd}$ is the plastic resistance moment of the composite section.

Determination of the resistance to normal forces and bending (example)



Position of the plastic neutral axis: $\sum N_i = N_{Ed}$

$$N_c + N_{aw,c} - N_{aw,t} = N_{Ed}$$

$$(b - t_w) z_{pl} 0,85 f_{cd} + t_w z_{pl} (1 - \rho) f_{yd} - t_w (h_w - z_{pl}) (1 - \rho) f_{yd} = N_{Ed}$$

$$z_{pl} = \frac{N_{Ed} + h_w t_w (1 - \rho) f_{yd}}{(b - t_w) 0,85 f_{cd} + 2 t_w (1 - \rho) f_{yd}}$$

Plastic resistance to bending $M_{pl,N,Rd}$ in case of the simultaneously acting compression force N_{Ed} and the vertical shear V_{Ed} :

$$M_{pl,N,Rd} = N_c z_c + N_{aw,c} z_{aw,c} + N_{aw,t} z_{aw,t} + N_{af} (h_w + t_f) + 2 N_s z_s$$

$$N_{aw,c} = z_{pl} t_w (1 - \rho) f_{yd}$$

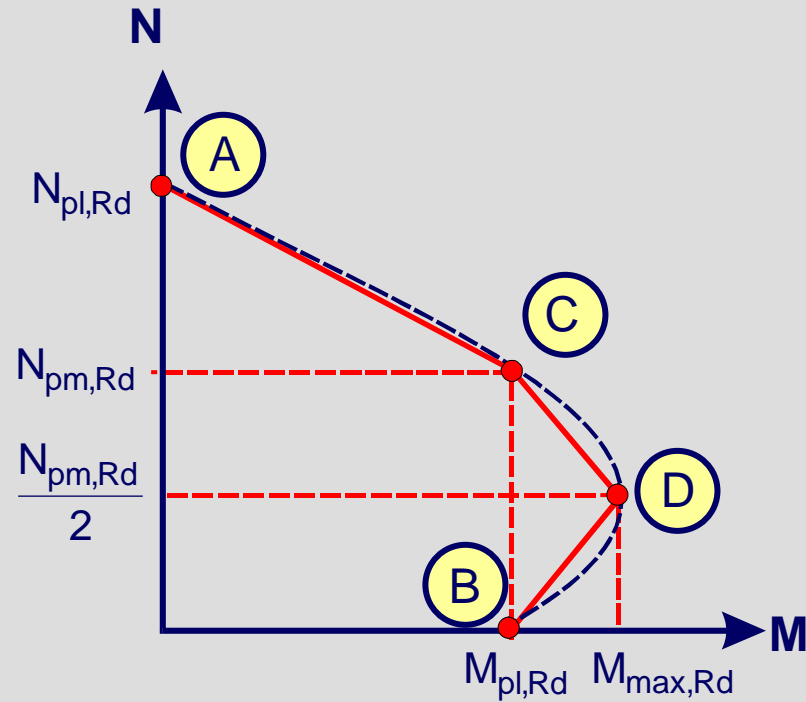
$$N_{aw,t} = (h_w - z_{pl}) t_w (1 - \rho) f_{yd}$$

$$N_{af} = b t_f f_{yd}$$

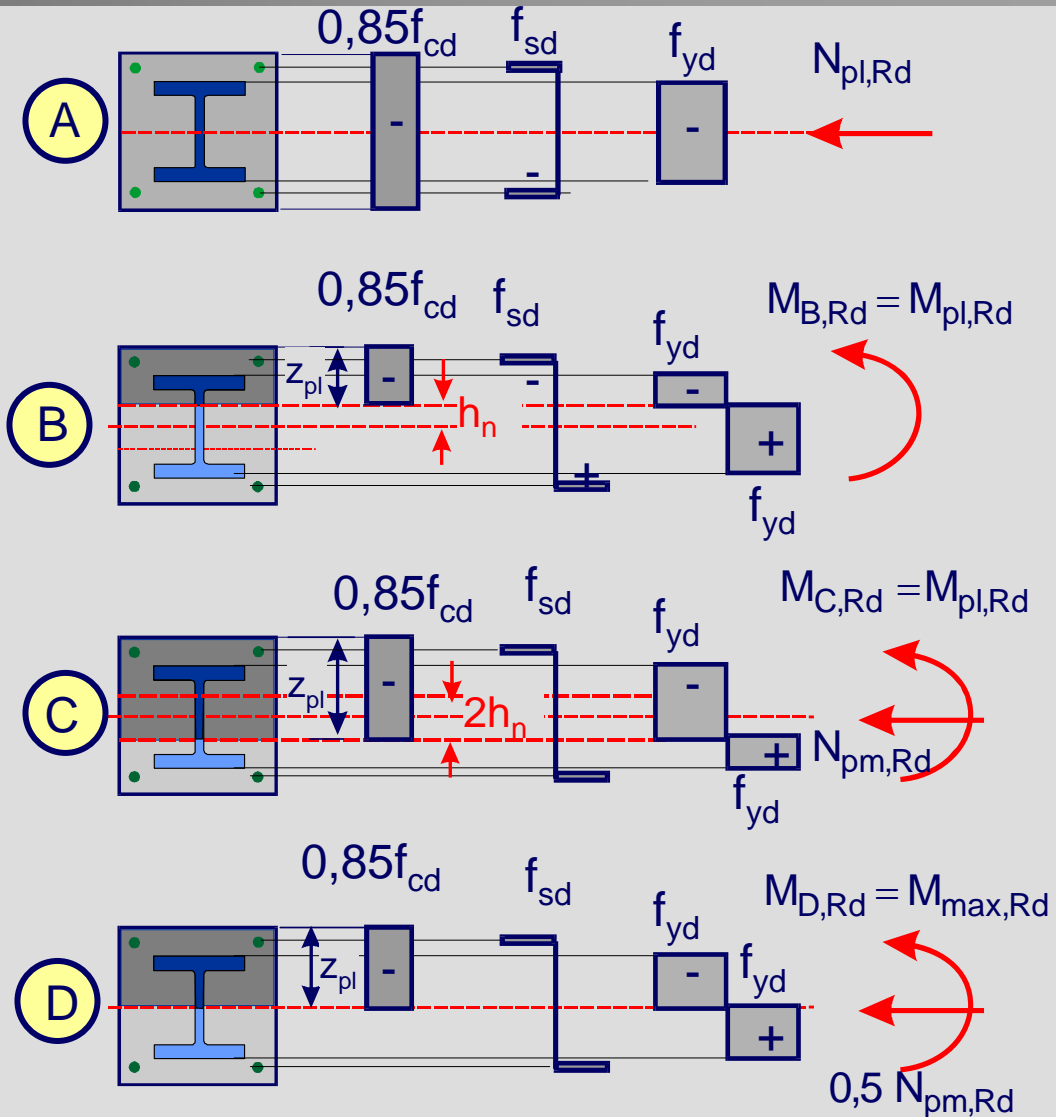
$$N_c = (b - t_w) z_{pl} 0,85 f_{cd}$$

$$N_s = 2 A_s f_{sd}$$

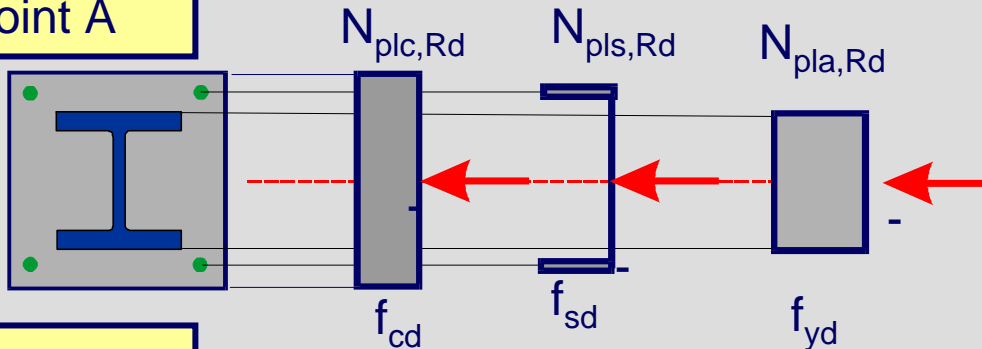
Simplified determination of the interaction curve



As a simplification, the interaction curve may be replaced by a polygonal diagram given by the points A to D.



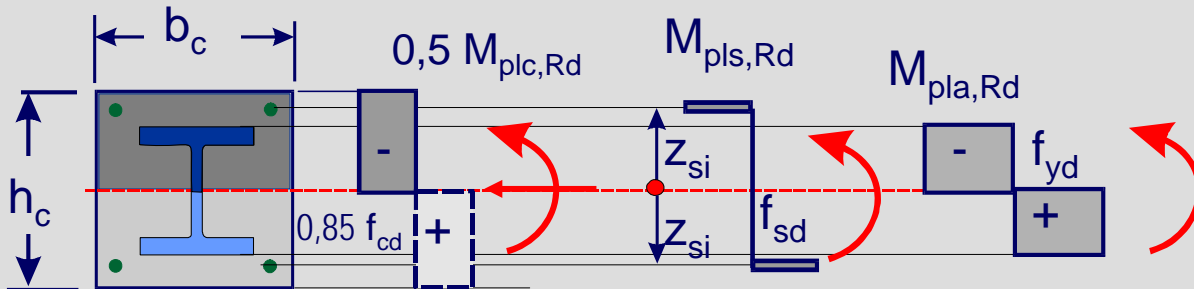
Point A



$$N_{pl,Rd} = N_{pla,Rd} + N_{plc,Rd} + N_{pls,Rd}$$

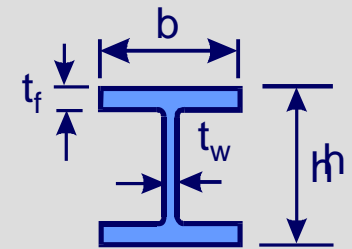
$$M_{A,Rd} = 0$$

Point D



$$N_{D,Rd} = 0,5 N_{plc,Rd}$$

$$M_{D,Rd} = M_{max,Rd}$$



$$M_{max,Rd} = M_{pla,Rd} + M_{pls,Rd} + 1/2 M_{plc,Rd}$$

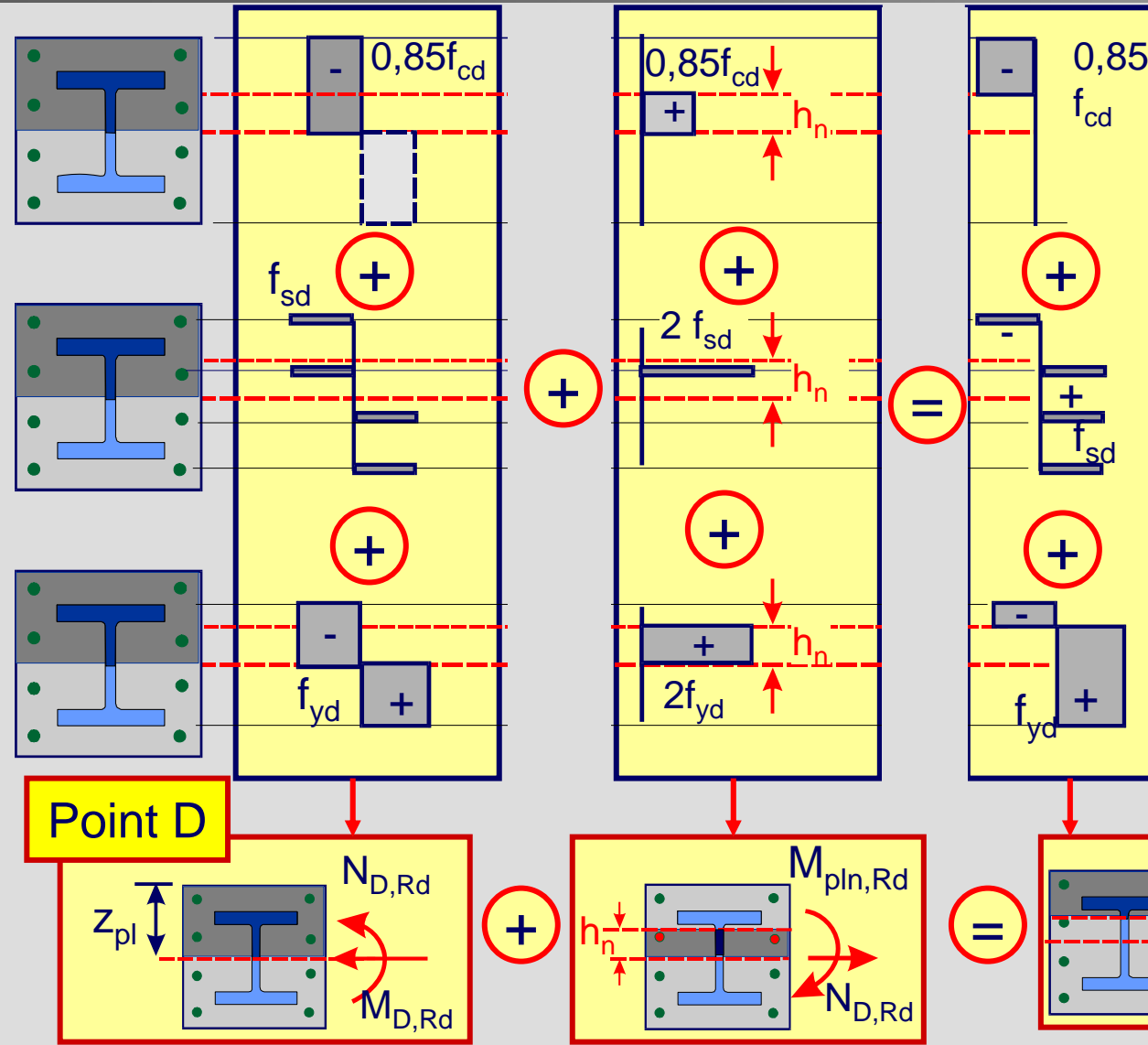
$$M_{pla,Rd} = W_{pl,a} f_{yd} = \left[\frac{(h - 2t_f)^2 t_w}{4} + b t_f (h - t_f) \right] f_{yd}$$

$$M_{pls,Rd} = W_{pl,s} f_{sd} = \left[\sum A_{si} z_{si} \right] f_{ys}$$

$$M_{plc,Rd} = W_{pl,c} 0,85 f_{cd} = \left[\frac{b_c h_c^2}{4} - W_{pl,a} - W_{pl,s} \right] 0,85 f_{cd}$$

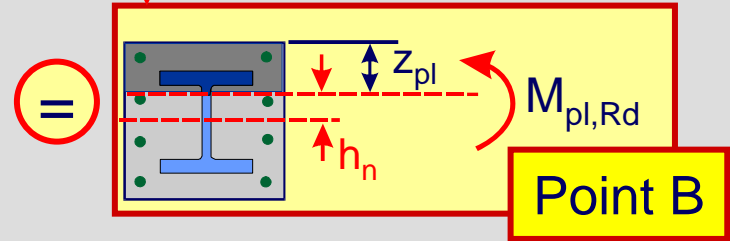
- $W_{pl,a}$ plastic section modulus of the structural steel section
- $W_{pl,s}$ plastic section modulus of the cross-section of reinforcement
- $W_{pl,c}$ plastic section modulus of the concrete section

Bending resistance at Point B ($M_{pl,Rd}$)

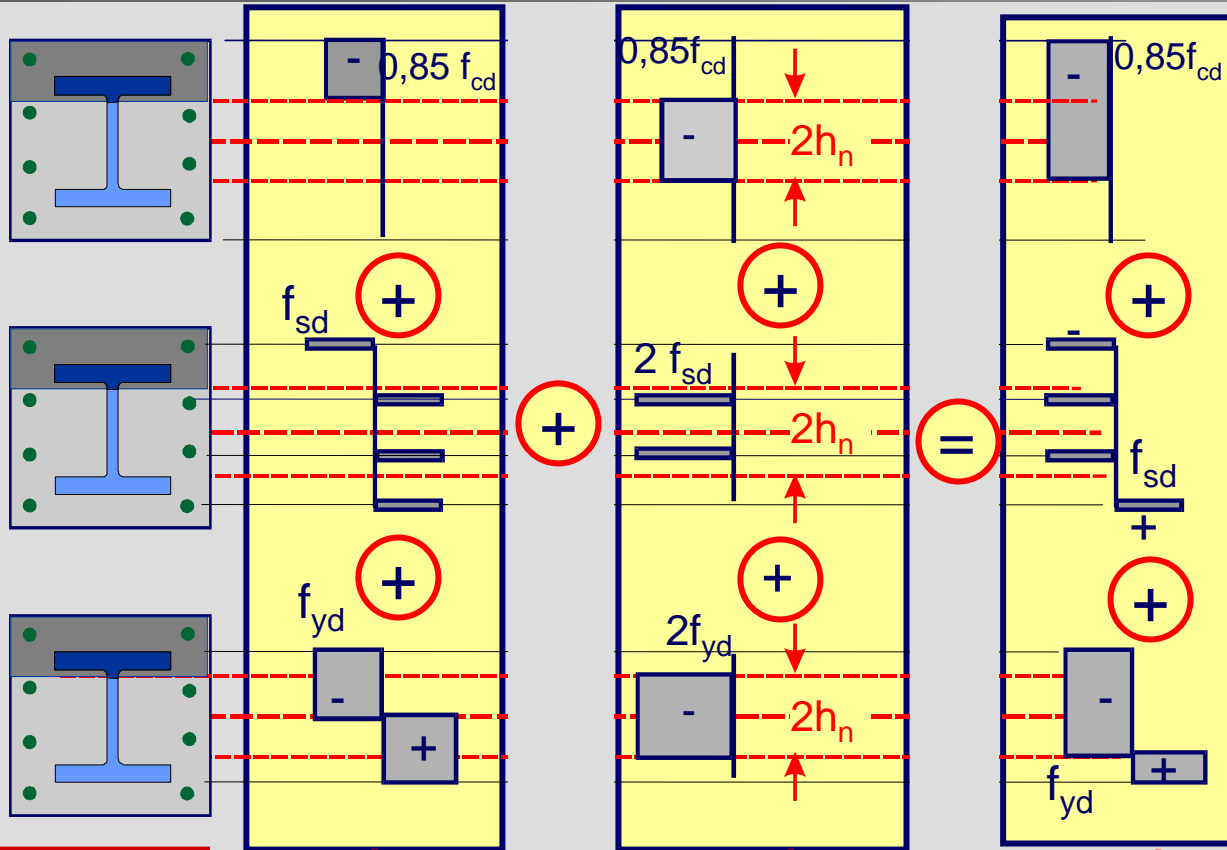


At point B is no resistance to compression forces. Therefore the resistance to compression forces at point D results from the additional cross-section zones in compression. With $N_{D,Rd}$ the depth h_n and the position of the plastic neutral axis at point B can be determined. With the plastic bending moment $M_{n,Rd}$ resulting from the stress blocks within the depth h_n the plastic resistance moment $M_{pl,Rd}$ at point B can be calculated by:

$$M_{pl,Rd} = M_{D,Rd} - M_{pln,Rd}$$



Plastic resistance moment at Point C



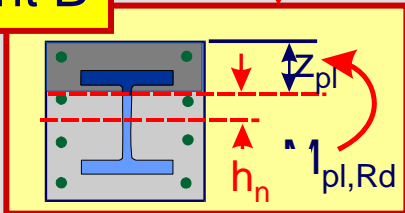
The bending resistance at point C is the same as the bending resistance at point B.

