EN1990
Basis of Structural Design

Annex B Reliability Differentiation
Annex C Reliability Theory

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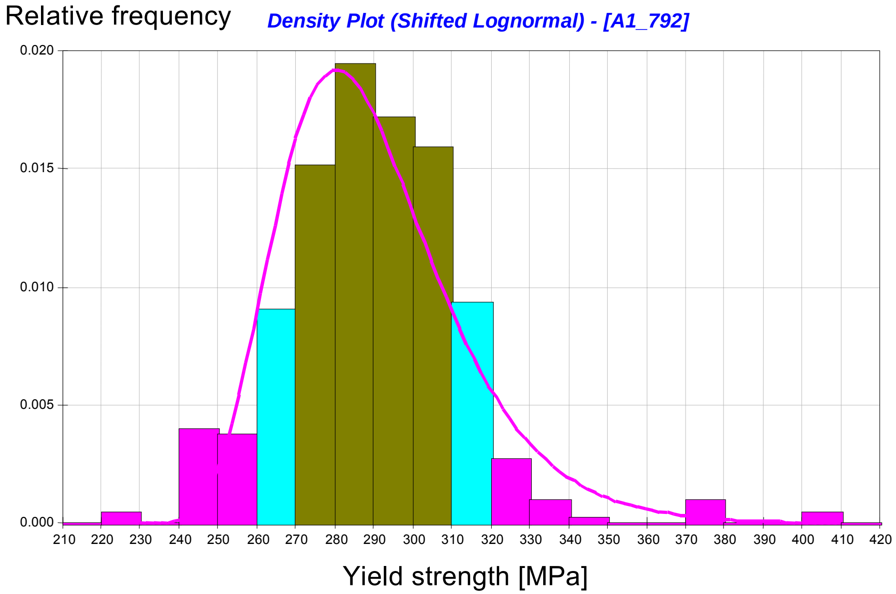
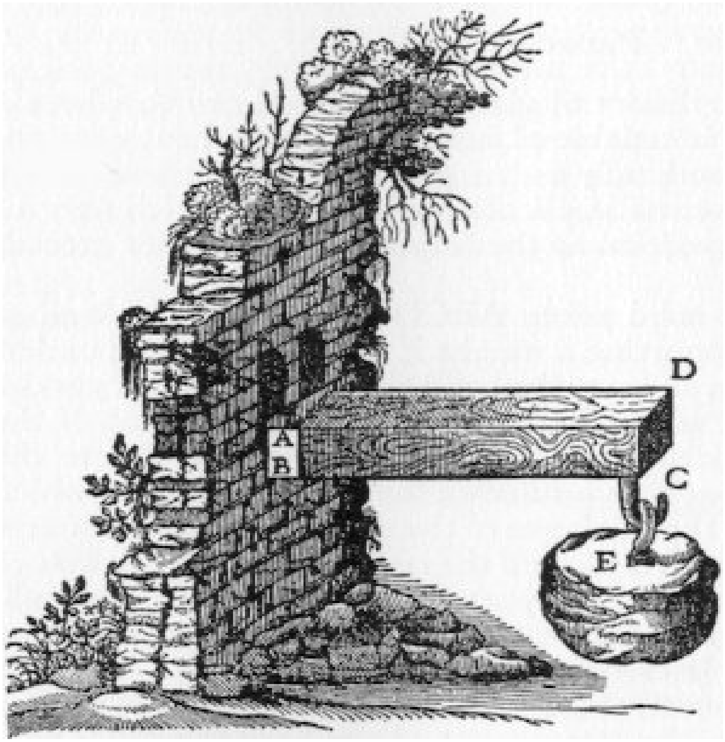
**Design =
decision making
under uncertainty**



- randomness - natural variability
- statistical uncertainties - lack of data
- model uncertainties - simplified models
- vagueness - imprecision in definitions
- gross errors - human factors
- ignorance - lack of knowledge

To get a grip:

- ❑ Mechanical models
- ❑ Statistical models
- ❑ Engineering judgement
- ❑ Robustness
- ❑ Quality Control



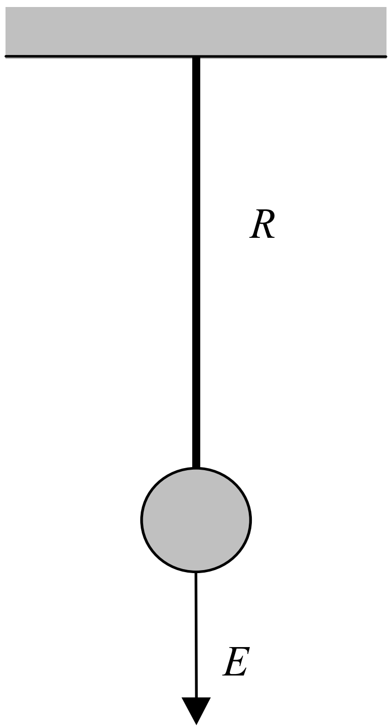
EN 1990: Annexes B / C (Informatieve)

- Consequences classes
- Differentiation of beta values and partial factors
- Background for Partial Factor design
- Background for Probabilistic design

EN 1990 Main Text:
3.5 (5) As an alternative, a design directly based on probabilistic methods may be used.