$$M_{C,Rd} = M_{pl,Rd}$$

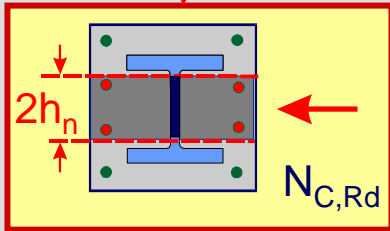
The normal force results from the stress blocks in the zone $2h_n$.

$$N_{C,Rd} = 2 N_{D,Rd} = N_{cpl,Rd} = N_{pm,Rd}$$

Point B

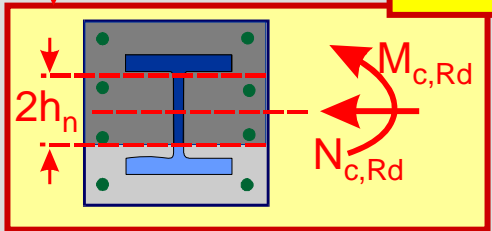


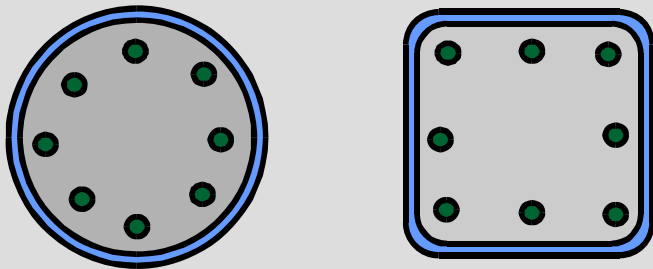
+



=

Point C



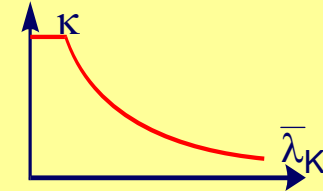


Part 4: Simplified design method

Methods of verification acc. to the simplified method

Axial
compression

Design based on the
European buckling curves

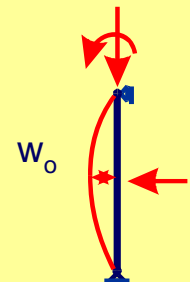


Design based on second order
analysis with equivalent geometrical
bow imperfections

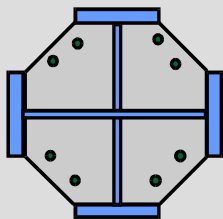
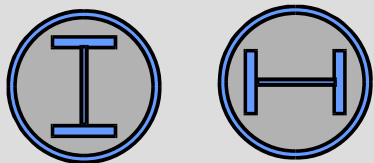
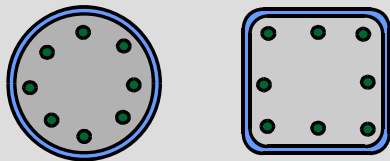
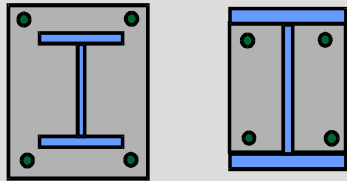


Resistance
of member
in combined
compression
and bending

Design based on second order
analysis with equivalent geometrical
bow imperfections



Scope of the simplified method



- double symmetrical cross-section
- uniform cross-sections over the member length with rolled, cold-formed or welded steel sections
- steel contribution ratio

$$0,2 \leq \delta \leq 0,9 \quad \delta = \frac{A_a f_{yd}}{N_{pl,Rd}}$$

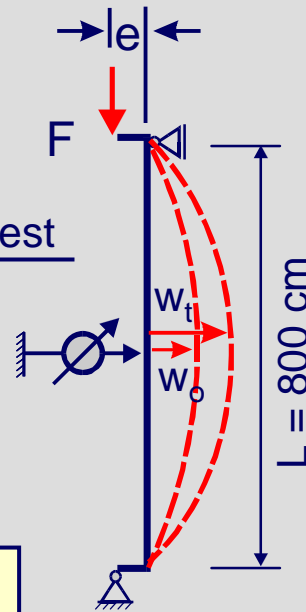
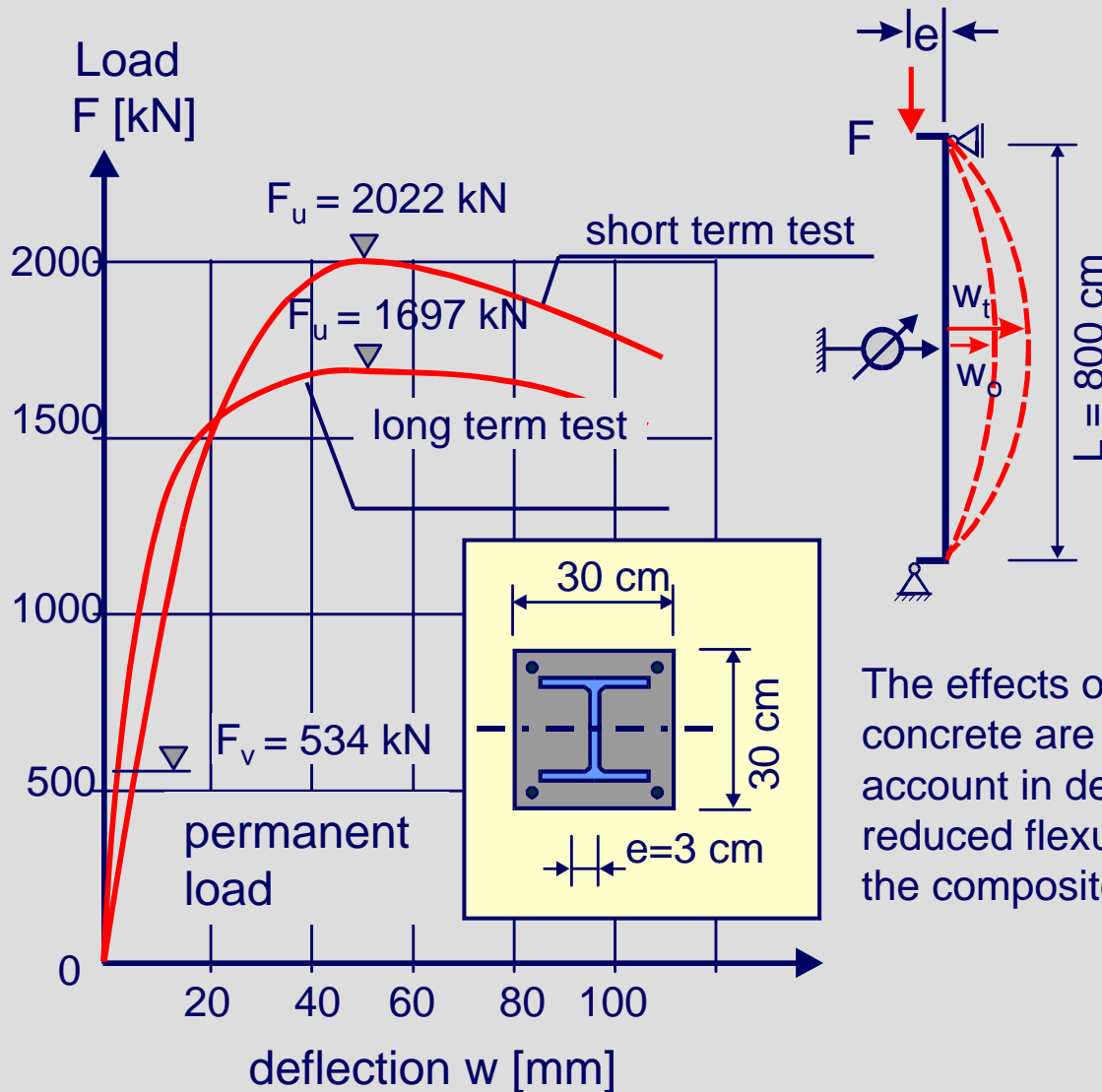
- relative slenderness

$$\bar{\lambda} = \sqrt{\frac{N_{pl,Rk}}{N_{cr}}} \leq 2,0$$

- longitudinal reinforcement ratio

$$0,3\% \leq \rho_s \leq 6,0\% \quad \rho_s = \frac{A_s}{A_c}$$

- the ratio of the depth to the width of the composite cross-section should be within the limits 0,2 and 5,0



The horizontal deflection and the second order bending moments increase under permanent loads due to creep of concrete. This leads to a reduction of the ultimate load.

The effects of creep of concrete are taken into account in design by a reduced flexural stiffness of the composite cross-section.



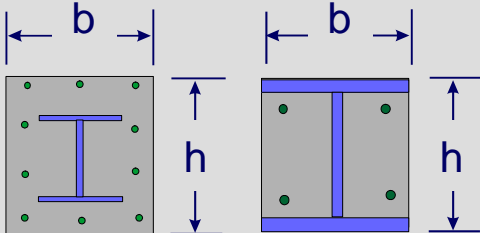
The effects of creep of concrete are taken into account by an effective modulus of elasticity of concrete

$$E_{c,eff} = \frac{E_{cm}}{1 + \frac{N_{G,Ed}}{N_{Ed}} \varphi(t, t_0)}$$

notional size of the cross-section for the determination of the creep coefficient $\varphi(t, t_0)$

$$h_0 = \frac{2 A_c}{U}$$

effective perimeter U of the cross-section

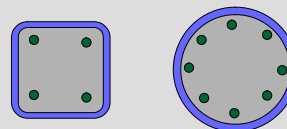


$$U = 2(b + h)$$

$$U \approx 2h + 0,5b$$

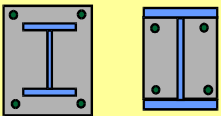
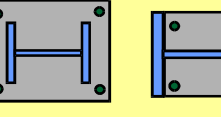
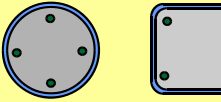
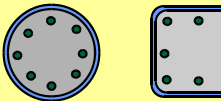
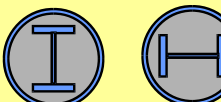
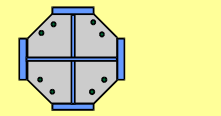
E_{cm}	Secant modulus of concrete
N_{Ed}	total design normal force
$N_{G,Ed}$	part of the total normal force that is permanent
$\varphi(t, t_0)$	creep coefficient as a function of the time at loading t_0 , the time t considered and the notional size of the cross-section

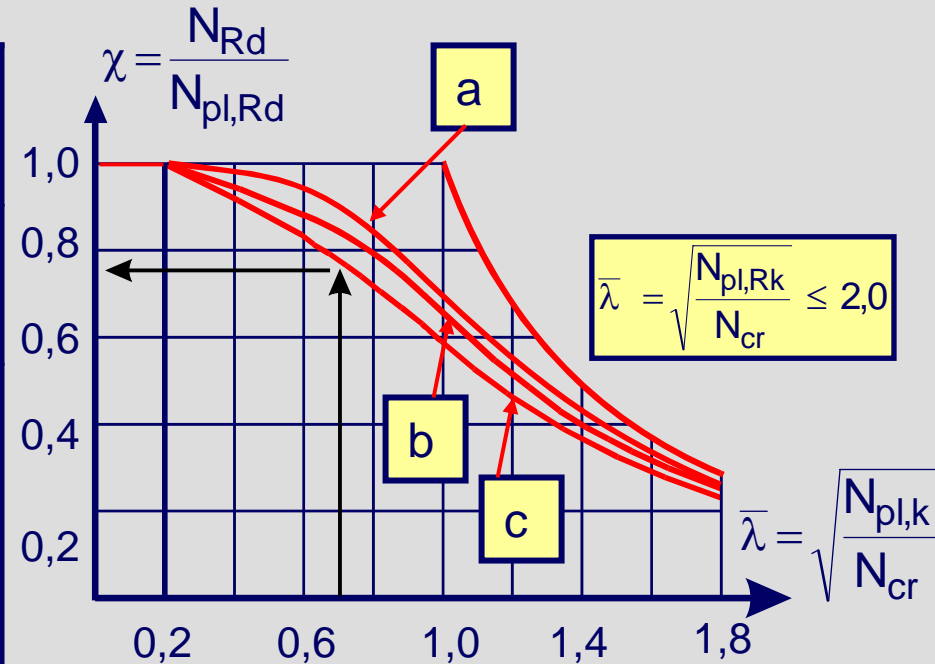
In case of concrete filled hollow section the drying of the concrete is significantly reduced by the steel section. A good estimation of the creep coefficient can be achieved, if 25% of that creep coefficient is used, which results from a cross-section, where the notional size h_0 is determined neglecting the steel hollow section.



$$\varphi_{t,eff} = 0,25 \varphi(t, t_0)$$

Verification for axial compression with the European buckling curves

cross-section		buckling curve
	buckling about strong axis $v = 0,85$	b
	buckling about weak axis $v = 0,85$	c
	$\rho_s \leq 3\%$ $v = 1,00$	a
	$3\% < \rho_s \leq 6\%$ $v = 1,00$	b
	$v = 1,00$	b
	$v = 0,85$	b



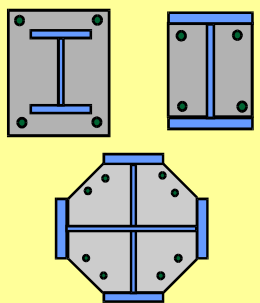
Verification: $\frac{N_{Ed}}{N_{Rd}} \leq 1,0$

Design value of resistance $N_{Rd} = \chi N_{pl,Rd}$

$N_{pl,Rd} = A_a f_{yd} + A_s f_{sd} + v A_c f_{cd}$

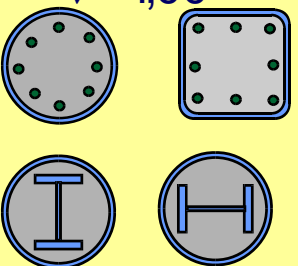
$$f_{cd} = \frac{f_{ck}}{\gamma_c}$$

$$v = 0,85$$



$$f_{cd} = \frac{f_{ck}}{\gamma_c}$$

$$v = 1,00$$



- relative slenderness:

$$\bar{\lambda} = \sqrt{\frac{N_{pl,Rk}}{N_{cr}}} \leq 2,0$$

- characteristic value of the plastic resistance to compressive forces

$$N_{pl,Rk} = A_a f_{yk} + A_c v f_{ck} + A_s f_{sk}$$

- elastic critical normal force

$$N_{cr} = \frac{\pi^2 (EJ)_{eff}}{(\beta L)^2}$$

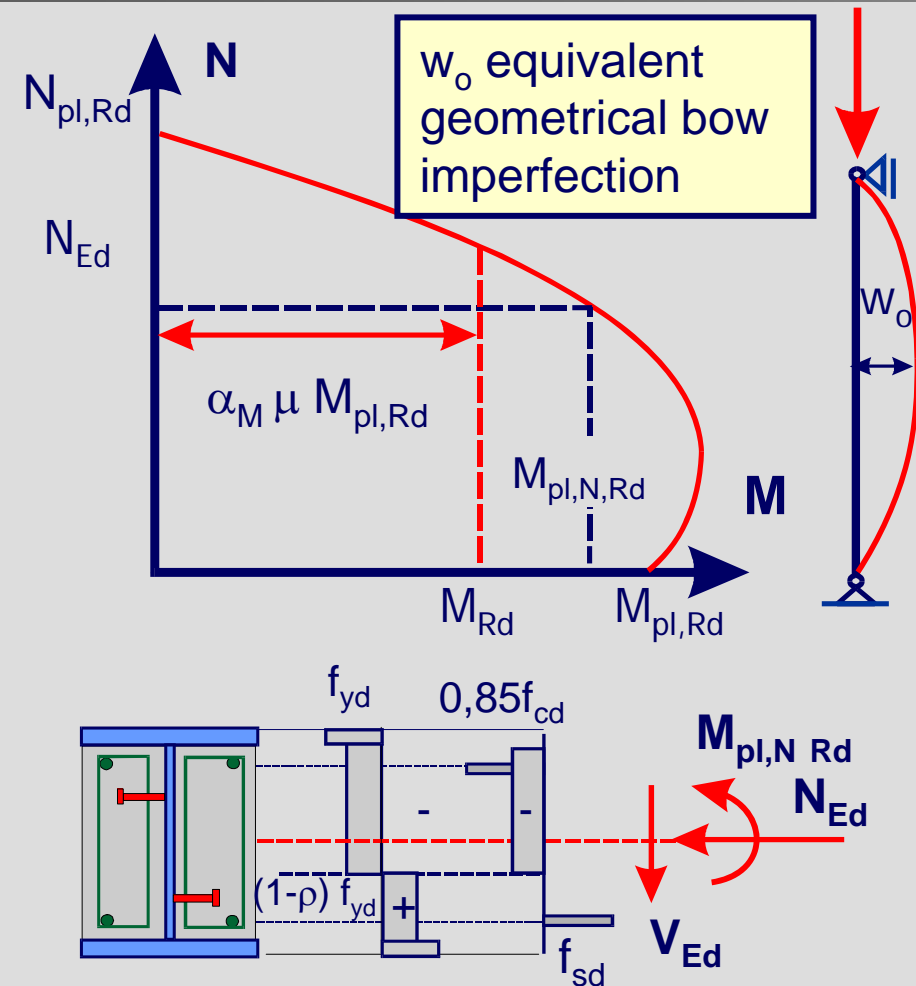
β - buckling length factor

- effective flexural stiffness

$$(EJ)_{eff} = (E_a J_a + K_e E_{c,eff} J_c + E_s J_s)$$

$$K_e = 0,6$$

Verification for combined compression and bending



The factor α_M takes into account the difference between the full plastic and the elasto-plastic resistance of the cross-section resulting from strain limitations for concrete.