Reliability calculation

$$P_f = P(R < E) = \iint_{R<E} \varphi_R(r)\varphi_E(e)\,dr\,de$$



JCSS Probabilistic Model Code

Part 1 Basis of Design
Part 2 Modeling of loads
Part 3 Modeling of structural properties

http://www.jcss.ethz.ch/
select "publications"
select "jcss model code"

Simple example

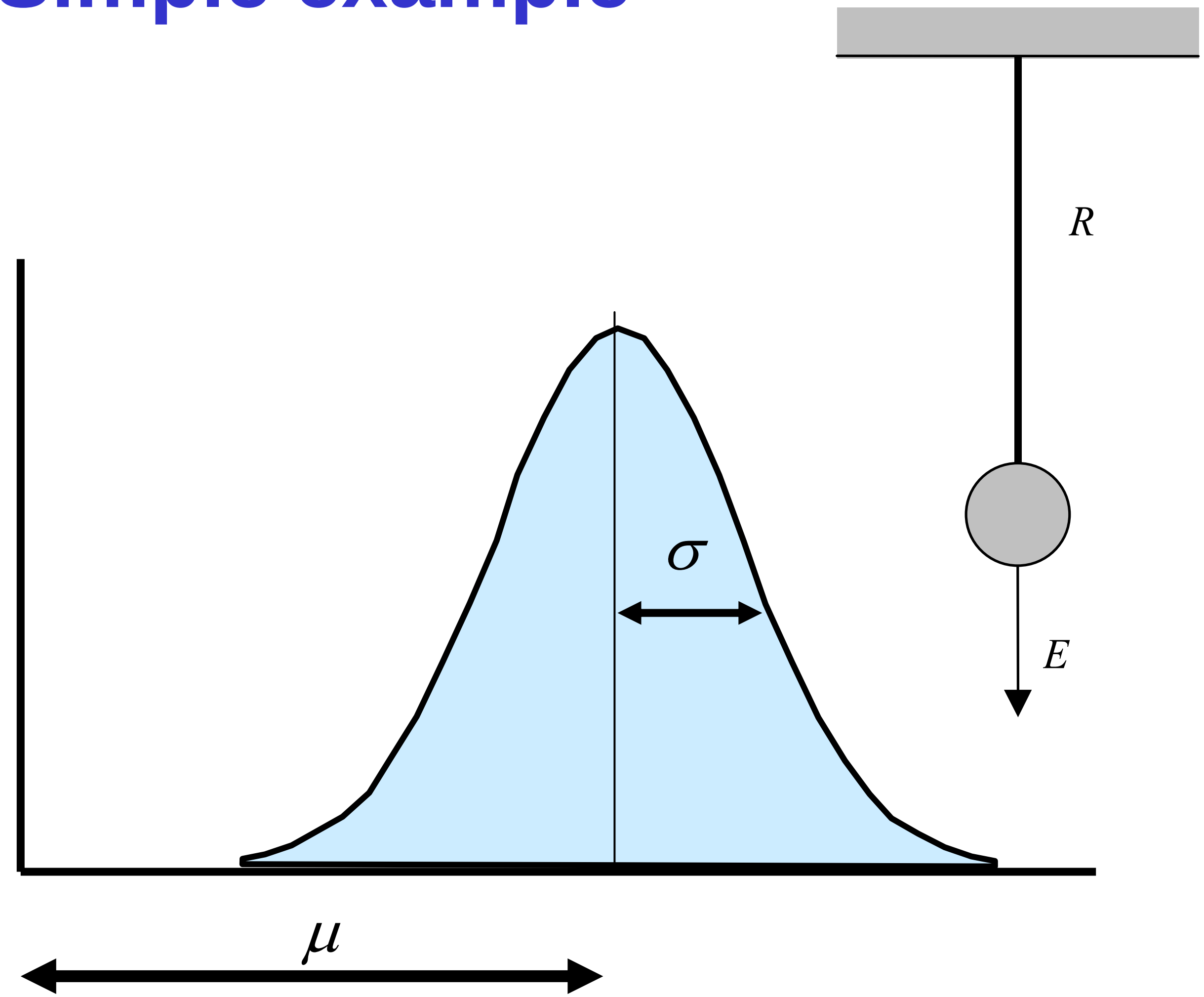
$$g = R - E$$

$$\mu_R = 100$$

$$\mu_E = 50$$

$$\sigma_R = 10$$

$$\sigma_E = 10$$



First Order Second Moment method

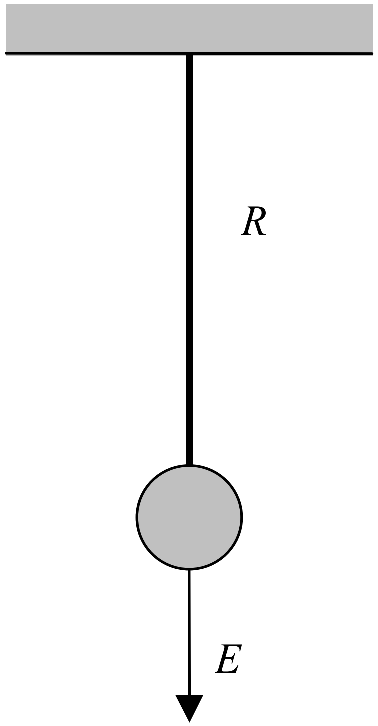
$$g = R - E$$

$$\mu_g = \mu_R - \mu_E = 100 - 50 = 50$$

$$\sigma_g^2 = \sigma_R^2 + \sigma_E^2 = 14^2$$

$$\beta = \mu_g / \sigma_g = 3.54$$

$$P_f = \mathrm{P}(Z < 0) = \Phi_Z(0) = 0.0002$$

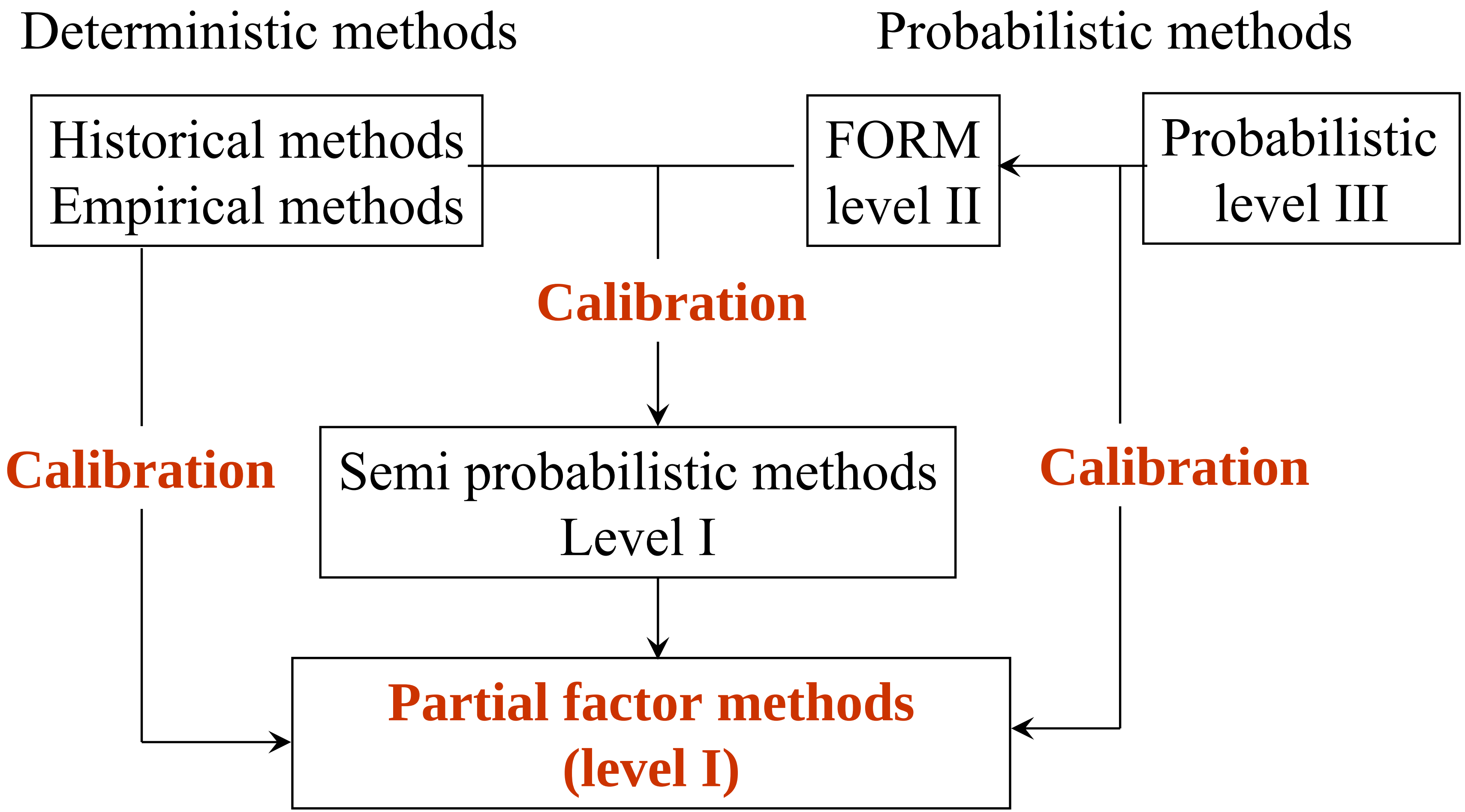


Reliability index β

β	1.3	2.3	3.1	3.7	4.2	4.7
$P(F)=\Phi(-\beta)$	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}