Verification

$\max M_{Ed} \leq M_{Rd} = \alpha_M \mu M_{pl,Rd}$

$\alpha_M = 0,9$ for S235 and S355

$\alpha_M = 0,8$ for S420 and S460

bending moments taking into account second order effects:

$$\max M_{Ed} = N_{Ed} w_o \frac{1}{1 - \frac{N_{Ed}}{N_{cr}}}$$

$$N_{cr} = \frac{\pi^2 (E J)_{eff,II}}{\beta^2 L^2}$$

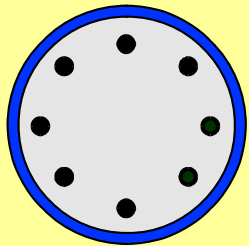
Effective flexural stiffness

$$(EI)_{eff,II} = K_o (E_a J_a + K_e E_{c,eff} J_c + E_s J_s)$$

with $K_{e,II} = 0,5$ $K_o = 0,9$

Buckling curve

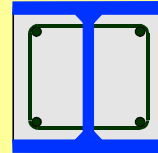
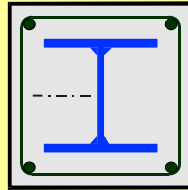
a



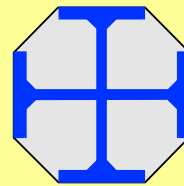
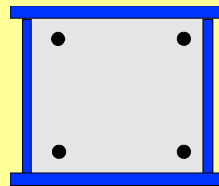
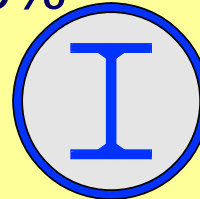
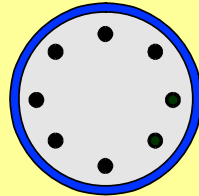
$$\rho_s \leq 3\%$$

$$w_o = L/300$$

b



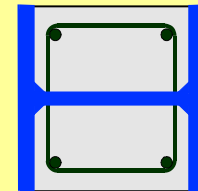
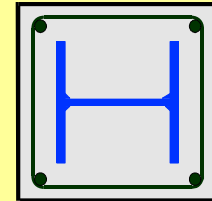
$$3\% < \rho_s \leq 6\%$$



Member imperfection

$$w_o = L/200$$

c



$$w_o = L/150$$

sway imperfection

Global initial sway imperfection acc. to EN 1993-1-1:

$$\phi = \phi_o \alpha_m \alpha_h$$

Φ_o basic value with $\Phi_o = 1/200$

α_h reduction factor for the height h in [m]

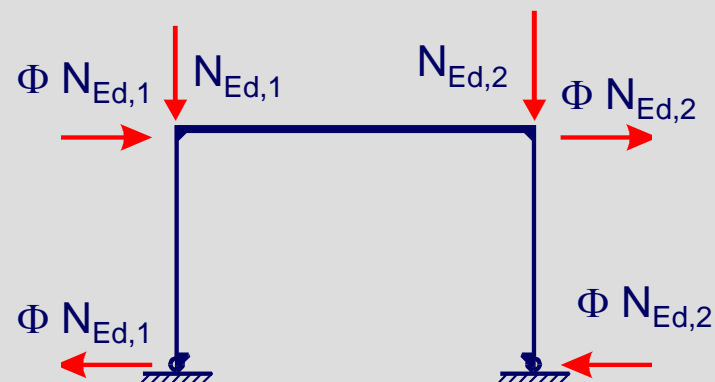
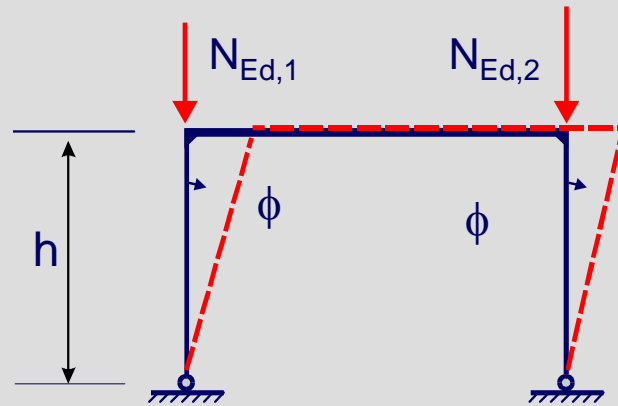
$$\alpha_h = \frac{2}{\sqrt{h}} \text{ but } \frac{2}{3} \leq \alpha_h \leq 1,0$$

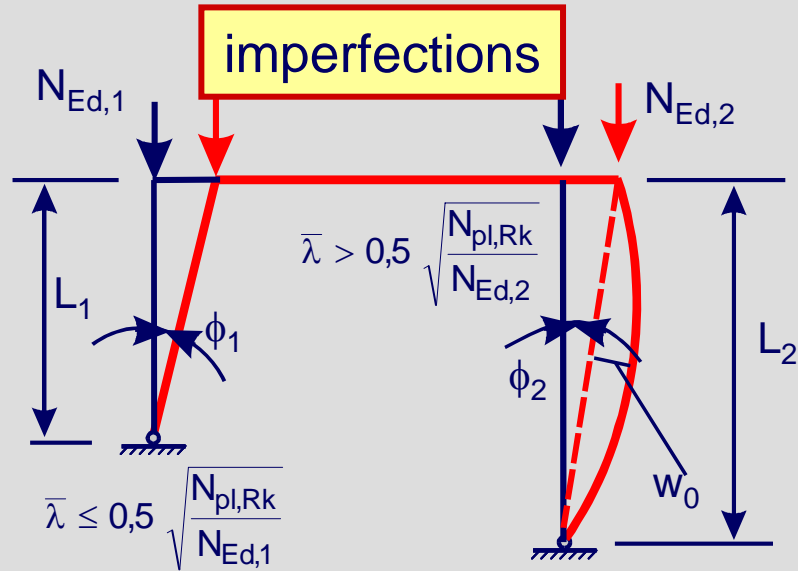
α_m reduction factor for the number of columns in a row

equivalent forces

$$\alpha_m = \sqrt{0,5 \left[1 + \frac{1}{m} \right]}$$

m is the number of columns in a row including only those columns which carry a vertical load N_{Ed} not less than 50% of the average value of the column in a vertical plane considered.





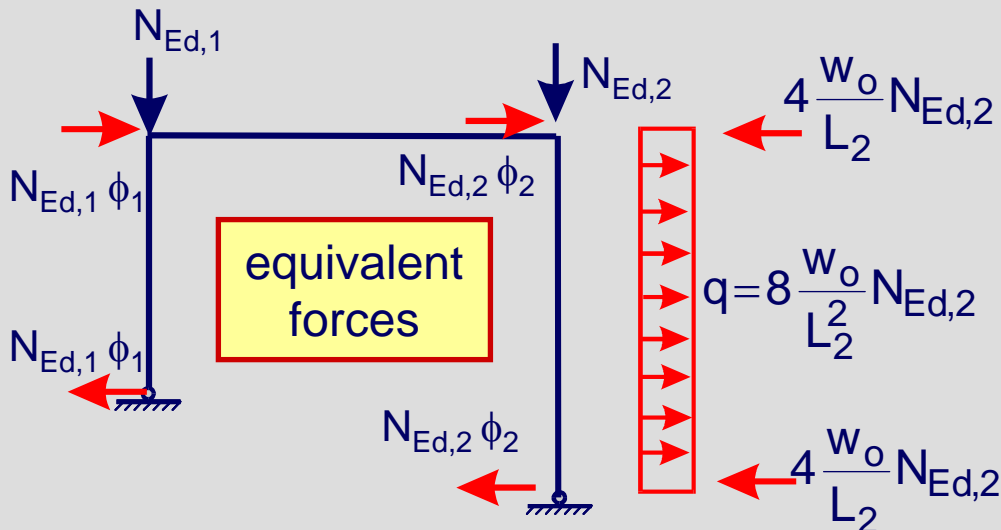
Within a global analysis, member imperfections in composite compression members may be neglected where first-order analysis may be used. Where second-order analysis should be used, member imperfections may be neglected within the global analysis if:

$$\bar{\lambda} \leq 0,5 \sqrt{\frac{N_{pl,Rk}}{N_{Ed,i}}}$$

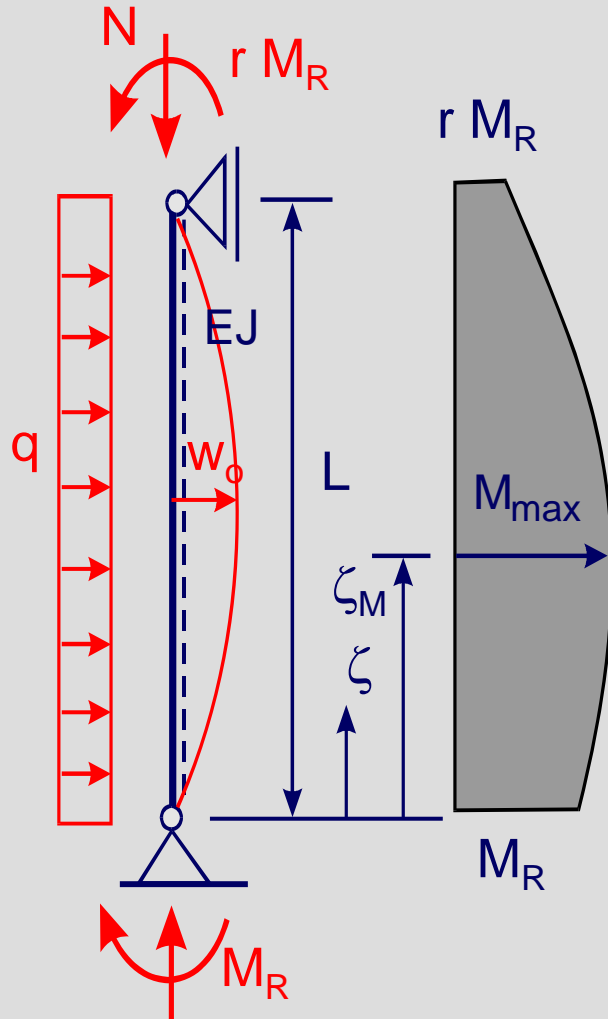
$$\bar{\lambda} = \sqrt{\frac{N_{pl,Rk}}{N_{cr}}}$$

$$N_{cr} = \frac{\pi^2 (EJ)_{eff}}{L_i^2}$$

$$(EJ)_{eff} = (E_a J_a + 0,6 E_{c,eff} J_c + E_s J_s)$$



Bending moments including second order effects:



$$M(\xi) = M_R \left(\frac{r \sin \varepsilon (1 - \xi) + \sin \varepsilon \xi}{\sin \varepsilon} \right) + \bar{M}_0 \left(\frac{\cos \varepsilon (0,5 - \xi)}{\cos (\varepsilon / 2)} - 1 \right)$$

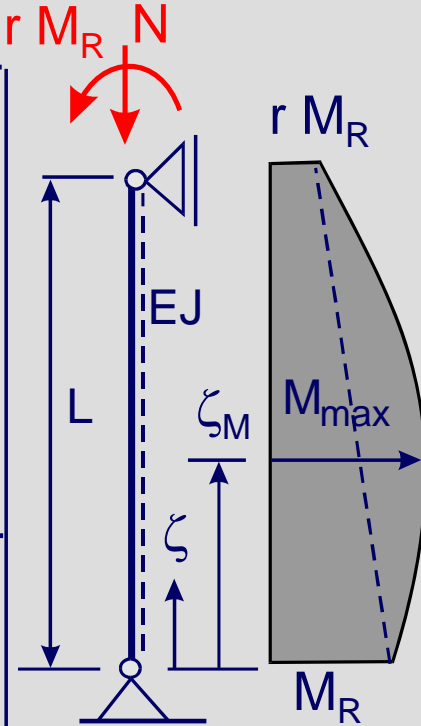
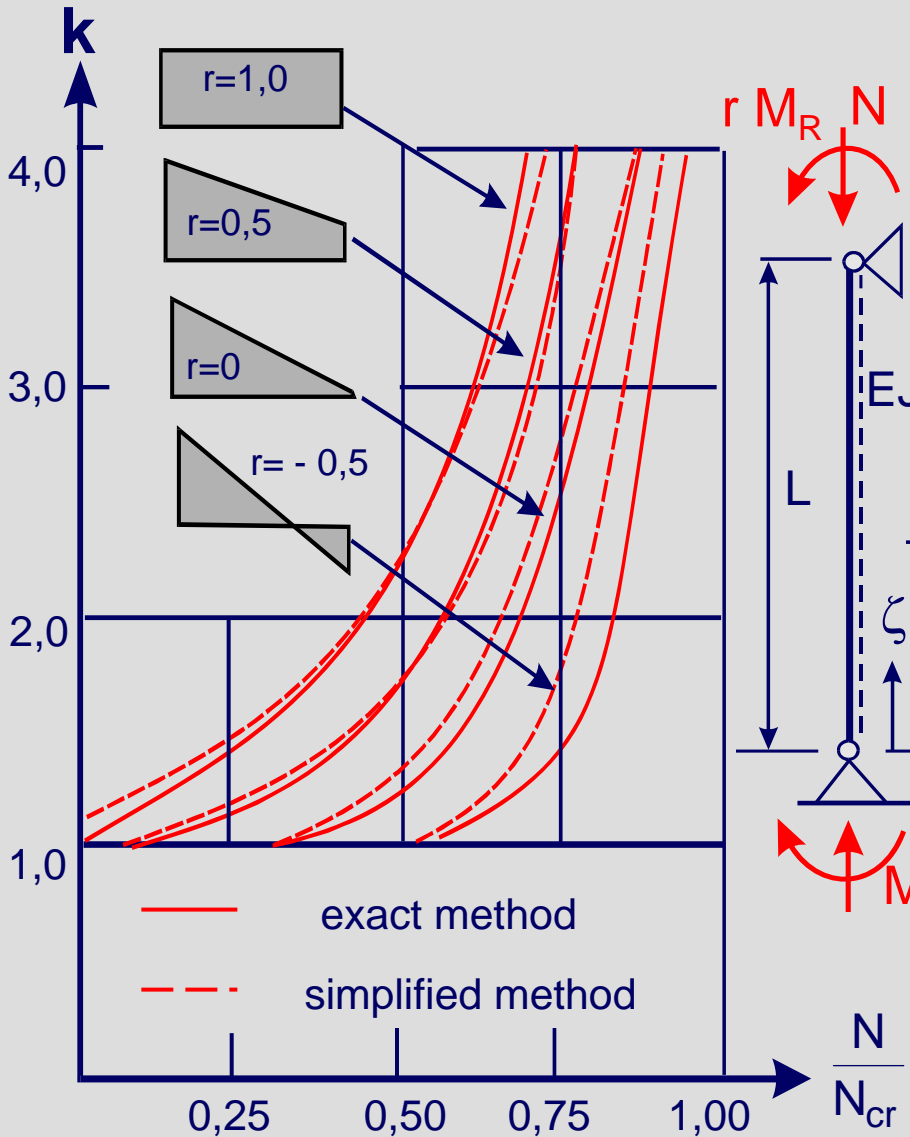
$$V_z(\xi) = \frac{M_R \varepsilon}{L} \left(\frac{r \cos \varepsilon (1 - \xi) + \cos \varepsilon \xi}{\sin \varepsilon} \right) + \bar{M}_0 \left(\frac{\sin \varepsilon (0,5 - \xi)}{\cos (\varepsilon / 2)} - 1 \right)$$

$$\bar{M}_0 = (q L^2 + 8 N w_0) \frac{1}{\varepsilon^2} \quad \varepsilon = L \sqrt{\frac{|N_{Ed}|}{(E J)_{eff,II}}}$$

Maximum bending moment at the point ξ_M : $\left(\frac{dM}{d\xi} = 0 \right)$

$$M_{max} = [0,5 M (1 + r) + M_0] \frac{\sqrt{1 + c^2}}{\cos(0,5 \varepsilon)} - M_0$$

$$c = \frac{M (r - 1)}{M (1 + r) + 2 M_0} \quad \frac{1}{\tan(0,5 \varepsilon)} \quad \xi_M = 0,5 + \frac{\arctan c}{\varepsilon}$$



Exact solution:

$$M_{\max} = 0,5 M_R (1+r) \frac{\sqrt{1+c^2}}{\cos(0,5\varepsilon)}$$

$$c = \frac{r-1}{1+r} \frac{1}{\tan(0,5\varepsilon)}$$

$$\xi_M = 0,5 + \frac{\arctan c}{\varepsilon} \quad \varepsilon = L \sqrt{\frac{|N_{Ed}|}{(EJ)_{\text{eff,II}}}}$$

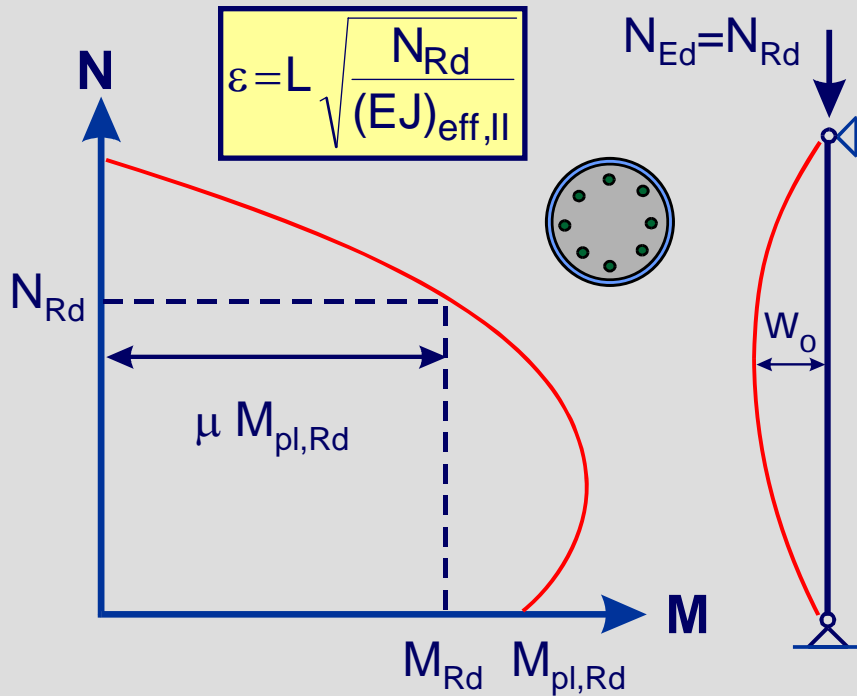
simplified solution:

$$k = \frac{M_{\max}}{M_R} = \frac{\beta}{1 - \frac{N_{Ed}}{N_{cr}}}$$

$$\beta = 0,66 + 0,44 r$$

$$\beta \geq 0,44$$

Background of the member imperfections



The initial bow imperfections were recalculated from the resistance to compression calculated with the European buckling curves.