Probability of Failure = $\Phi(-\beta) \approx 10^{-\beta}$

Reliability Methods



Format Partiel Factor Design

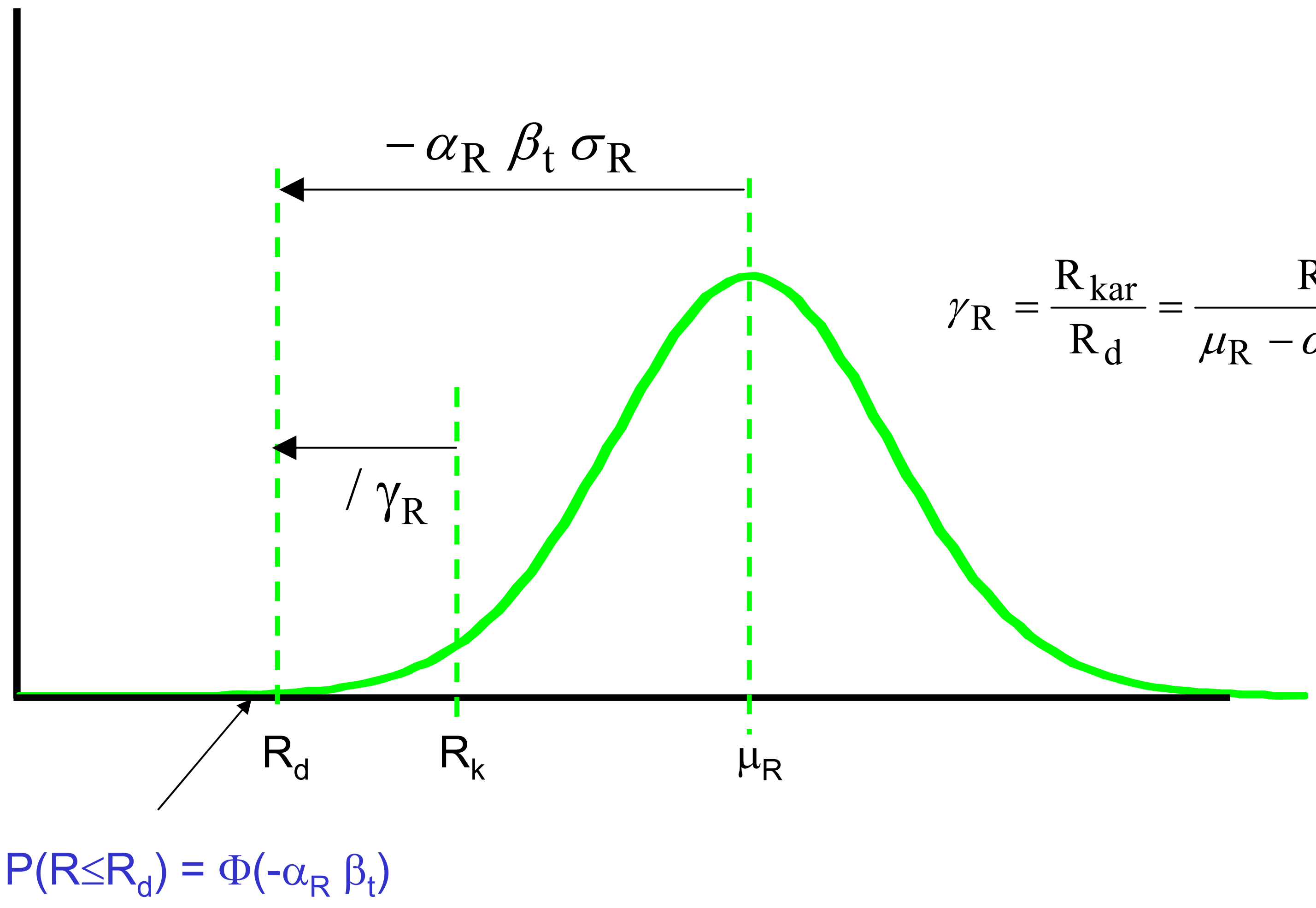
$$E_d < R_d$$

$$E_d = E(\gamma_G G_k, \gamma_Q Q_k)$$

$$R_d = R_k / \gamma_M$$

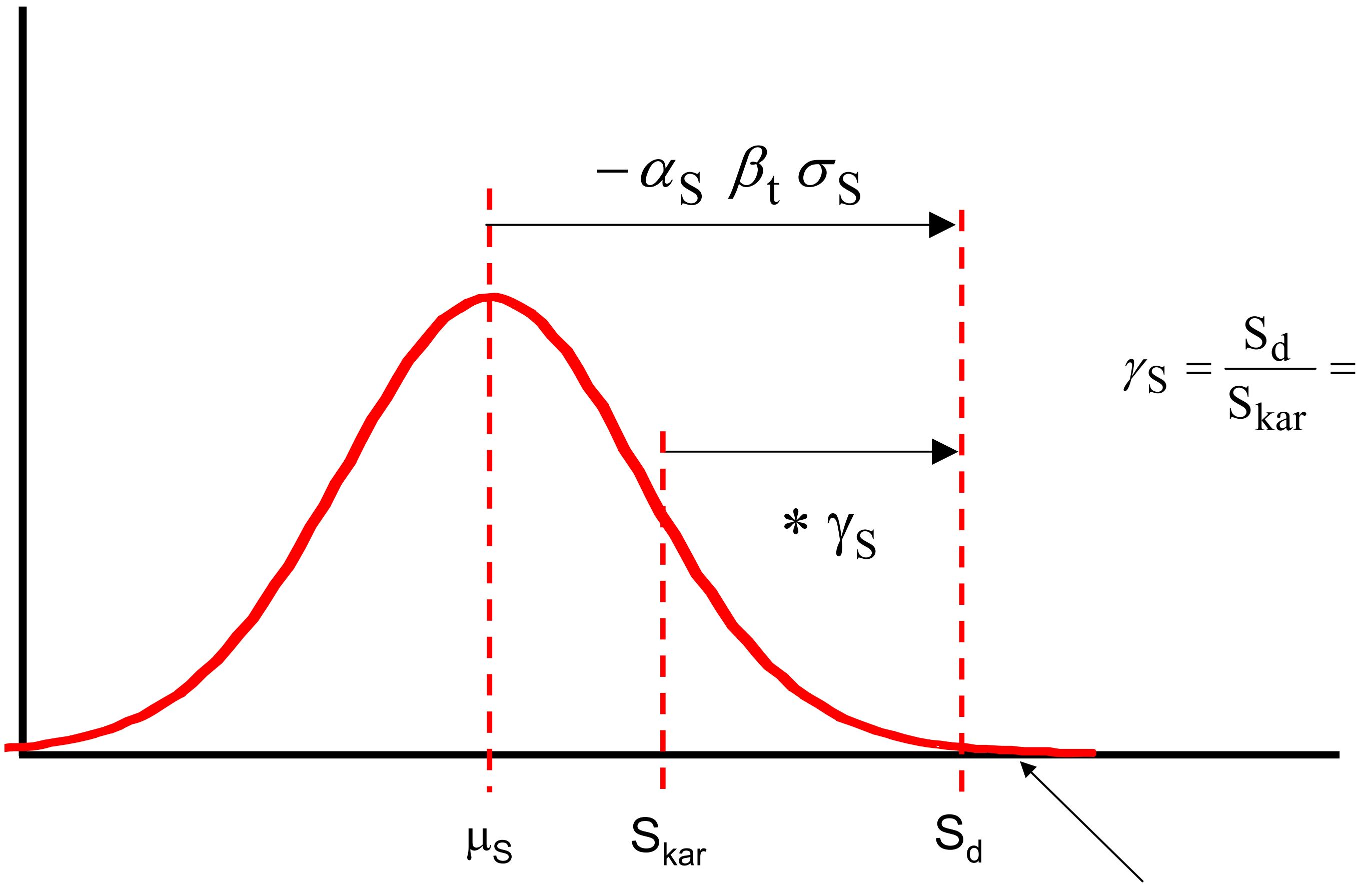
R_k	**characteristic value of resistance**
G_k Q_k	**charactistic value of load**
γ_M, γ_G ,γ_Q	**partial factors**

partiale factor for R



$$\gamma_R = \frac{R_{kar}}{R_d} = \frac{R_{kar}}{\mu_R - \alpha_R \beta_t \sigma_R}$$

partial factor for S



$$\gamma_S = \frac{S_d}{S_{kar}} = \frac{\mu_S - \alpha_S \beta_t \sigma_S}{S_{kar}}$$

$P(S > S_d) = 1- \Phi(-\alpha_S \beta_t)$

JCSS *Code Calibration Program* - Input Sheet

Exit CodeCal

Check Design Situation | Clear Sheet | Fill in Example Values

Compute

1. General Input
2. Design Situation
3. Time Invariant Variables
4. Variable Loads
5. Partial Safety Factors
6. Computation Options

1. General Input

Editor : Editor's Name ?

Date : 11.03.2003 Today ?

Comment : Example values for Codecal, the JCSS ?

code calibration program.

2. Design Situation

Design Situation 1: One Variable Load

Design Equation