Bending moment based on second order analysis:

$$M = \frac{8 w_0 (EJ)_{\text{eff,II}}}{L^2} \left[\frac{1}{\cos(\varepsilon/2)} - 1 \right]$$

Resistance to axial compression based on the European buckling curves:

$$N_{Rd} = \chi N_{pl,Rd}$$

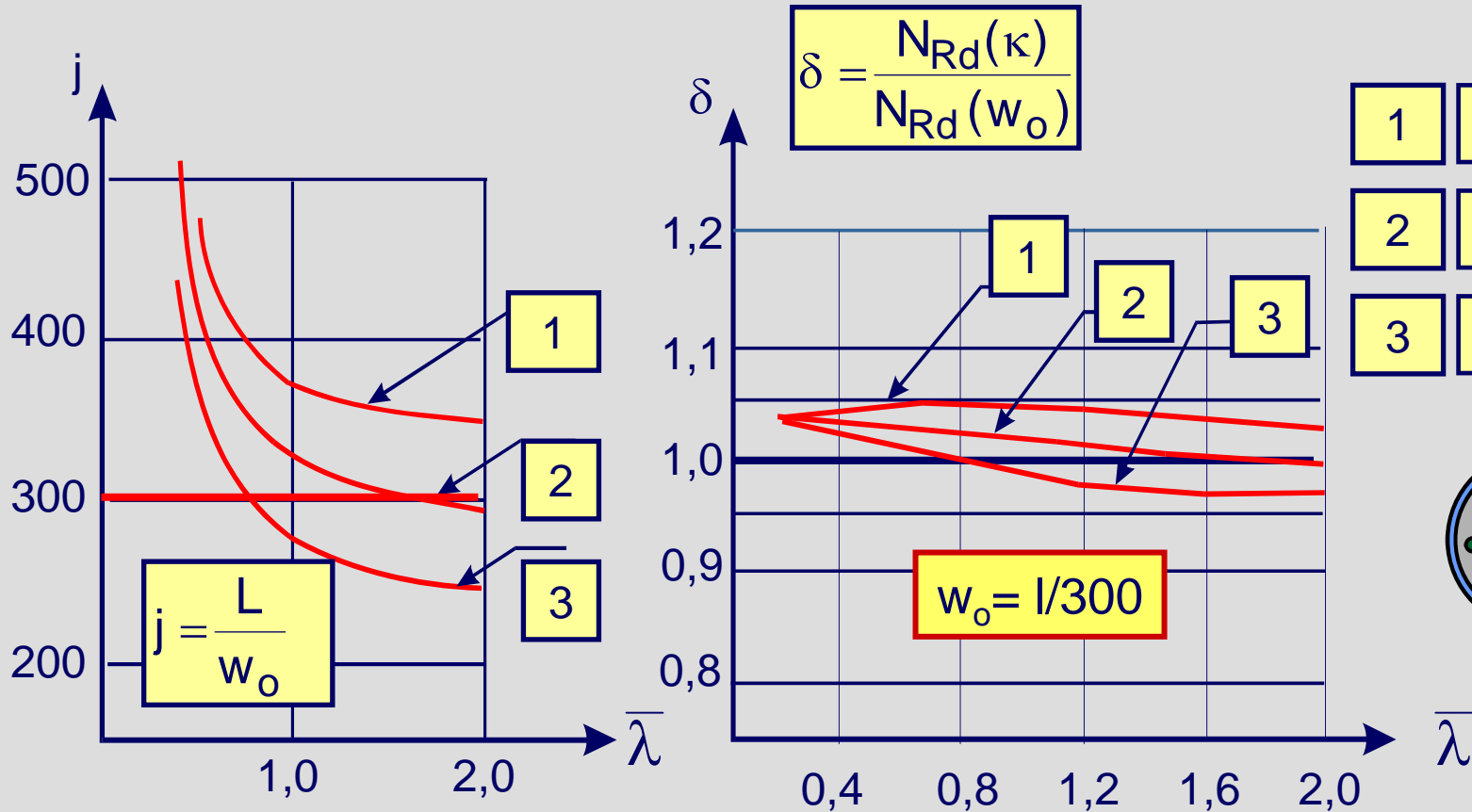
Bending resistance:

$$M_{Rd} = \alpha_M \mu M_{pl,Rd}$$

Determination of the equivalent bow imperfection:

$$w_0 = \frac{\alpha_M \mu_d M_{pl,Rd} L^2}{8 (EJ)_{\text{eff,II}} \left[\frac{1}{1 - \cos(\varepsilon/2)} - 1 \right]}$$

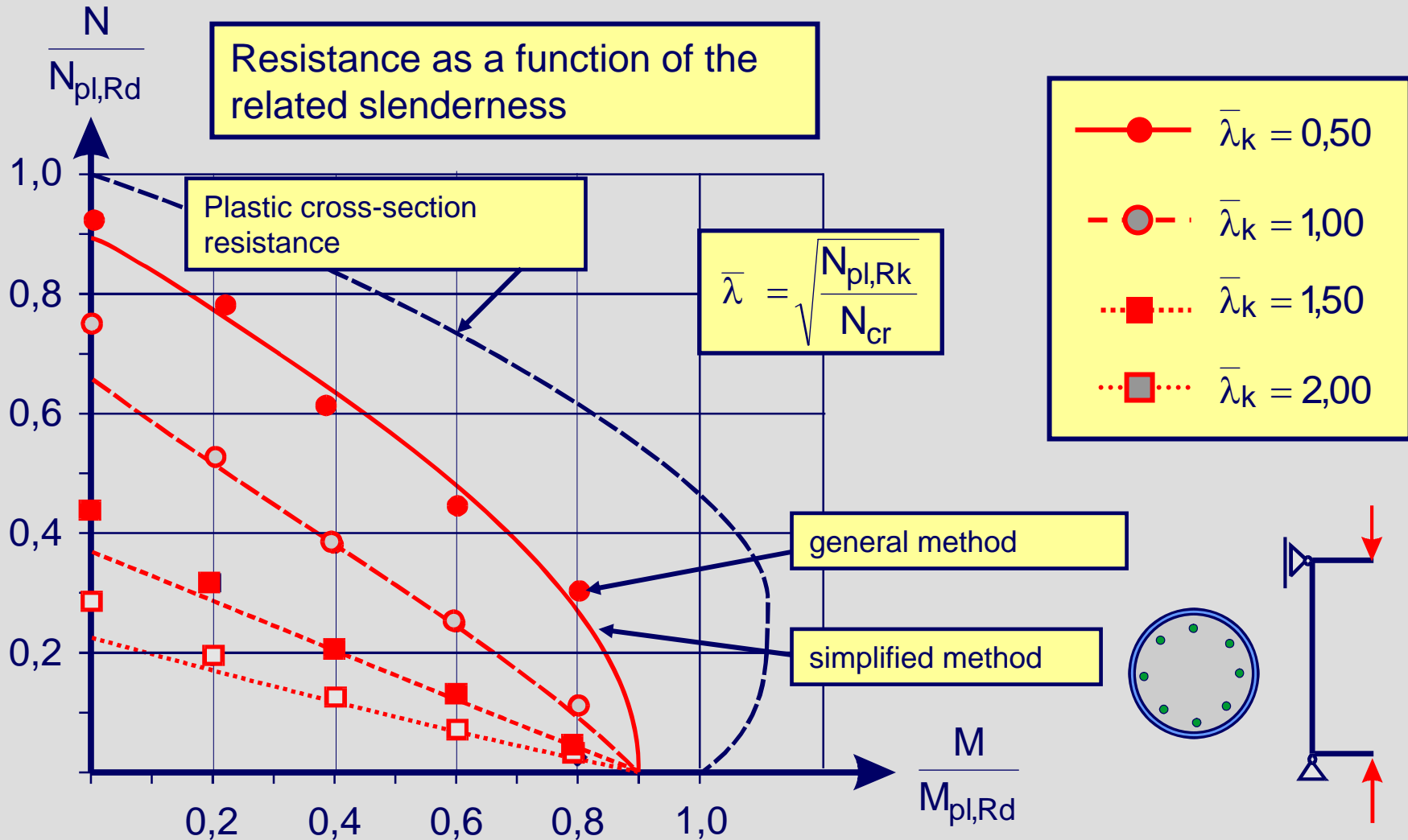
Geometrical bow imperfections – comparison with European buckling curves for axial compression



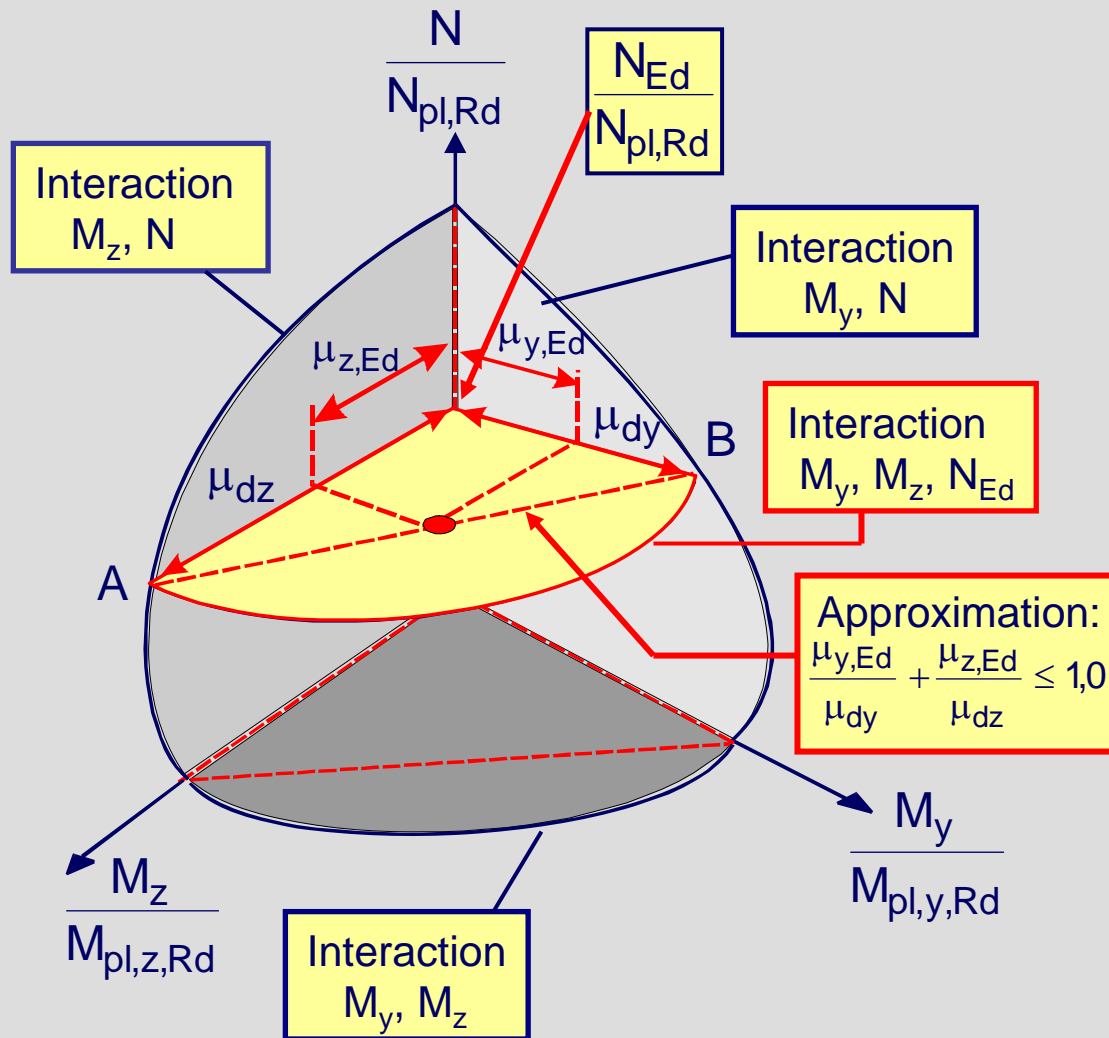
The initial bow imperfection is a function of the related slenderness and the resistance of cross-sections. In Eurocode 4 constant values for w_0 are used.

The use of constant values for w_0 leads to maximum differences of 5% in comparison with the calculation based on the European buckling curves.

Comparison of the simplified method with non-linear calculations for combined compression and bending



Resistance to combined compression and biaxial bending



The resistance is given by a three-dimensional interaction relation. For simplification a linear interaction between the points A and B is used.

$$M_{y,Rd}(N_{Ed}) = \mu_{dy} M_{pl,y,Rd}$$

$$M_{z,Rd}(N_{Ed}) = \mu_{dz} M_{pl,z,Rd}$$

$$M_{y,Ed} = \mu_{y,Ed} M_{y,Rd}$$

$$M_{z,Ed} = \mu_{z,Ed} M_{z,Rd}$$

approximation for the interaction curve:

$$\frac{M_{y,Ed}}{\mu_{dy} M_{pl,y,Rd}} + \frac{M_{z,Ed}}{\mu_{dz} M_{pl,z,Rd}} \leq 1,0$$

$$\frac{M_{y,Ed}}{\mu_{dy} M_{pl,y,Rd}} + \frac{M_{z,Ed}}{\mu_{dz} M_{pl,z,Rd}} \leq 1,0$$

Verification in case of compression and biaxial bending

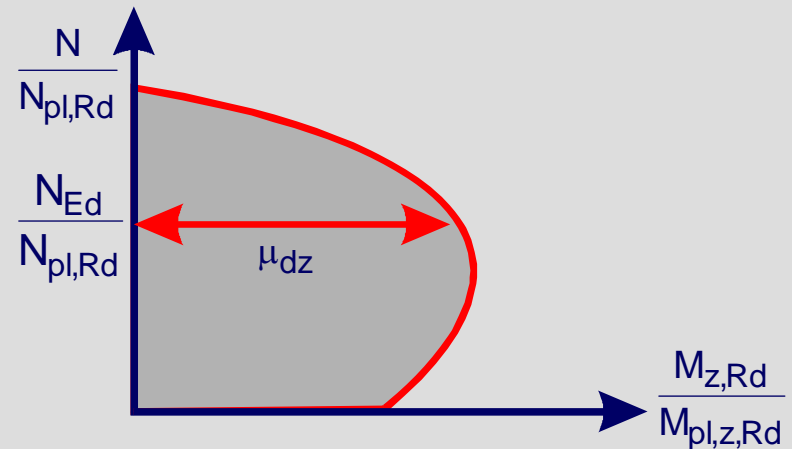
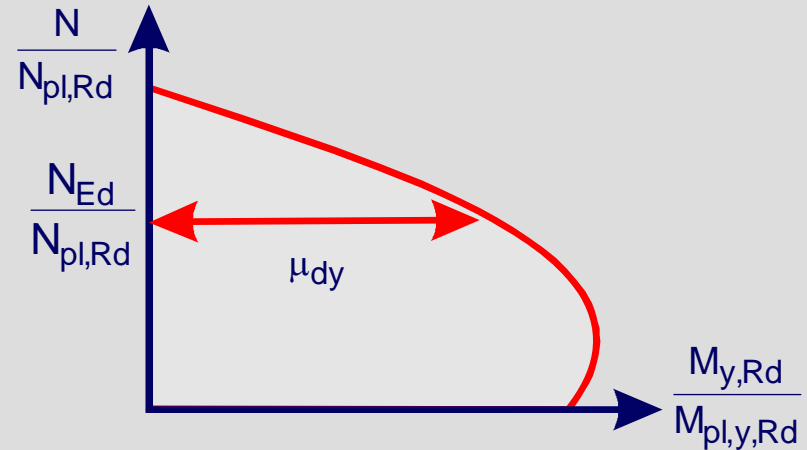
For both axis a separate verification is necessary.

$$\frac{M_{y,Ed}}{\mu_{dy} M_{pl,y,Rd}} \leq \alpha_M \quad \frac{M_{z,Ed}}{\mu_{dz} M_{pl,y,Rd}} \leq \alpha_M$$

Verification for the interaction of biaxial bending.

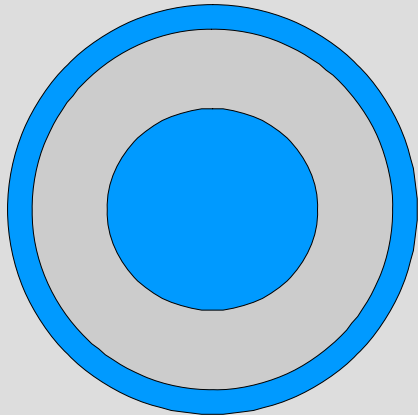
$$\frac{M_{y,Ed}}{\mu_{dy} M_{pl,y,Rd}} + \frac{M_{z,Ed}}{\mu_{dz} M_{pl,y,Rd}} \leq 1,0$$

Imperfections should be considered only in the plane in which failure is expected to occur. If it is not evident which plane is the most critical, checks should be made for both planes.



$$\alpha_M = 0,9 \text{ for S235 and S355}$$

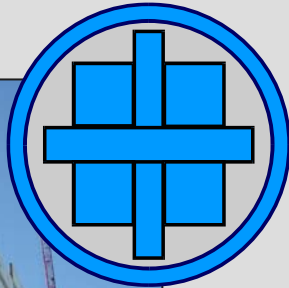
$$\alpha_M = 0,8 \text{ for S420 and S460}$$



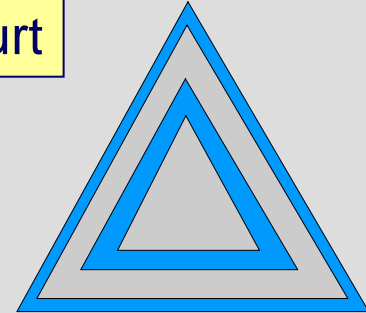
Part 5:

Special aspects of columns with inner core profiles

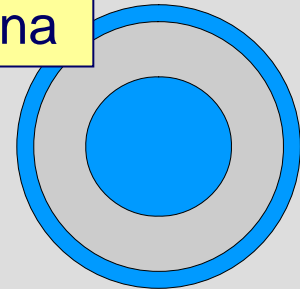
Composite columns – General Method



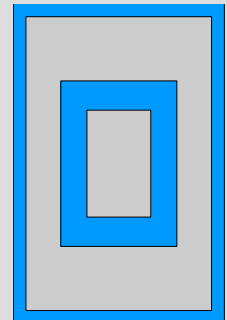
Commerzbank Frankfurt



Millennium Tower Vienna



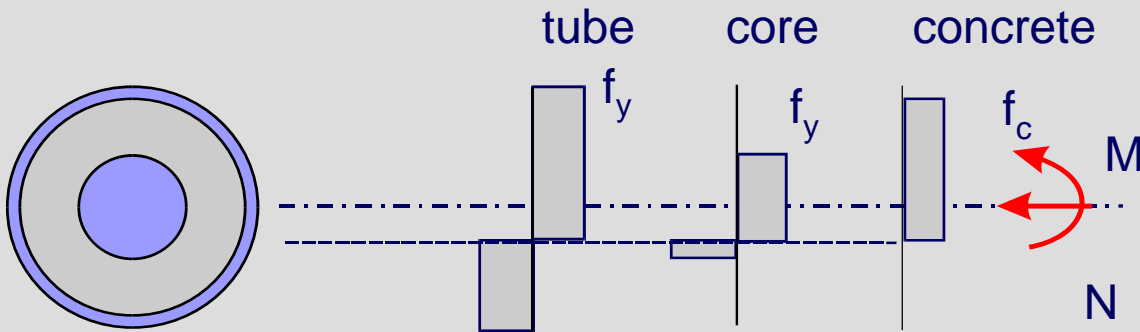
New railway station in Berlin
(Lehrter Bahnhof)



Highlight Center
Munich

Composite columns with concrete filled tubes and steel cores – special effects

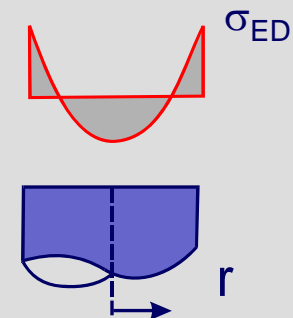
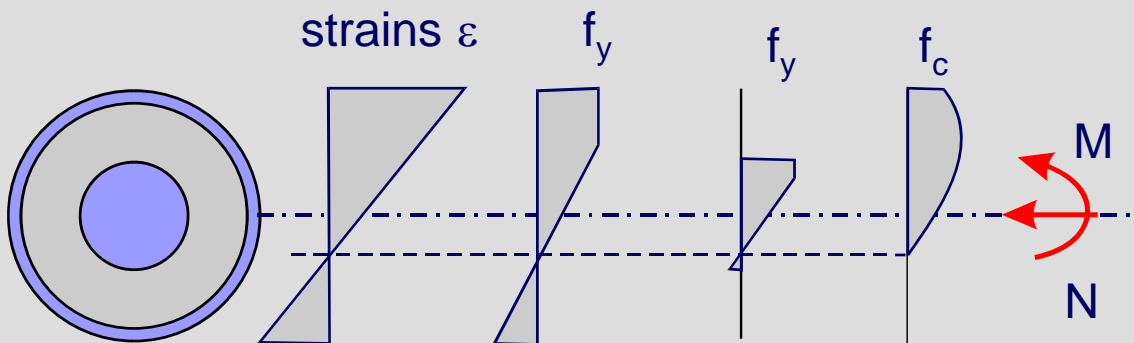
Resistance based on stress blocks (plastic resistance)



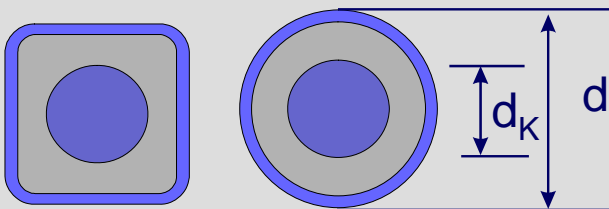
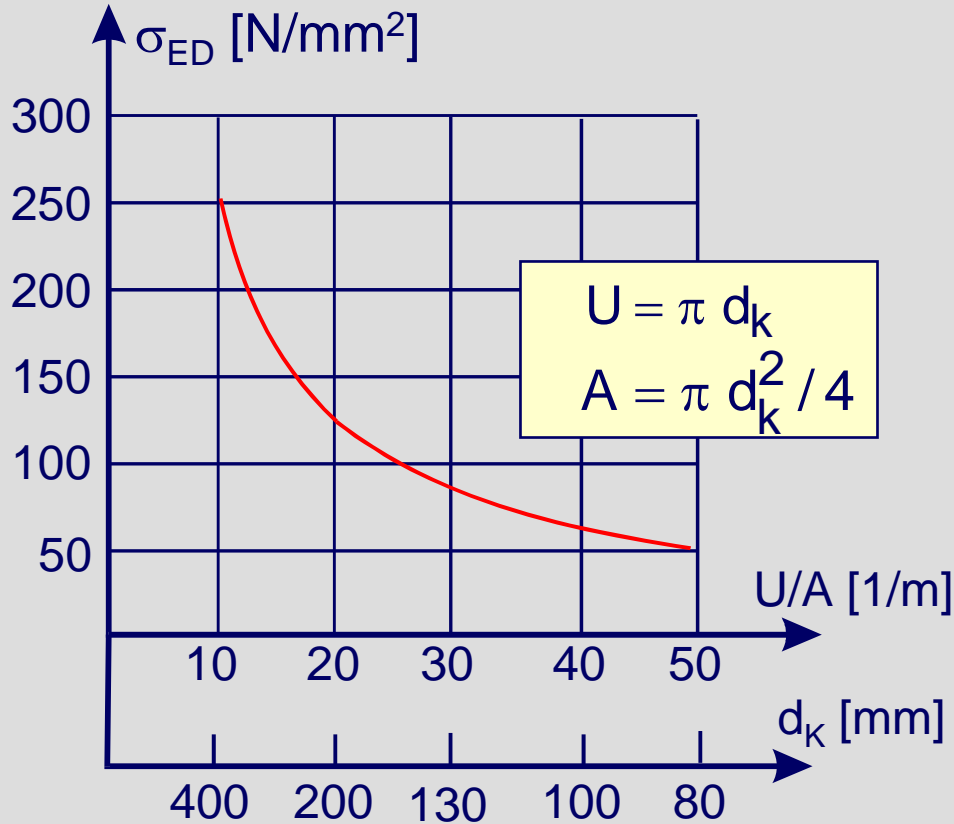
$$\alpha_M = \frac{M_{Rd}}{\mu M_{pl,Rd}}$$

Cross-sections with massive inner cores have a very high plastic shape factor and the cores can have very high residual stresses. Therefore these columns can not be design with the simplified method according to EN 1944-1-1.

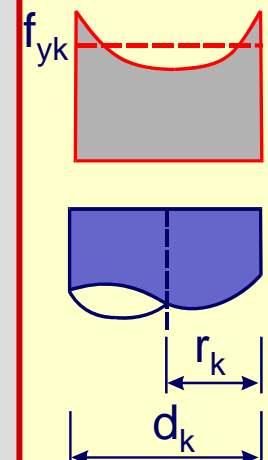
Non linear resistance with strain limitation for concrete



Residual stresses and distribution of the yield strength



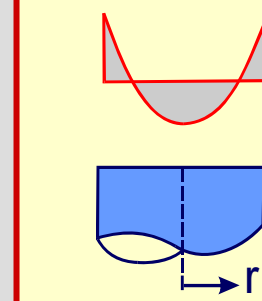
distribution of yield strength



f_{yk} – characteristic value of the yield strength

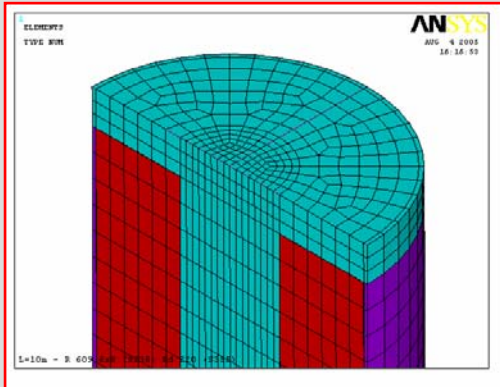
$$\frac{f_y(r)}{f_{yk}} = 0,95 + 0,1 \left(\frac{r}{r_k} \right)^2$$

residual stresses:

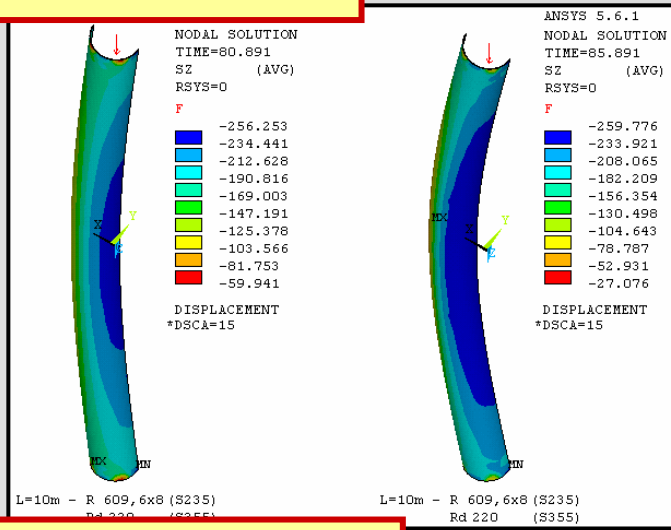


$$\sigma_E(r) = \sigma_{ED} \left[1 - \frac{2r^2}{r_k^2} \right]$$

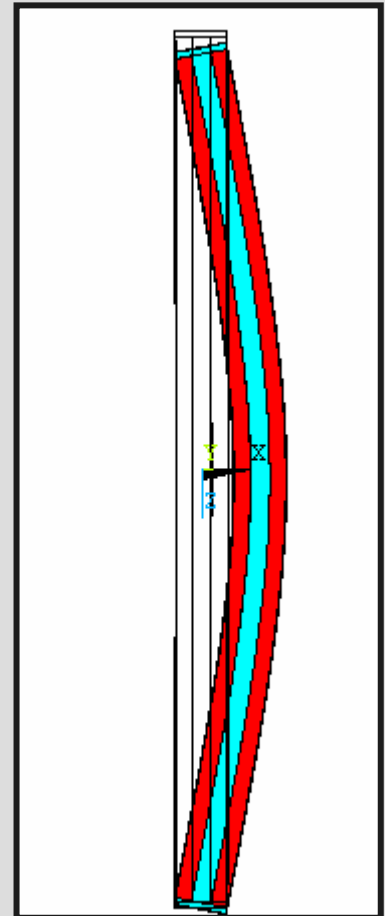
load introduction



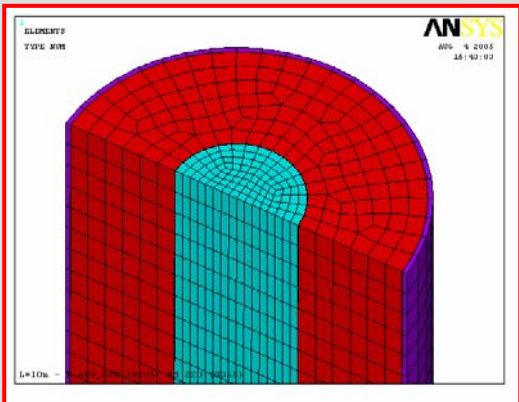
stresses in the tube



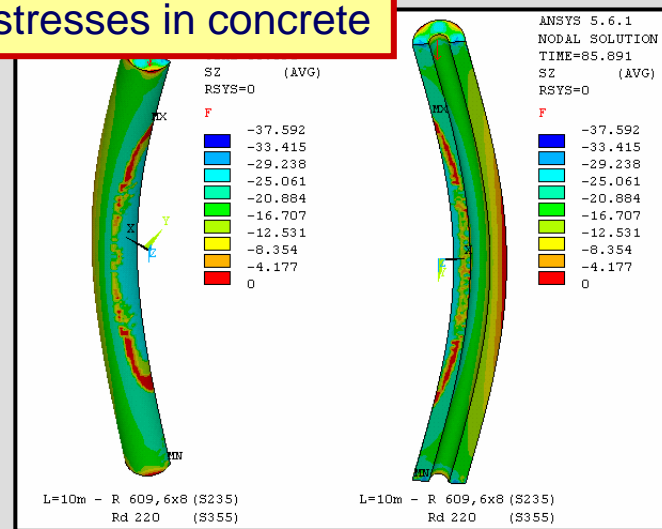
initial bow imperfection

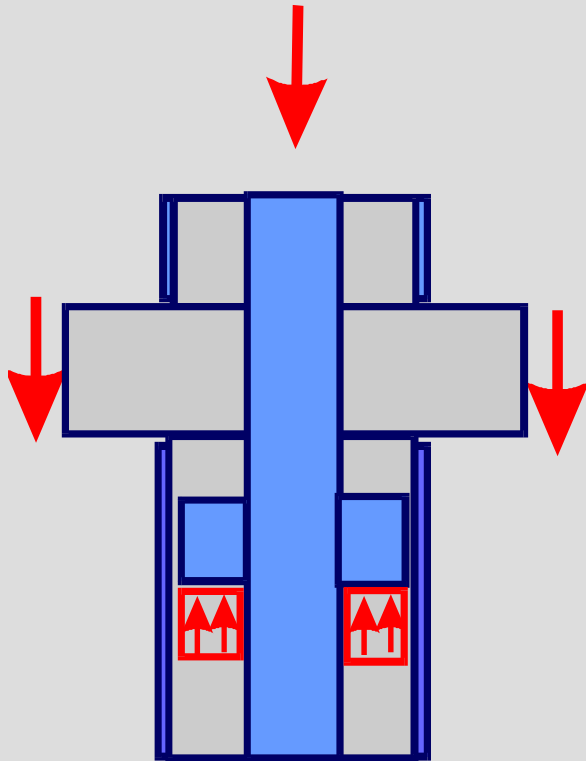


cross-section

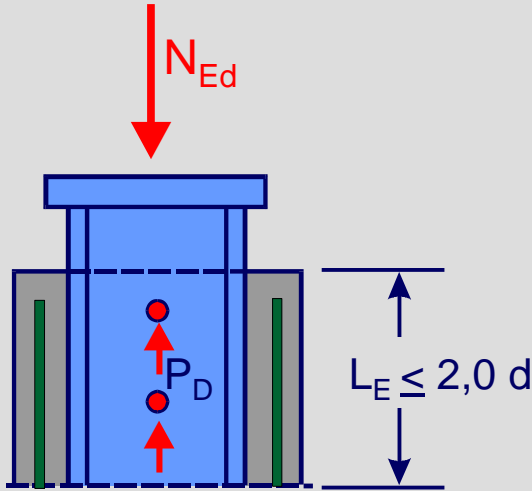


stresses in concrete





Part 6: Load introduction and longitudinal shear



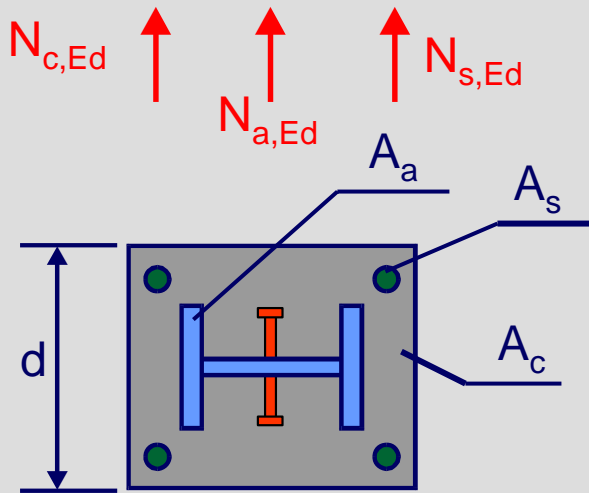
load introduction by headed studs within the load introduction length L_E

$$L_E \leq \begin{cases} 2d \\ L/3 \end{cases}$$

d minimum transverse dimension of the cross-section
 L member length of the column

sectional forces of the cross-section :

$$N_{a,Ed} = N_{Ed} \frac{N_{pl,a}}{N_{pl,Rd}} \quad N_{s,Ed} = N_{Ed} \frac{N_{pl,s}}{N_{pl,Rd}} \quad N_{c,Ed} = N_{Ed} \frac{N_{pl,c}}{N_{pl,Rd}}$$



required number of studs n resulting from the sectional forces $N_{Ed,c} + N_{Ed,s}$:

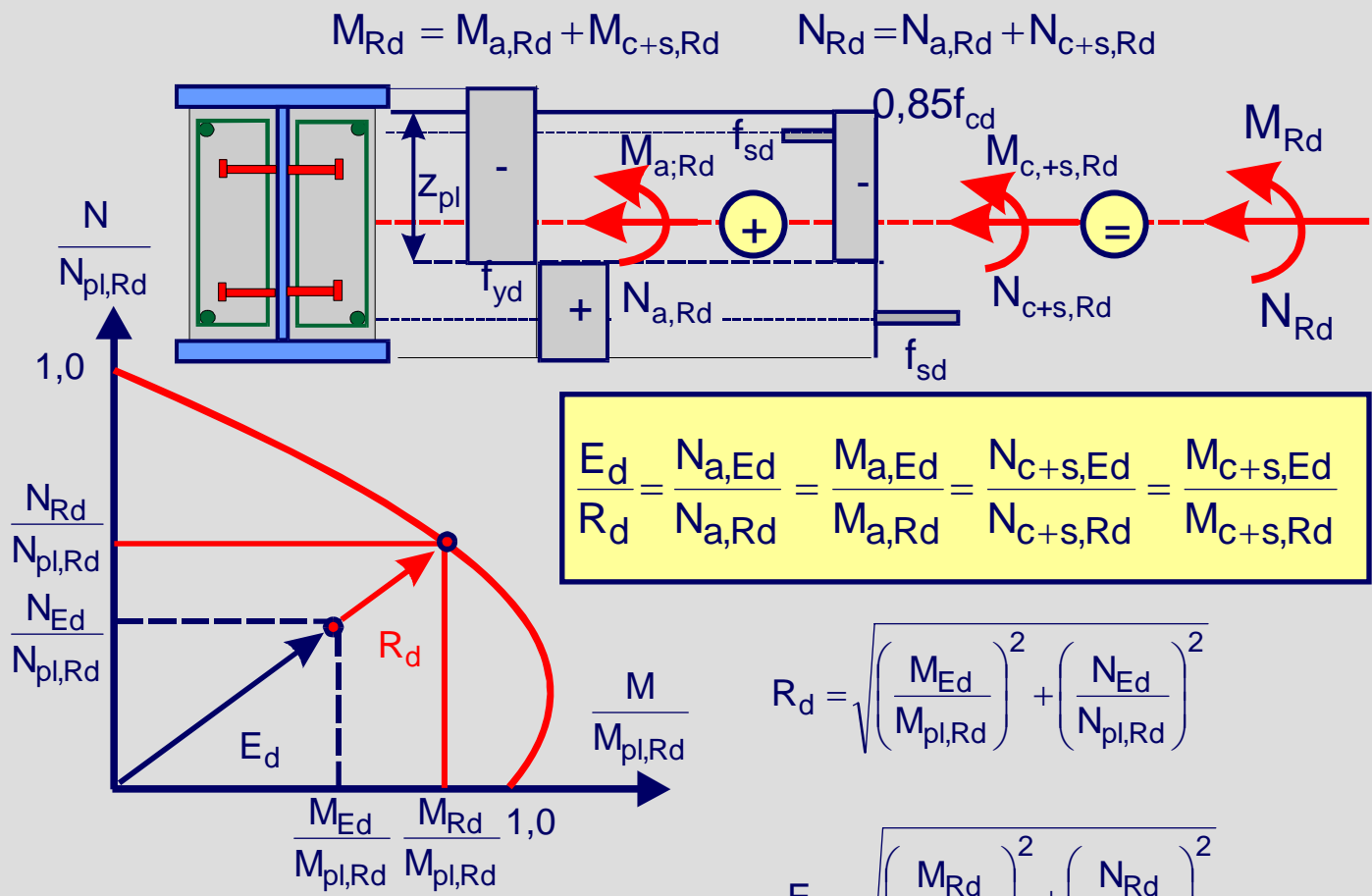
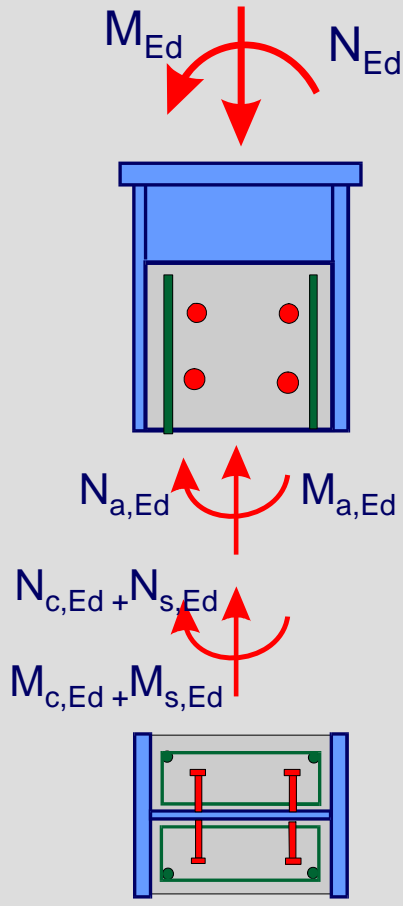
$$V_{L,Ed} = N_{c,Ed} + N_{s,Ed} = N_{Ed} \left[1 - \frac{N_{pl,a}}{N_{pl,Rd}} \right] \quad V_{L,Rd} = n P_{Rd}$$