$$z = \frac{\gamma_m}{R_k}\left(\alpha\ \gamma_G\ G_k + (1-\alpha)\gamma_Q\ Q_k\right)$$

Limit State Function

$$g(\mathbf{X}) = z\ R\ \xi - \alpha\ G - (1-\alpha)\ Q \le 0$$

Design Situation 2: Two Variable Loads

Design Equation ?

$$z = \max\{z_1, z_2\}$$

$$z_1 = \frac{\gamma_m}{R_k}\left(\alpha_G\ \gamma_G\ G_k + (1-\alpha_G)\left[\alpha_Q\ \gamma_Q\ Q_{k1} + (1-\alpha_Q)\ \gamma_Q\ \psi_{02}\ Q_{k2}\right]\right)$$

$$z_2 = \frac{\gamma_m}{R_k}\left(\alpha_G\ \gamma_G\ G_k + (1-\alpha_G)\left[\alpha_Q\ \gamma_Q\ Q_{k2} + (1-\alpha_Q)\ \gamma_Q\ \psi_{01}\ Q_{k1}\right]\right)$$

Limit State Function ?

$$g_1(\mathbf{X}) = z\ R\ \xi - \alpha_G\ G - (1-\alpha_G)\ \left[\alpha_Q\ Q_1\right] \le 0$$

$$g_2(\mathbf{X}) = z\ R\ \xi - \alpha_G\ G - (1-\alpha_G)\ \left[\alpha_Q\ Q_{1L} + (1-\alpha_Q)\ Q_{2A}\right] \le 0$$

$$g_3(\mathbf{X}) = z\ R\ \xi - \alpha_G\ G - (1-\alpha_G)\ \left[\alpha_Q\ Q_{1A} + (1-\alpha_Q)\ Q_{2L}\right] \le 0$$

$$\beta = \min\{\beta_1, \beta_2, \beta_3\}$$

Design Situation 1: One Variable Load

Design Situation 2: Two Variable Loads

3. Stochastic Model of Time Invariant Variables

Resistance R

Material : Structural Steel, etc. ?