P_{Rd} – design resistance of studs

Load introduction for combined compression and bending

sectional forces due to N_{Ed} und M_{Ed}

sectional forces based on plastic theory

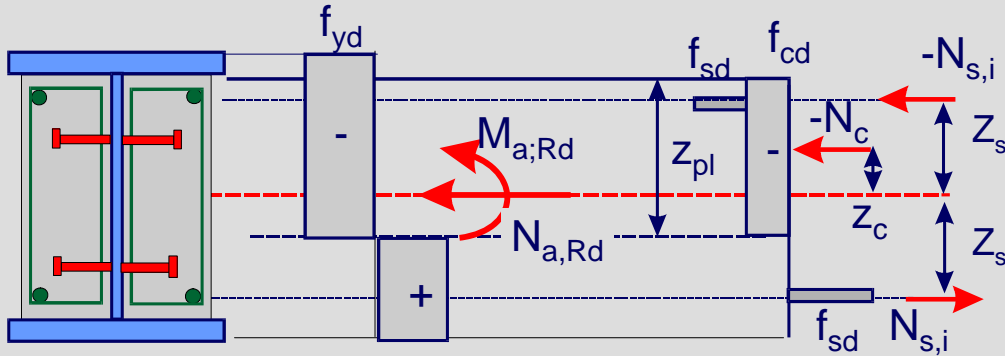


$$\frac{E_d}{R_d} = \frac{N_{a,Ed}}{N_{a,Rd}} = \frac{M_{a,Ed}}{M_{a,Rd}} = \frac{N_{c+s,Ed}}{N_{c+s,Rd}} = \frac{M_{c+s,Ed}}{M_{c+s,Rd}}$$

$$R_d = \sqrt{\left(\frac{M_{Ed}}{M_{pl,Rd}}\right)^2 + \left(\frac{N_{Ed}}{N_{pl,Rd}}\right)^2}$$

$$E_d = \sqrt{\left(\frac{M_{Rd}}{M_{pl,Rd}}\right)^2 + \left(\frac{N_{Rd}}{N_{pl,Rd}}\right)^2}$$

sectional forces based on stress blocks:



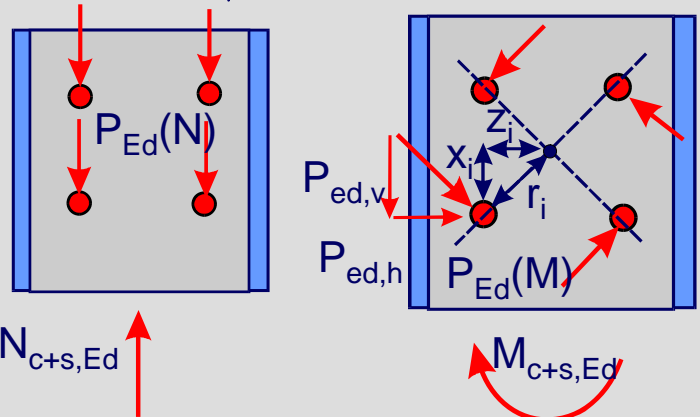
$$N_{C+s,Rd} = N_C + \sum N_{Si}$$

$$M_{C+s,Rd} = N_C z_C + \sum N_{Si} z_{Si}$$

shear forces of studs based on elastic theory

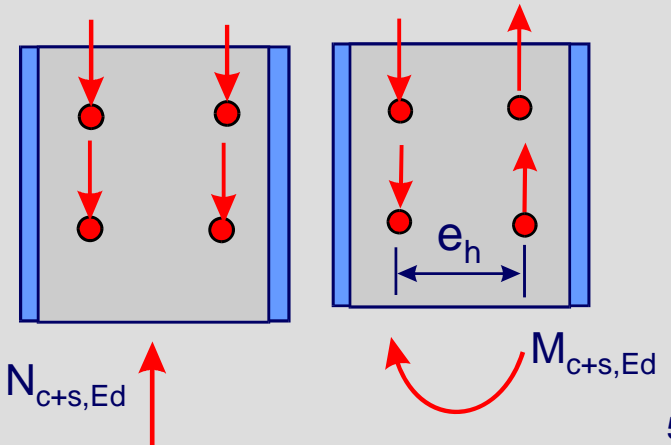
shear forces of studs based on plastic theory

$$\max P_{Ed} = \sqrt{\left[\frac{N_{C+s,Ed}}{n} + \frac{M_{C+s,Ed}}{\sum r_i^2} x_i \right]^2 + \left[\frac{M_{C+s,Ed}}{\sum r_i^2} z_i \right]^2}$$

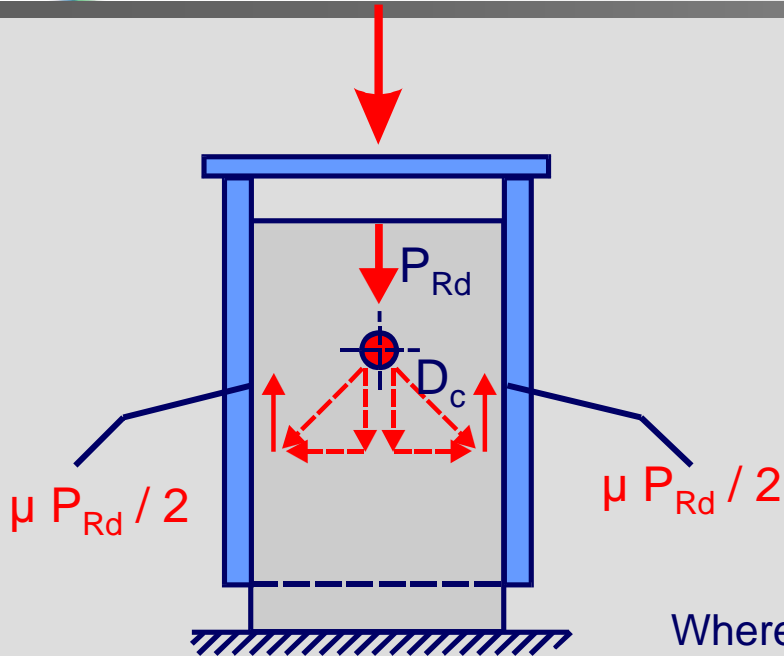


n – number of studs within the load introduction length

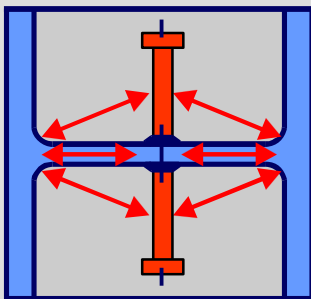
$$\max P_{Ed} = \frac{N_{C+s,Ed}}{n} + \frac{M_{C+s,Ed}}{e_h 0,5n}$$



Shear resistance of stud connectors welded to the web of partially encased I-Sections



$$V_{L,Rd} = P_{Rd} + V_{LR,Rd}$$

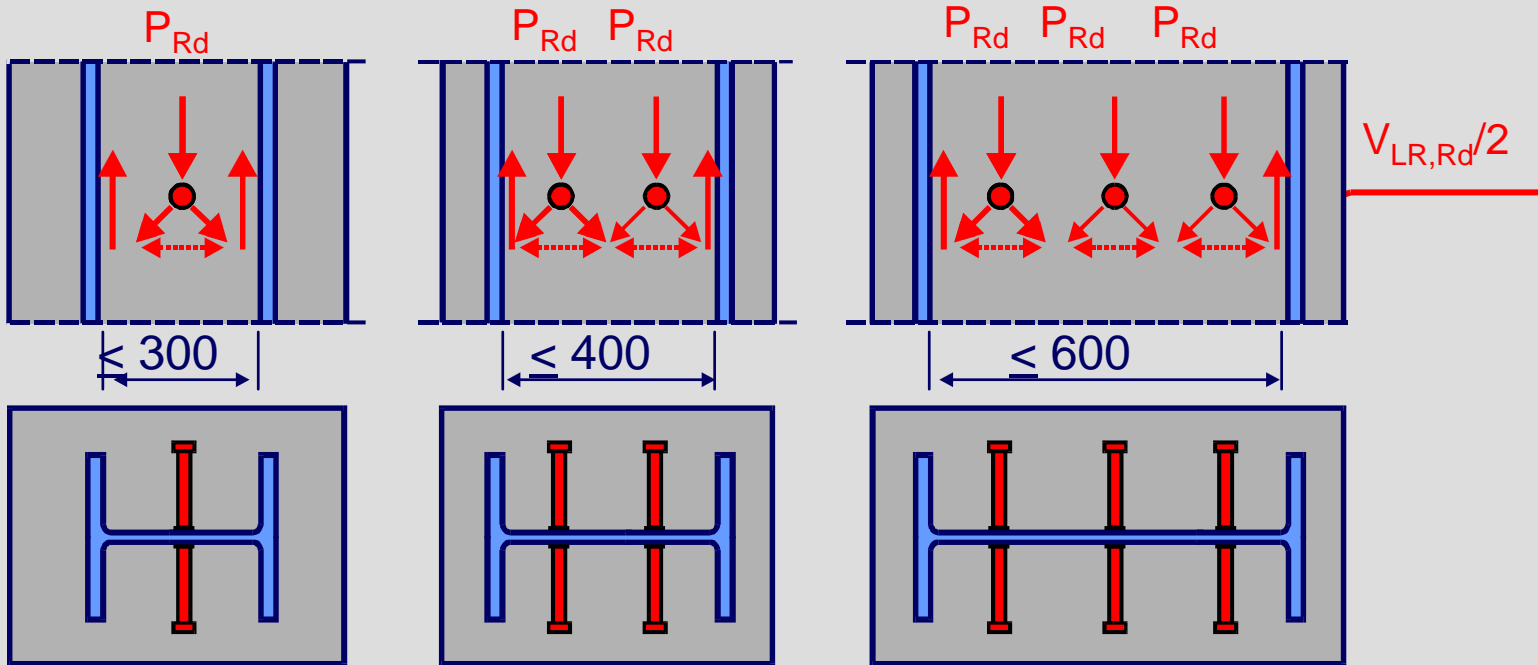


$$V_{LR,Rd} = \mu P_{Rd}$$



Where stud connectors are attached to the web of a fully or partially concrete encased steel I-section or a similar section, account may be taken of the frictional forces that develop from the prevention of lateral expansion of the concrete by the adjacent steel flanges. This resistance may be added to the calculated resistance of the shear connectors. The additional resistance may be assumed to be on each flange and each horizontal row of studs, where μ is the relevant coefficient of friction that may be assumed. For steel sections without painting, μ may be taken as 0,5. P_{Rd} is the resistance of a single stud.

Shear resistance of stud connectors welded to the web of partially encased I-Sections

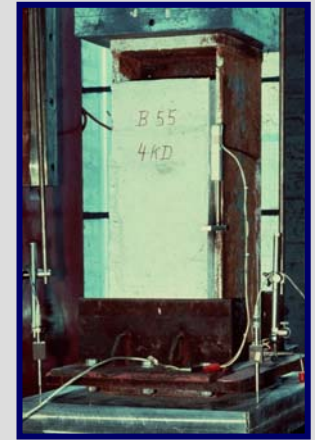
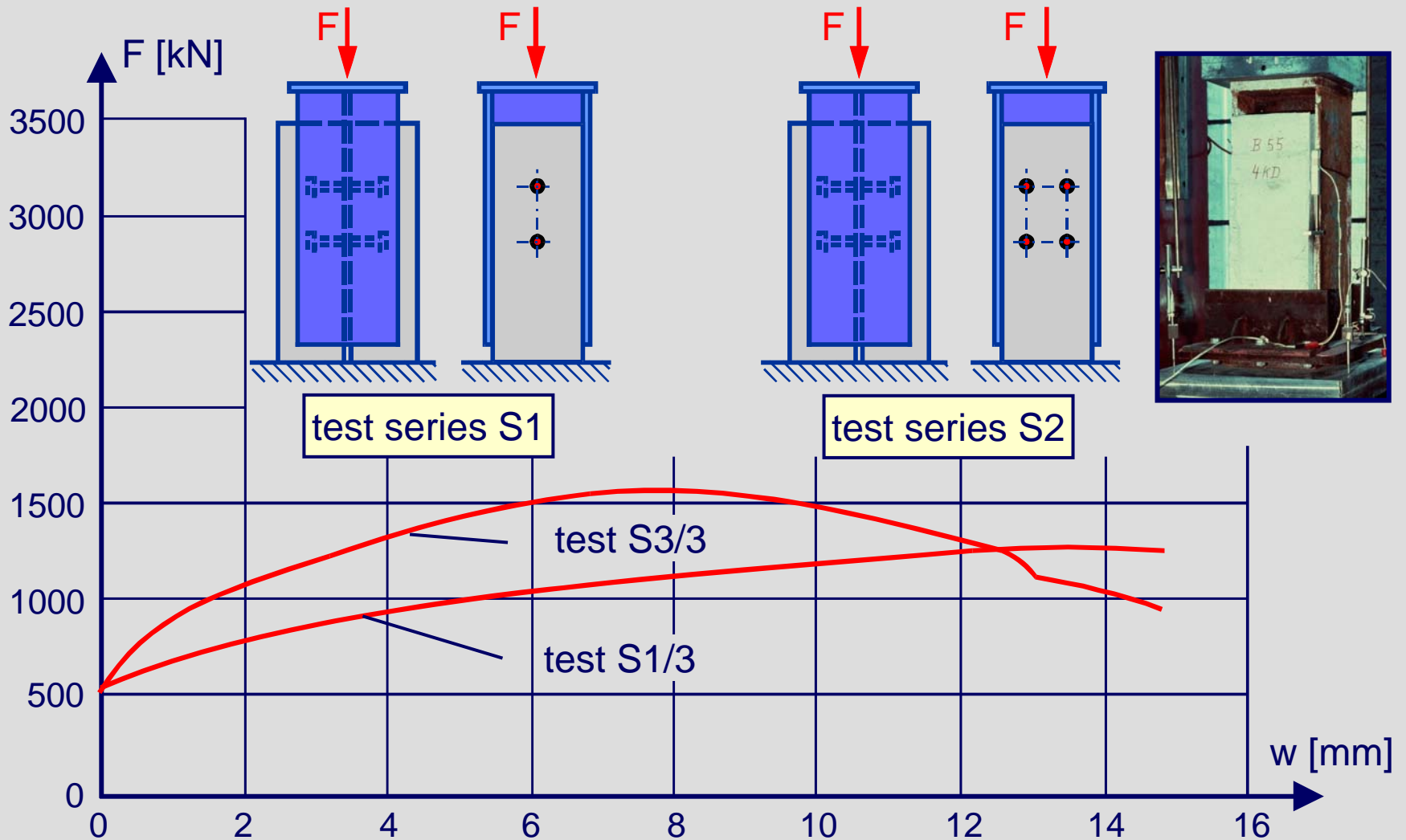


$$V_{L,Rd} = nP_{Rd} + V_{LR,Rd} \quad V_{LR,Rd} = \mu P_{Rd}$$

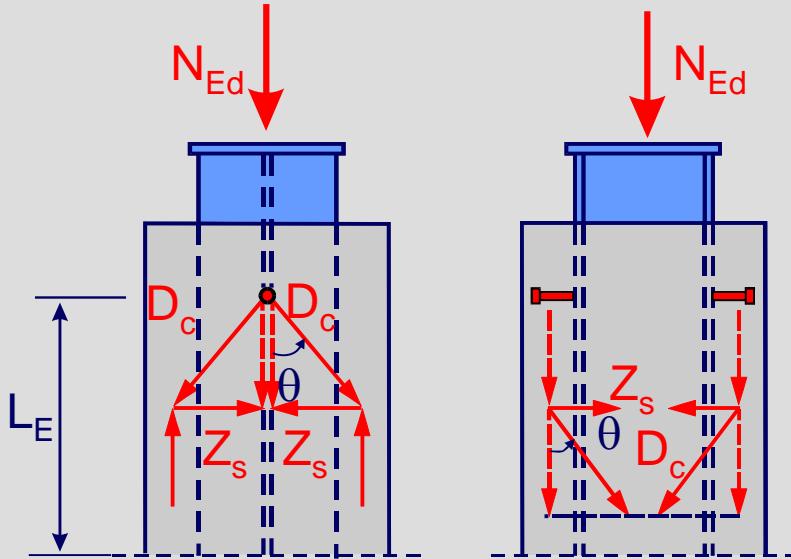
$$P_{Rd} = \min \left\{ \begin{array}{l} P_{Rd,1} = 0,29 \alpha d^2 \sqrt{f_{ck} E_{cm}} \frac{1}{\gamma_v} \\ P_{Rd,2} = 0,8 \cdot f_u \left(\frac{\pi d^2}{4} \right) \frac{1}{\gamma_v} \end{array} \right.$$

In absence of better information from tests, the clear distance between the flanges should not exceed the values given above.

Shear resistance of stud connectors welded to the web of partially encased I-sections



Load introduction – longitudinal shear forces in concrete



Longitudinal shear force in section I-I:

$$V_{L,Ed} = N_{Ed} \left[1 - \frac{N_{pl,a}}{N_{pl,Rd}} \right] \frac{A_{c1} 0,85 f_{cd} + A_{s1} f_{sd}}{A_c 0,85 f_{cd} + A_s f_{sd}}$$

Longitudinal shear resistance of concrete struts:

$$V_{L,Rd,max} = 4 \frac{c_y v 0,85 f_{cd}}{\cot \theta + \tan \theta} L_E \quad \theta = 45^\circ$$

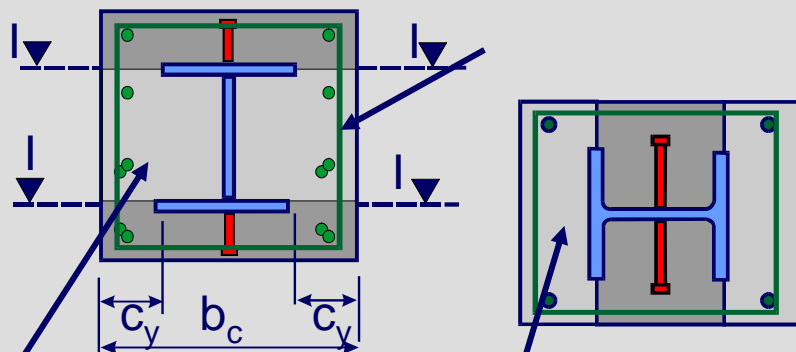
$$v = 0,6 (1 - (f_{ck} / 250)) \text{ with } f_{ck} \text{ in } N/mm^2$$

longitudinal shear resistance of the stirrups:

$$V_{L,Rd,s} = 4 \frac{A_s f_{yd}}{s_w} \cot \theta L_E$$

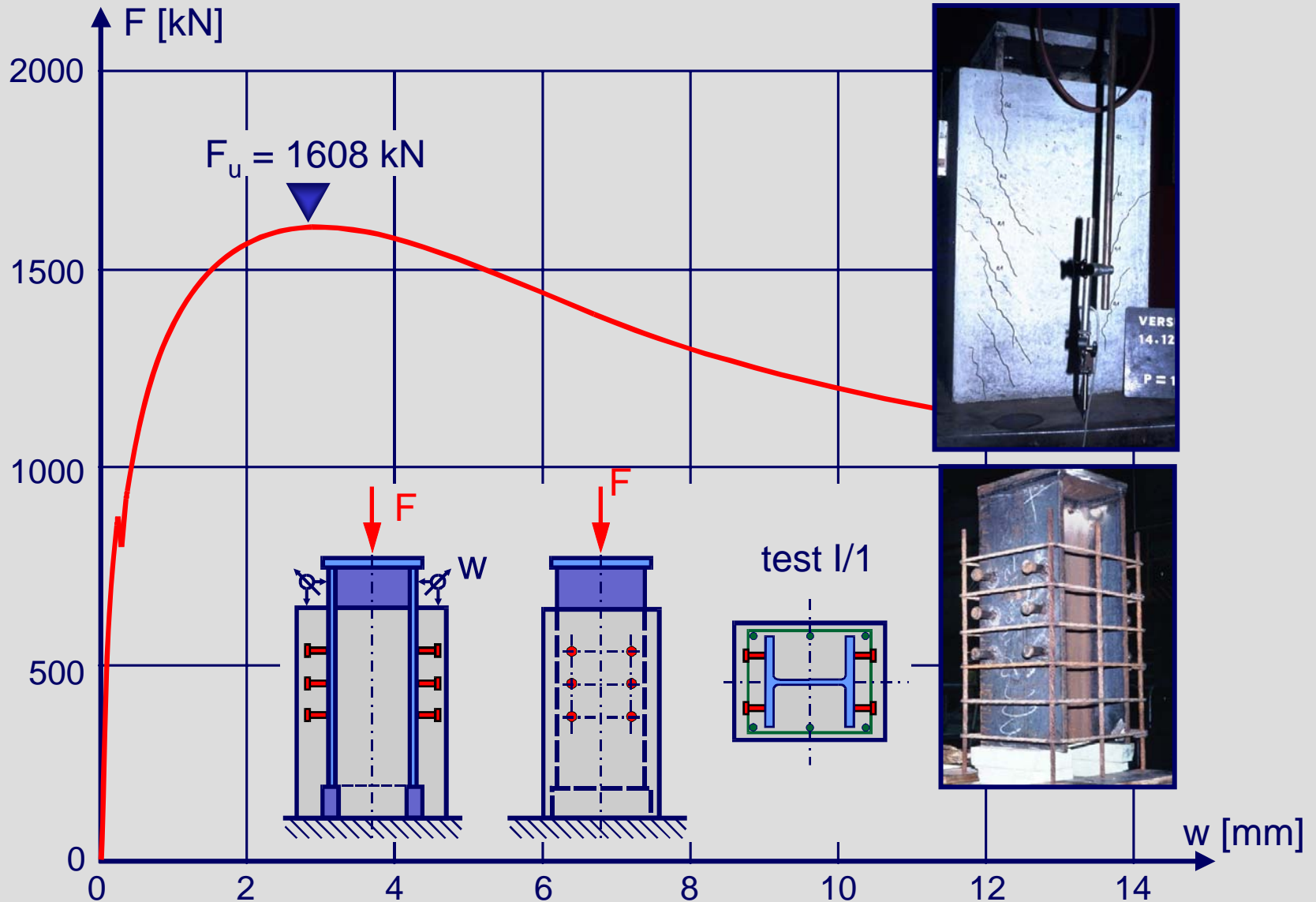
s_w - spacing of stirrups

A_s - cross-section area of the stirrups

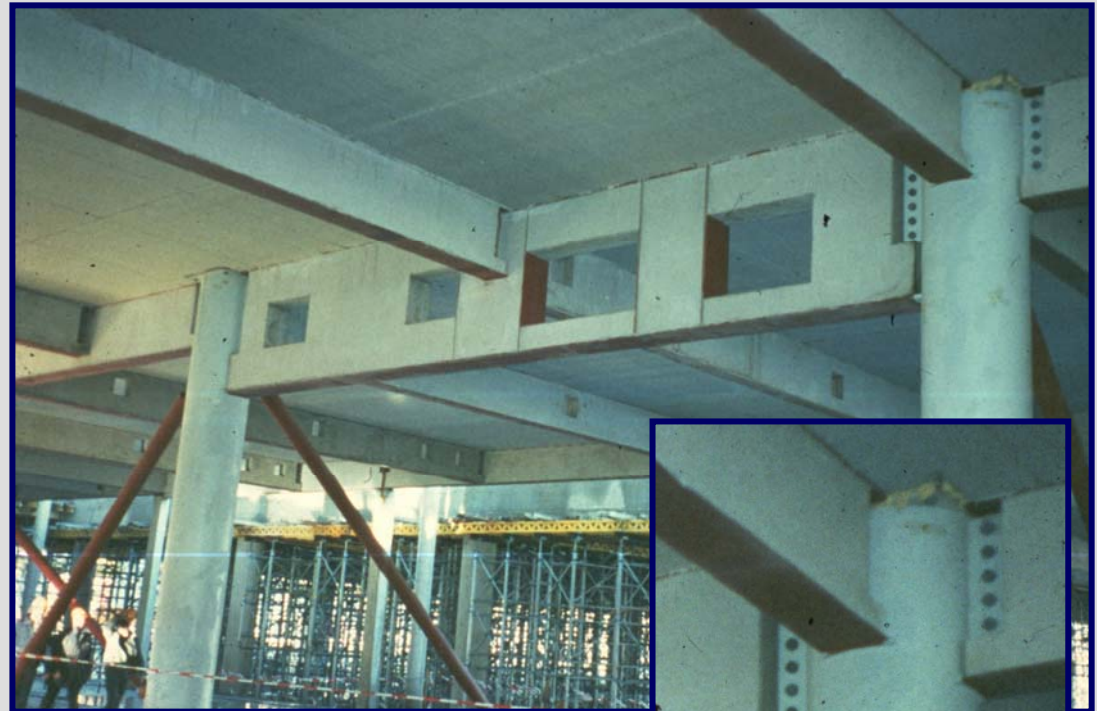
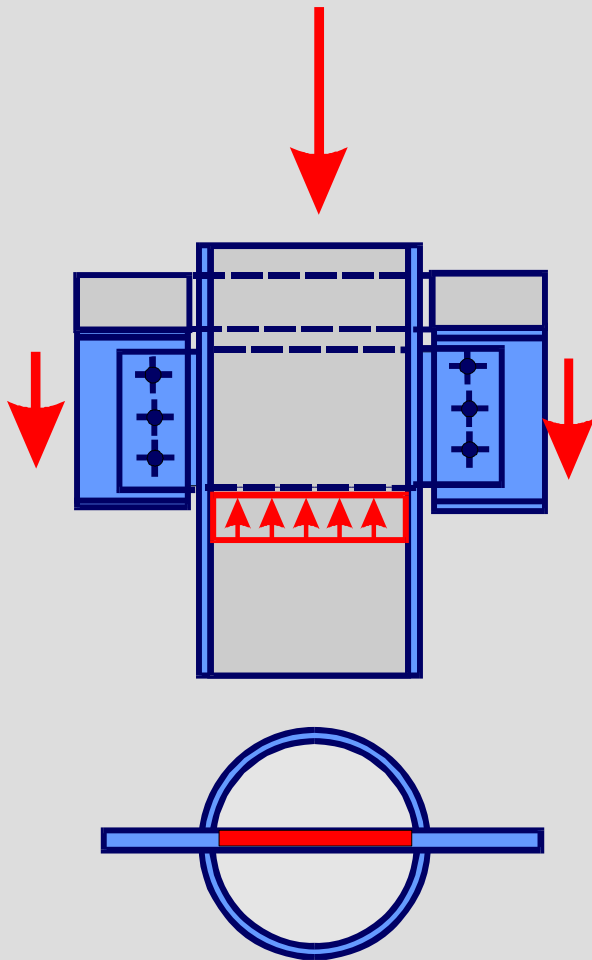


not directly connected concrete area A_{s1}

Load introduction – longitudinal shear forces in concrete – test results

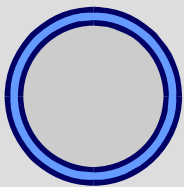
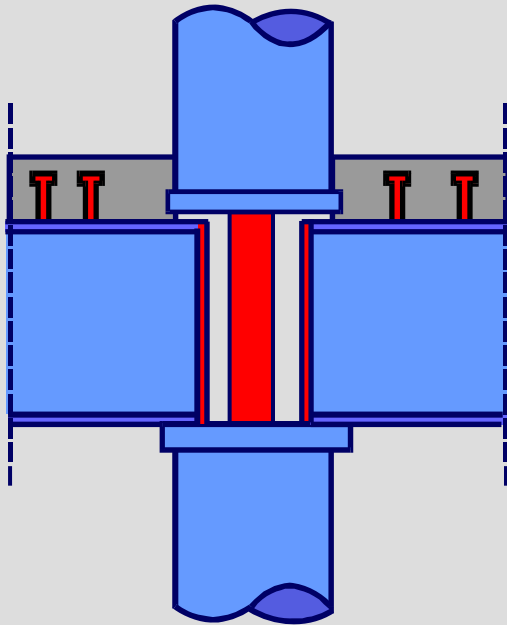


Load introduction – Examples (Airport Hannover)



Load introduction with gusset plates

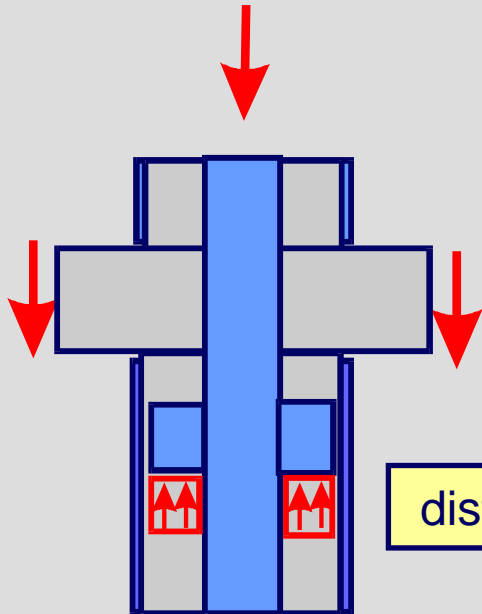
Load introduction with partially loaded end plates



Load introduction with
partially loaded end plates



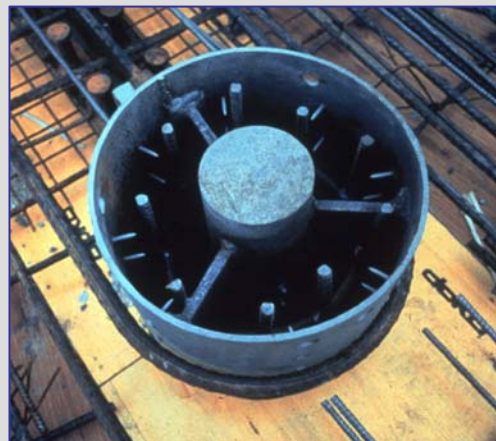
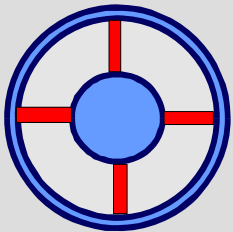
Load introduction with distance plates for columns with inner steel cores



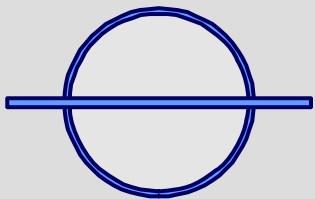
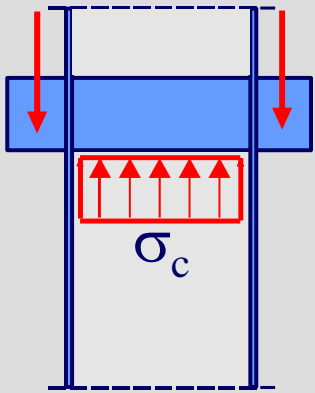
distance plates



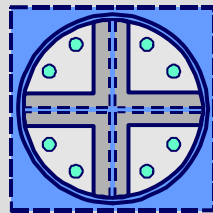
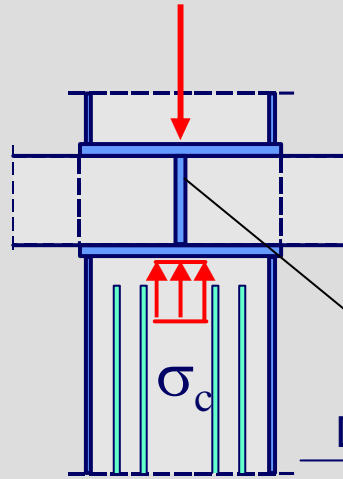
Post Tower Bonn



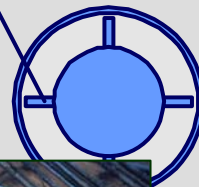
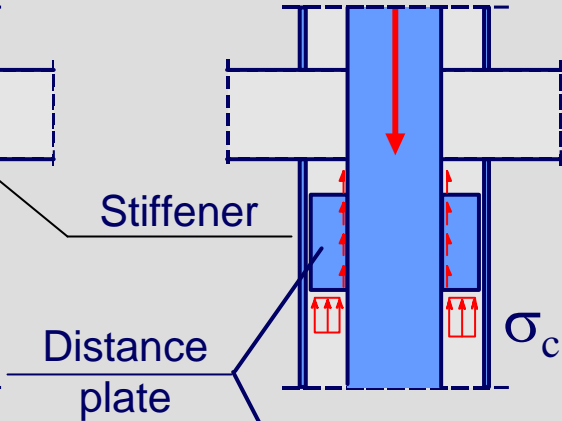
gusset plate



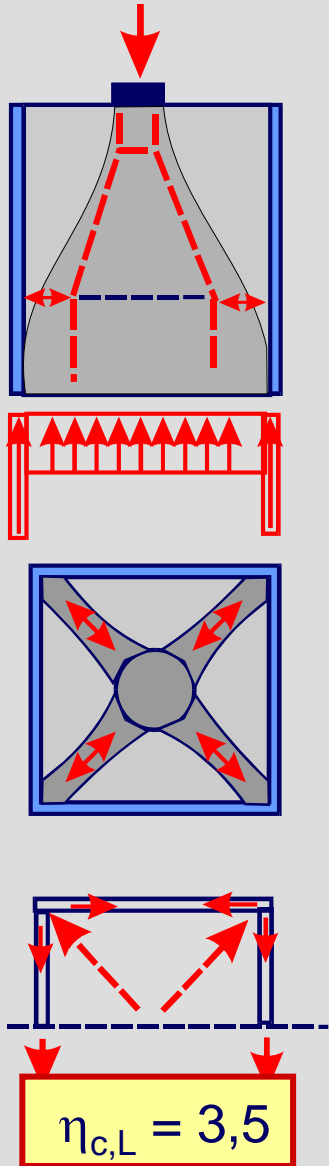
stiffeners and end plates



distance plates



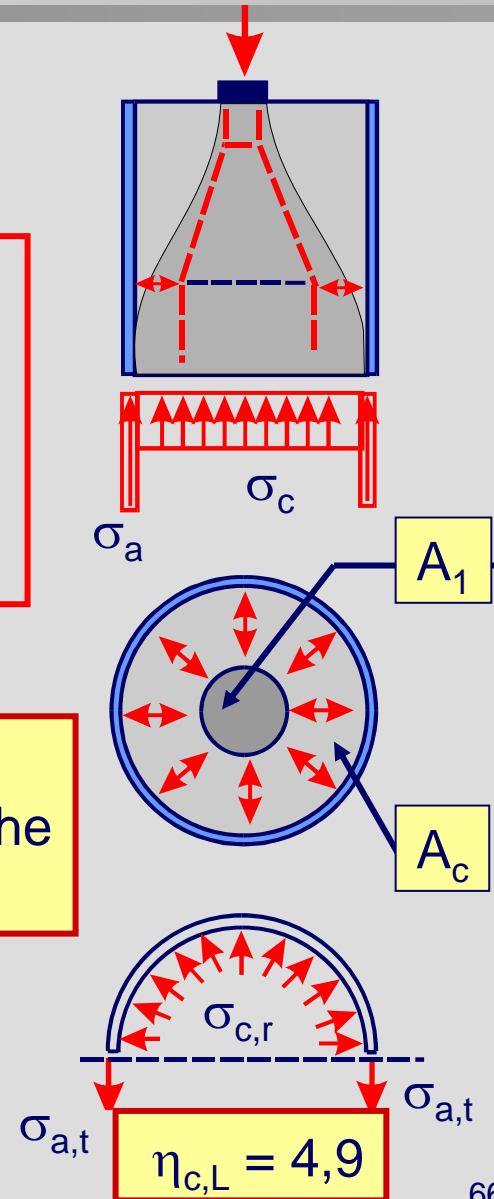
Mechanical model



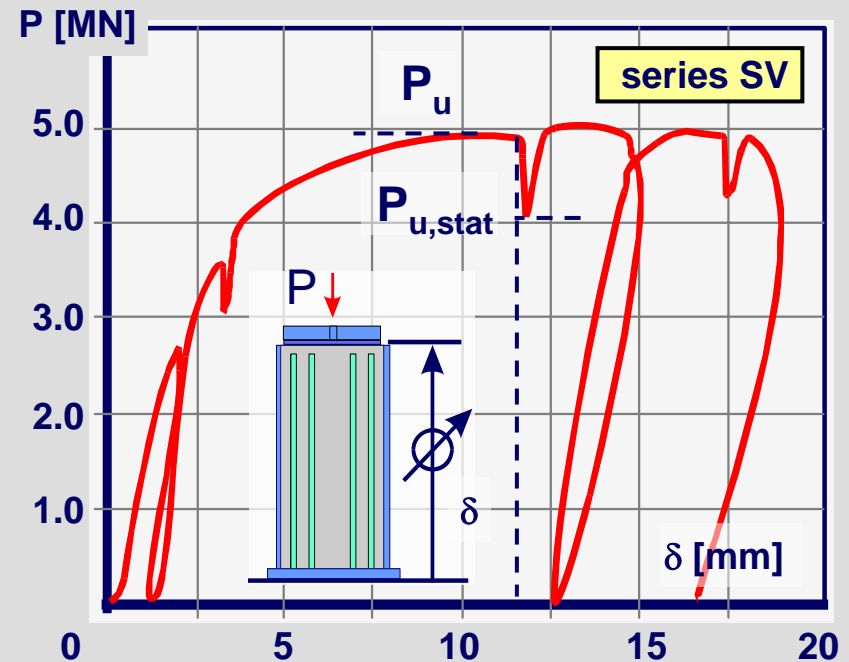
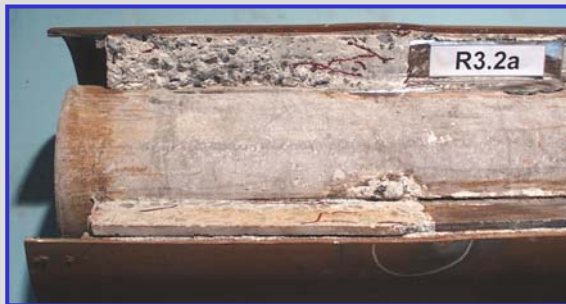
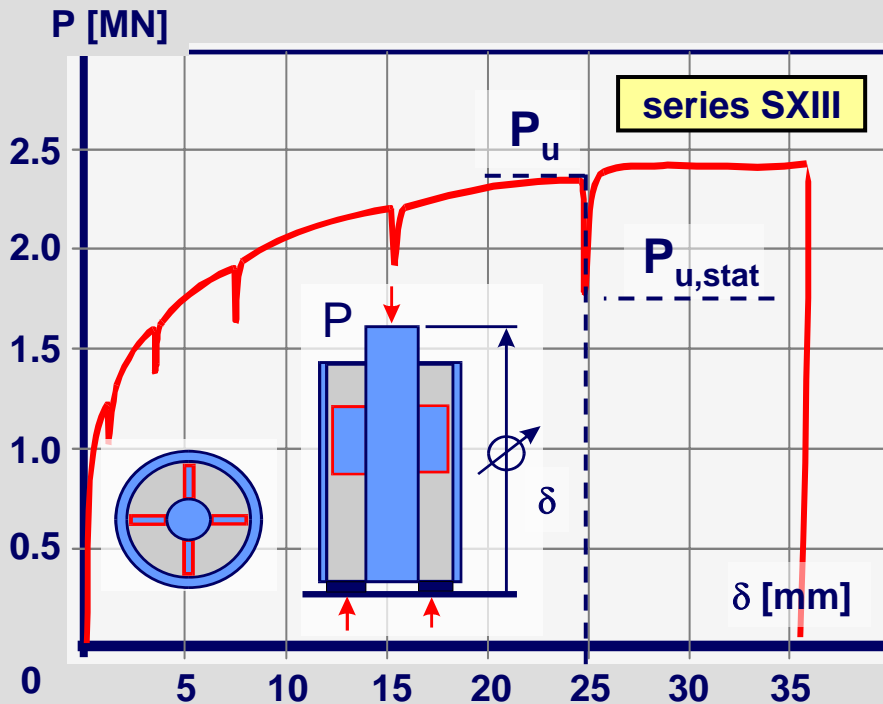
$$P_{cR,m} = f_c A_1 \sqrt{\frac{A_c}{A_1}} \left[1 + \eta_{cL} \frac{t}{d} \frac{f_y}{f_c} \right]$$