Distribution Type : Weibull ?

E[R] = 1 ?

D[R] = 0.05 ?

$F[R_k]$ = 5 % ?

Model Uncertainty ξ

Distribution Type : Lognormal ?

ISO 2394 STANDARD ALFA-values

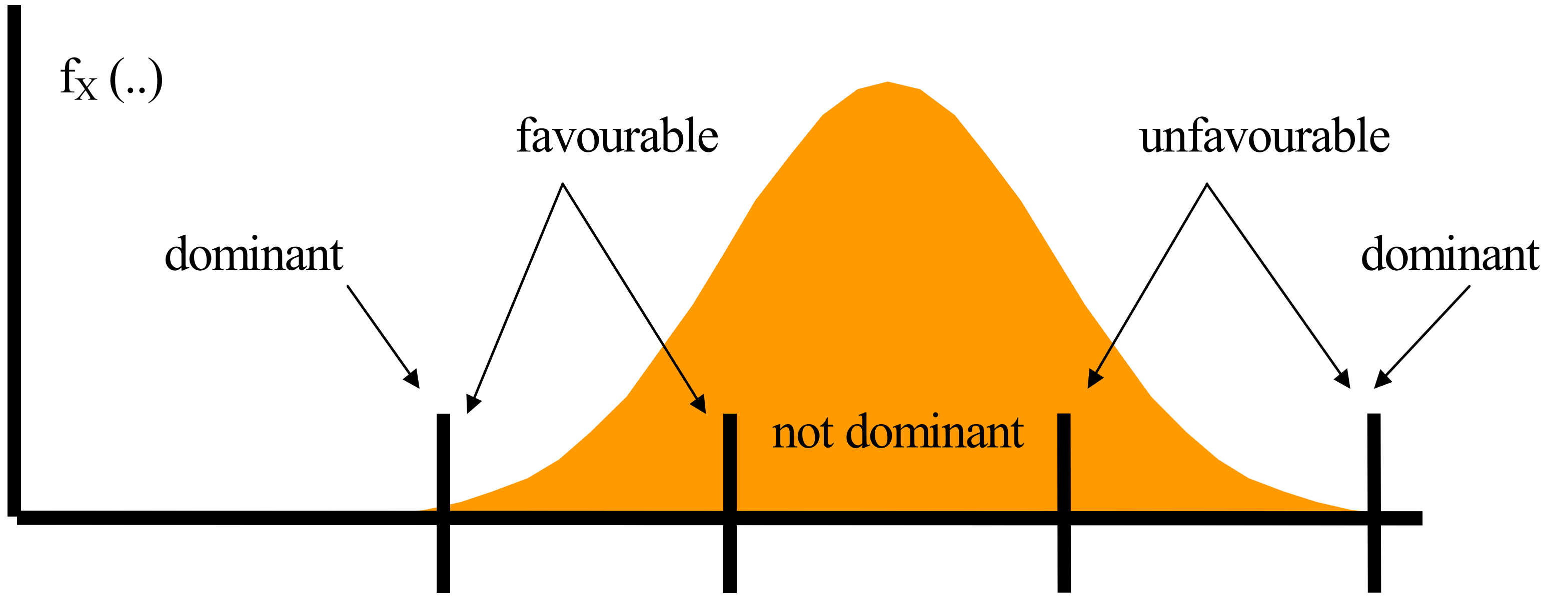
	load	resistance
Dominant Variable	α = - 0.70	α = 0.80
Other variables	α = - 0.28	α = 0.32

Table 2 : Target reliability index β for Class RC 2 structural members

Limit state	Target reliability index	
	1 year	50 year
Ultimate	4,7	3,8
Fatigue		1,5 to 3,8 [2)]
Serviceability (irreversible)	2,9	1,5

[1)] See Annex B
[2)] Depends on degree of inspectability, reparability and damage tolerance.

Four design values for ULS



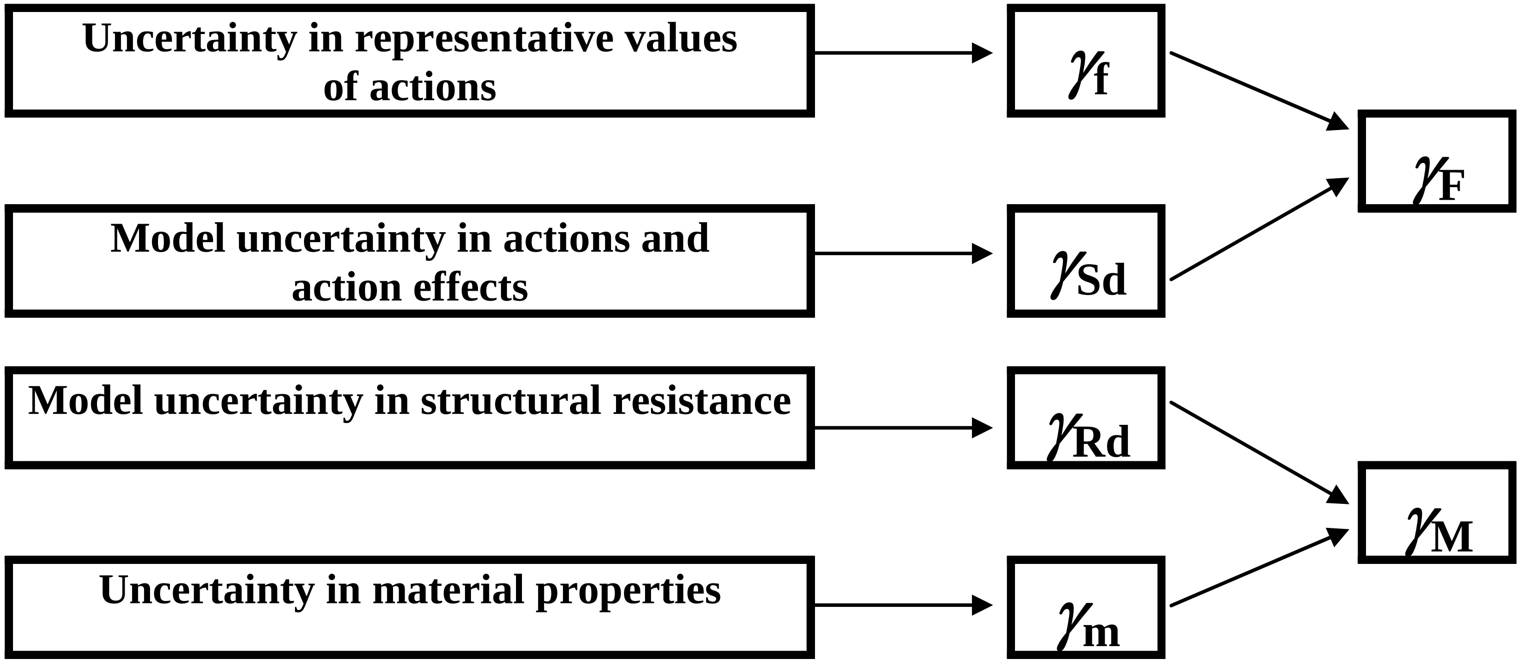


Figure 3 : Relation between individual partial factors

Permant loads

$$G_d = \mu_G (1 - \alpha_E \beta V_G)$$

$$G_k = \mu_G$$

μ_G = mean value

$V_G = \sigma_G / \mu_G$

α_E = FORM factor

β = reliability index

unfavorable	dominant	θ_d/θ_{nom}	α_E	γ_G ($V_G = 0.10$)
yes	yes	1.05	-0.70	(1.05)(1+3.8*0.70*0.10) =1.33
yes	no	1.05	-0.28	(1.05)(1+3.8*0.28*0.10) =1.16
no	no	1.00	+0.32	1-3.8*0.32*0.10 =0.88
no	yes	1.00	+0.80	1-3.8*0.80*0.10 =0.70

In EN 1990 one finds: 1.35, 1.35ξ = 1.15, 1.00 and 1.00.

Variable loads

$$Q_d = \mu_G (1 - \alpha_E \beta V_Q)$$

$$Q_k = \mu_G$$

$\beta = 3.8$

$\theta_d/\theta_{nom} = 1.05$

$\alpha_E = -0.7$ (dominant)

$\alpha_E = -0.7x0.4$ (combination)

unfavorable	dominant	θ_d/θ_{nom}	α_E	γ_Q ($V_Q = 0.2$)
yes	yes	1.05	-0.70	(1.05)(1+3.8*0.70*0.20) =1.61
yes	no	1.05	-0.28	(1.05)(1+3.8*0.28*0.20) =1.27

In EN 1990 one finds 1.5.