Effect of partially loaded area

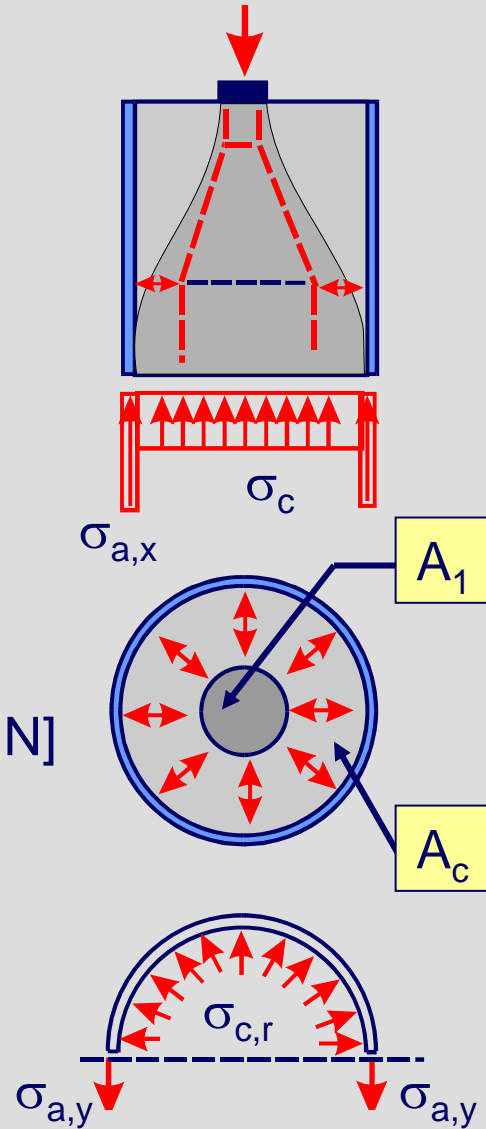
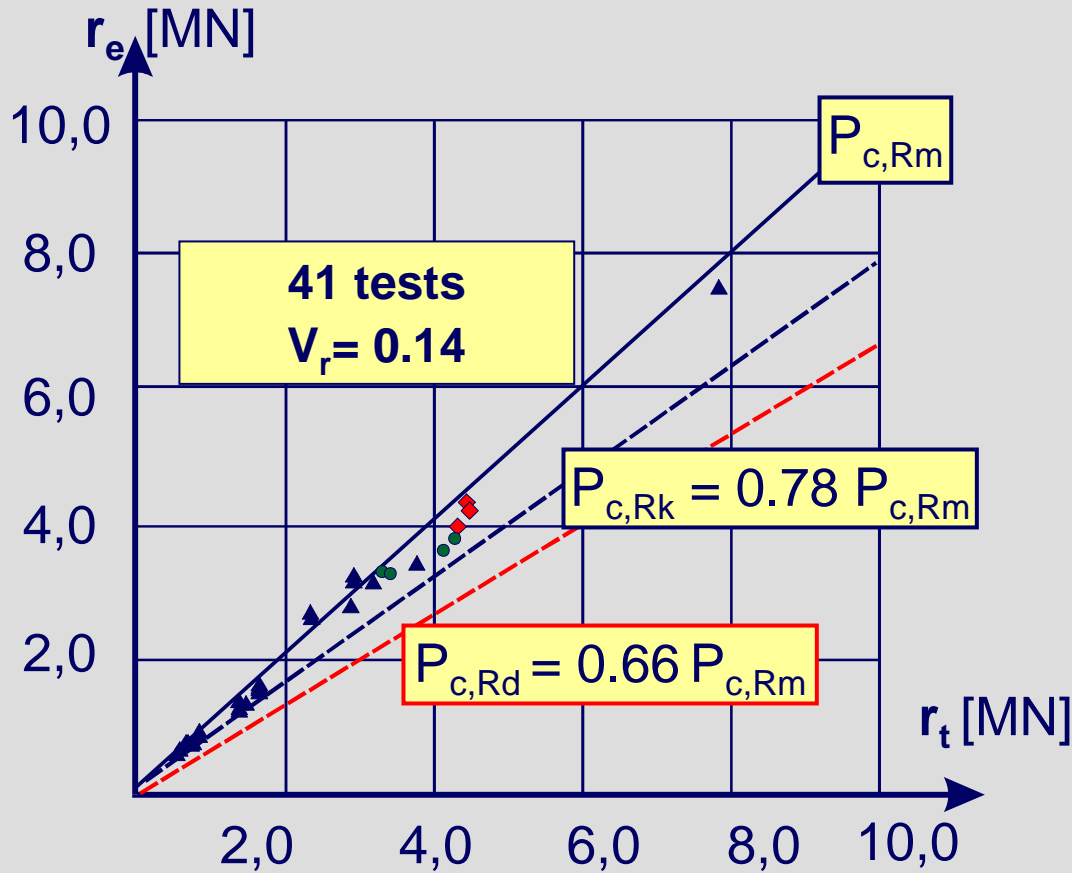
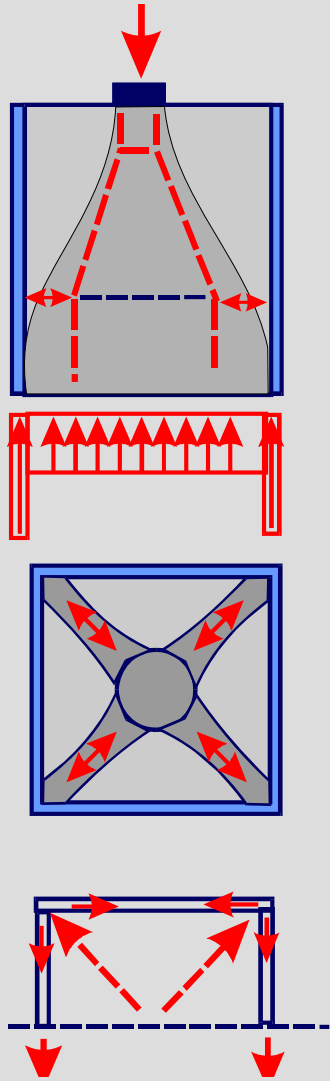
Effect of confinement by the tube



Typical load-deformation curves

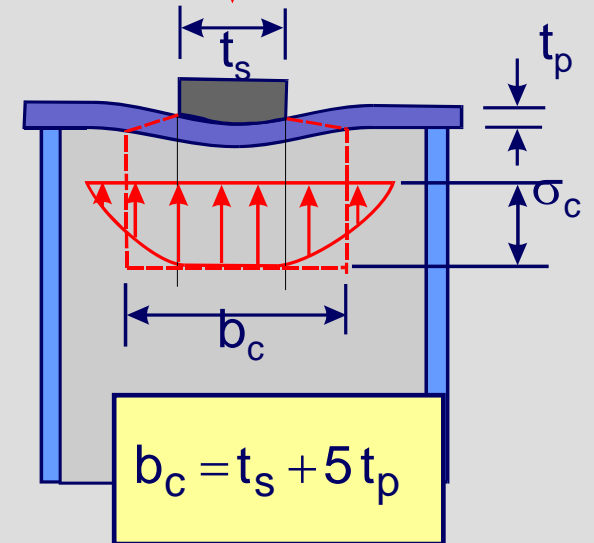
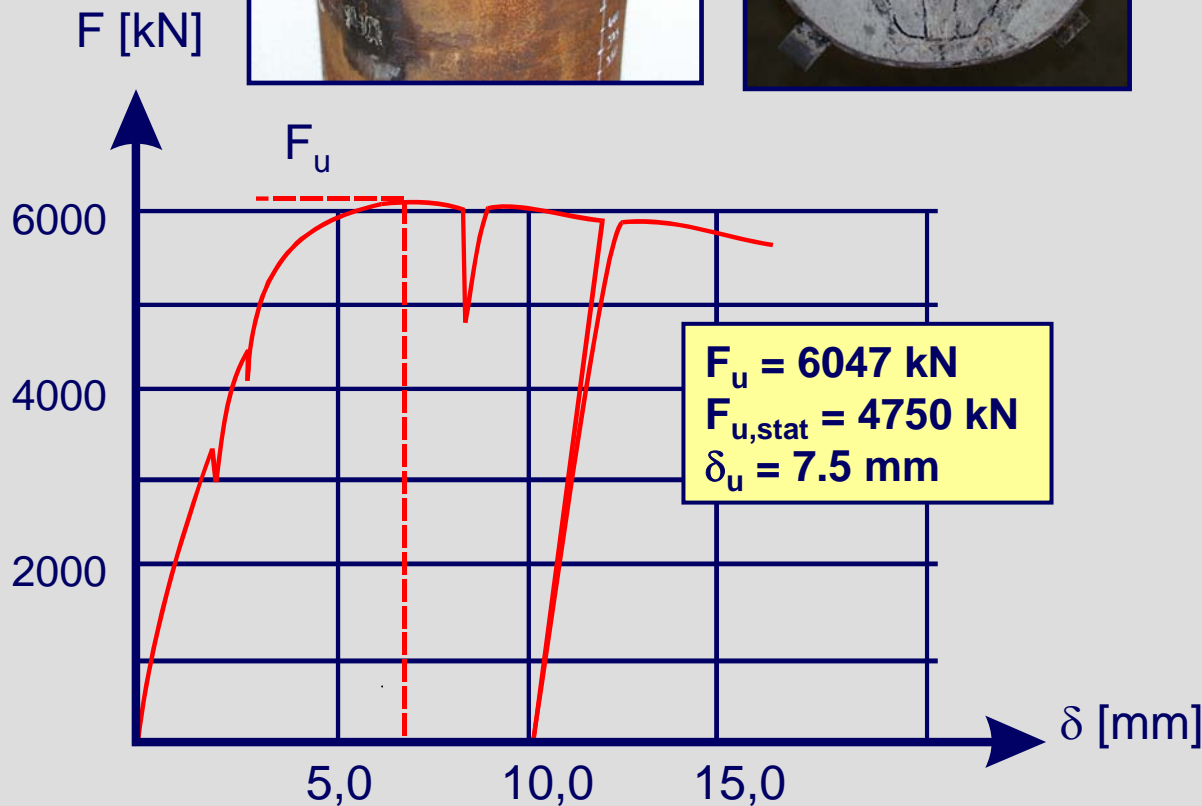


Test evaluation according to EN 1990



$$P_{cR,m} = f_c A_1 \left[1 + \eta_{cL} \frac{t}{d} \frac{f_y}{f_c} \right] \sqrt{\frac{A_c}{A_1}}$$

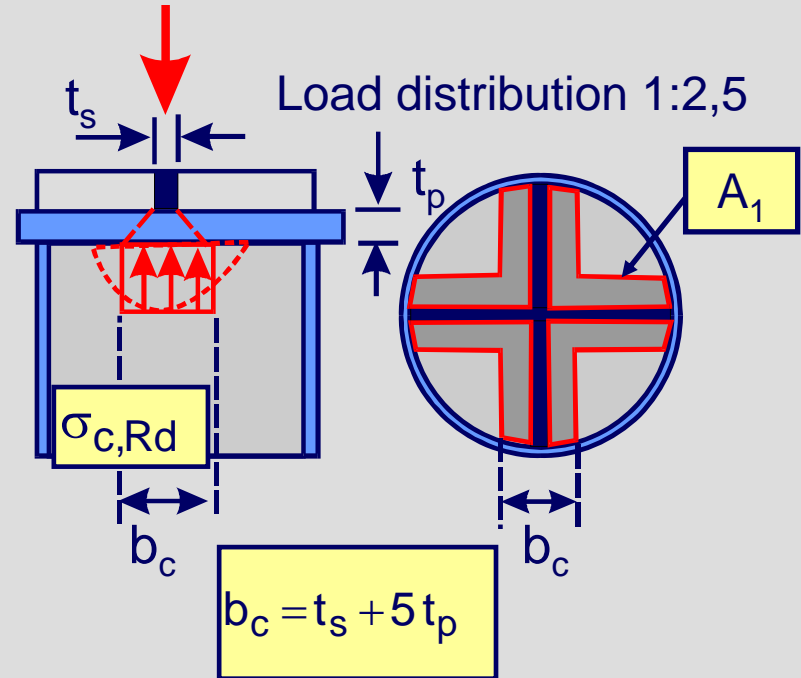
Load distribution by end plates

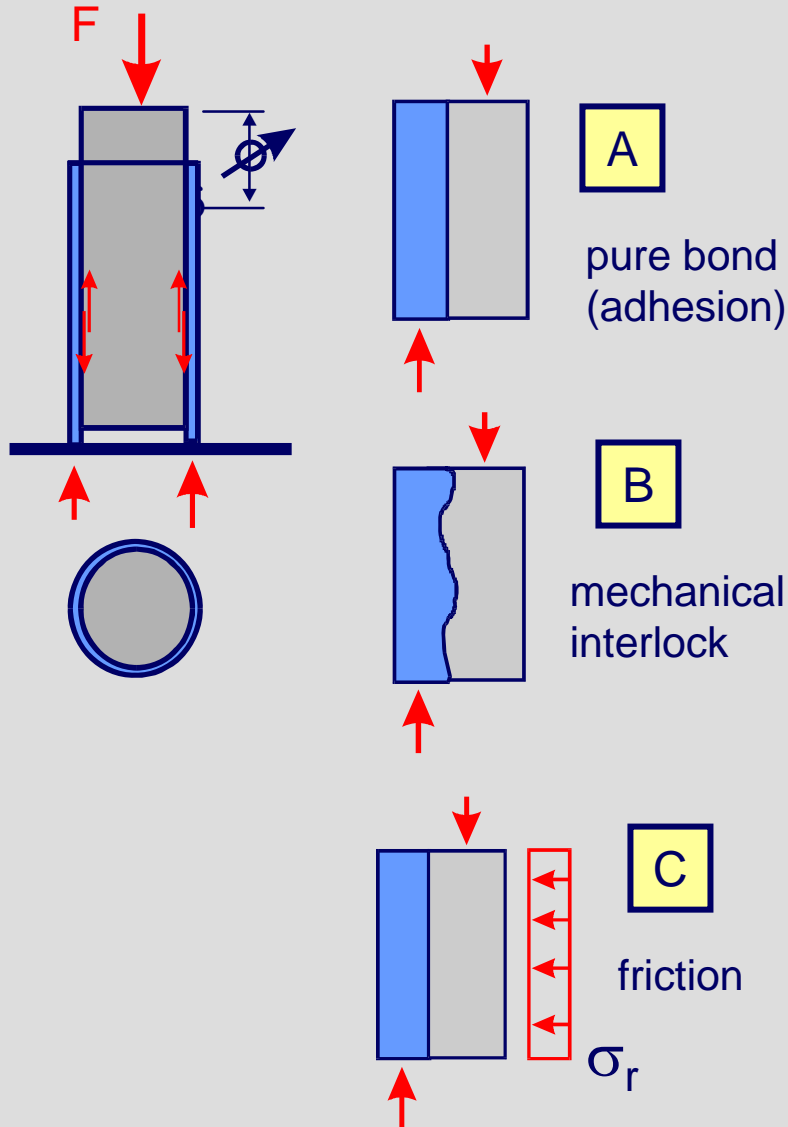


$$\sigma_{c,Rd} = f_{cd} \left[1 + \eta_{cL} \frac{t}{d} \frac{f_{yk}}{f_{ck}} \right] \sqrt{\frac{A_c}{A_1}} \leq \frac{A_c f_{cd}}{A_1} \leq f_{yd}$$

- f_{ck} concrete cylinder strength
- t wall thickness of the tube
- d diameter of the tube
- f_{yk} yield strength of structural steel
- A_1 loaded area
- A_c cross section area of the concrete
- $\eta_{c,L}$ confinement factor
 - $\eta_{c,L} = 4,9$ (tube)
 - $\eta_{c,L} = 3,5$ (square hollow sections)

$$\frac{A_c}{A_1} \leq 20$$

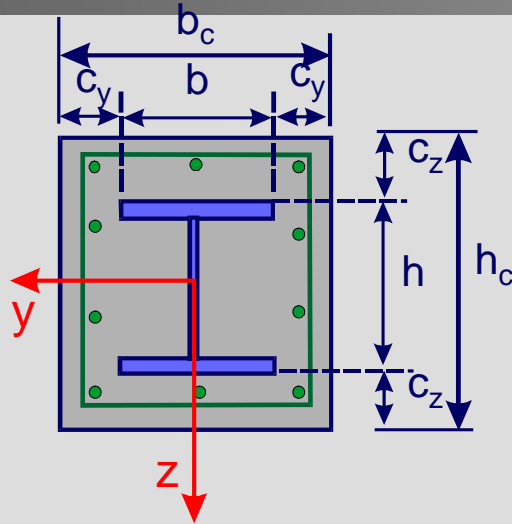




Outside the area of load introduction, longitudinal shear at the interface between concrete and steel should be verified where it is caused by transverse loads and / or end moments. Shear connectors should be provided, based on the distribution of the design value of longitudinal shear, where this exceeds the design shear strength τ_{Rd} .

In absence of a more accurate method, elastic analysis, considering long term effects and cracking of concrete may be used to determine the longitudinal shear at the interface.

concrete encased sections



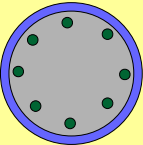
$$\tau_{Rd,o} = 0,30 \text{ N/mm}^2$$

$$\tau_{Rd} = \tau_{Rd,o} \beta_c$$

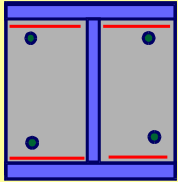
$$\beta_c = 1 + 0,02 c_z \left[1 - \frac{c_{z,min}}{c_z} \right] \leq 2,5$$

c_z - nominal concrete cover [mm]

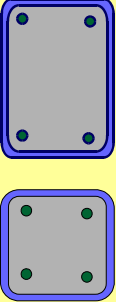
$c_{z,min} = 40 \text{ mm}$ (minimum value)



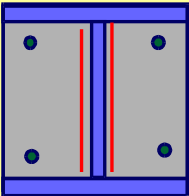
concrete filled tubes

$$\tau_{Rd} = 0,55 \text{ N/mm}^2$$


flanges of partially encased I-sections

$$\tau_{Rd} = 0,20 \text{ N/mm}^2$$


concrete filled rectangular hollow sections

$$\tau_{Rd} = 0,40 \text{ N/mm}^2$$


webs of partially encased I-sections

$$\tau_{Rd} = 0,0 \text{ N/mm}^2$$



**Thank you very
much for your kind
attention**