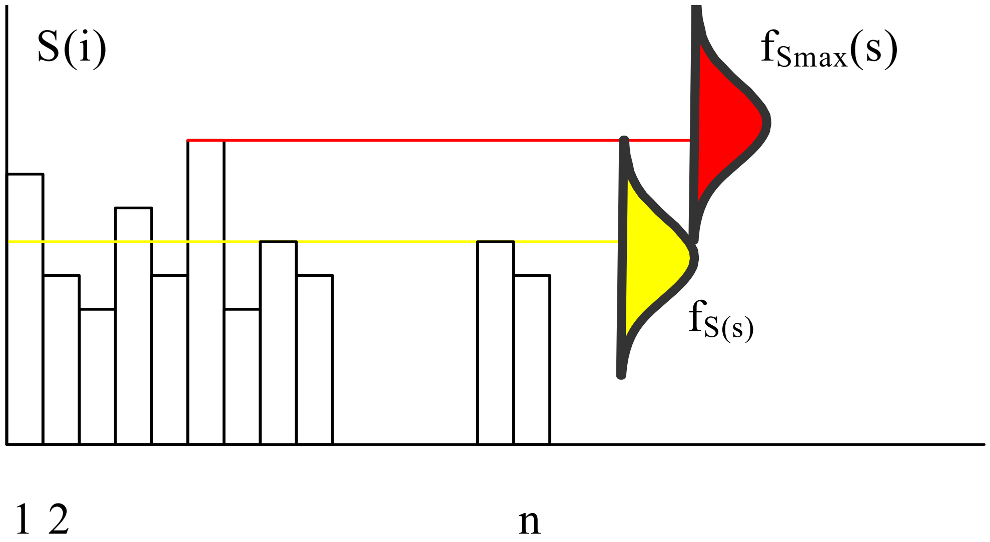
$\Psi_o = 1,27/1,61 = 0,8.$

Variable load / Gumbel distribution

load	V_Q (T = 50 jaar)	γ_Q
wind	0.20	2.04
snow (land climate)	0.15	1.77
snow (sea climate)	0.30	2.67
Floor load (20 m^2)	0.30	1.50

So maybe beta = 3.8 is not really true.

Stochastic variable load Model



N independent loads of duration Δt:

$$F_{Smax}(s) = \{F_{Si}(s)\}^N$$

Variable loads / PSI values

Theory

dominant	$\psi_{o,eg}$	$\psi_{o,floor}$	$\psi_{o,snow}$	$\psi_{o,wind}$	$\psi_{o,acc}$
permanent	-	0.55	0.60	0.68	0
floor	0.87	-	0.40	0.51	0
snow	0.87	0.34	-	0.31	0
wind	0.87	0.34	0.15	-	0
accidental	0.87	0.34	0.15	0.04	-

EN 1990

dominant	$\psi_{o,eg}$	$\psi_{o,vloerr}$	$\psi_{o,sneeuw}$	$\psi_{o,wind}$	$\psi_{o,bb}$
permanent	-	0.7	0.7	0.6	0
floor	ξ =0.85-1.0	-	0.7	0.6	0
snow	ξ =0.85-1.0	0.7	-	0.6	0
wind	ξ =0.85-1.0	0.7	0.7	-	0
accidental	0.74**	0.33*	0.13*	0.33*	-

* corresponding with ψ_1/γ_Q

** corresponding wiht $1/\gamma_G = 1/1.35$

Annex B Management of Structural Reliability for Construction Works (Informative)

Reliability differentiation
Design supervision differentiation
Inspection during execution

Table 1: Definition of consequences classes

Consequences Class	Description	Examples of buildings and civil engineering works
CC3	**High** consequence for loss of human life, *or* economic, social or environmental consequences **very great**	Grandstands, public buildings where consequences of failure are high
CC2	**Medium** consequence for loss of human life, economic, social or environmental consequences **considerable**	Residential and office buildings, public buildings where consequences of failure are medium
CC1	**Low** consequence for loss of human life, *and* economic, social or environmental consequences **small or negligible**	Agricultural buildings where people do not normally enter (e.g. storage buildings), greenhouses

Particular members of the structure may be designated at the same, higher or lower consequences class than for the entire structure.

Reliabiilty differentiation (beta, parial factors)

Table 2 : Recommended minimum values for reliability index β (ultimate limit states)

Reliability Class	Minimum values for β	
	1 year reference period	50 years reference period
RC3	5,2	4,3
RC2	4,7	3,8
RC1	4,2	3,3

Table 3 : K_{FI} factor for actions

K_{FI} factor for actions	Reliability class		
	RC1	RC2	RC3
K_{FI}	0,9	1,0	1,1

K_{FI} should be applied only to unfavourable actions.

Zwolle

London Eye

Table 4 : Design supervision levels (DSL)

Design Supervision Levels	Characteristics	Minimum recommended requirements for checking of calculations, drawings and specifications
DSL3 relating to RC3	Extended supervision	Third party checking : Checking performed by an organisation different from that which has prepared the design
DSL2 relating to RC2	Normal supervision	Checking by different persons than those originally responsible and in accordance with the procedure of the organisation.
DSL1 Relating to RC1	Normal supervision	Self-checking: Checking performed by the person who has prepared the design

Table 5 : Inspection levels (IL)

Inspection Levels	Characteristics	Requirements
IL3 Relating to RC3	Extended inspection	Third party inspection
IL2 Relating to RC2	Normal inspection	Inspection in accordance with the procedures of the organisation
IL1 Relating to RC1	Normal inspection	Self inspection

The rules are to be given in the relevant execution standards.

Table 5 : Inspection levels (IL)

Inspection Levels	Characteristics	Requirements
IL3 Relating to RC3	Extended inspection	Third party inspection
IL2 Relating to RC2	Normal inspection	Inspection in accordance with the procedures of the organisation
IL1 Relating to RC1	Normal inspection	Self inspection

The rules are to be given in the relevant execution standards.

B6(1) A partial factor for a material or product property or a member resistance can be reduced if an inspection class higher than that required according to Table B5 and/or more severe requirements are used

Relevant Background Documents

ISO 2394

JCSS documents (http://www.jcss.ethz.ch/)

Background document for the ENV-version of Basis of Design, ECCS/JCSS, 1996

IABSE Conferences Delft (1996) and Malta (2001)

Leonardo da Vinci Project CZ/02/B/F/PP-134007
Handbooks Implementtion of Eurocodes (2005)