NCCI: Elastic critical moment for lateral torsional buckling

This NCCI gives the expression of the elastic critical moment for doubly symmetric cross-sections. Values of the factors involved in the calculation are given for common cases. For a beam under a uniformly distributed load with end moments or a concentrated load at mid-span with end moments, the values for the factors are given in graphs.

Contents

1. General

For doubly symmetric cross-sections, the elastic critical moment M_{cr} may be calculated by the method given in paragraph 2.

For cases not covered by the method given in paragraph 2, the elastic critical moment may be determined by a buckling analysis of the beam provided that the calculation accounts for all the parameters liable to affect the value of M_{cr} :

- geometry of the cross-section
- warping rigidity
- position of the transverse loading with regard to the shear centre
- restraint conditions

The ***LTBeam*** software is specific software for the calculation of the critical moment M_{cr}. It may be downloaded free of charge from the following web site:

http://www.cticm.com

2. Method for doubly symmetric sections

The method given hereafter only applies to uniform straight members for which the cross-section is symmetric about the bending plane.

The conditions of restraint at each end are at least :

- restrained against lateral movement
- restrained against rotation about the longitudinal axis

The elastic critical moment may be calculated from the following formula derived from the buckling theory :

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{(kL)^2} \left\{ \sqrt{\left(\frac{k}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 GI_t}{\pi^2 EI_z} + (C_2 z_g)^2} - C_2 z_g \right\} \qquad \textbf{(1)}$$

where

E is the Young modulus (E = 210000 N/mm^2)

G is the shear modulus (G = 80770 N/mm^2)

I_z is the second moment of area about the weak axis

I_t is the torsion constant

I_w is the warping constant

L is the beam length between points which have lateral restraint

k and k_w are effective length factors

z_g is the distance between the point of load application and the shear centre.

Note : for doubly symmetric sections, the shear centre coincides with the centroid.

C_1 and C_2 are coefficients depending on the loading and end restraint conditions (see §3).

The factor k refers to end rotation on plan. It is analogous to the ratio of the buckling length to the system length for a compression member. k should be taken as not less than 1,0 unless less than 1,0 can be justified.

The factor k_w refers to end warping. Unless special provision for warping fixity is made, k_w should be taken as 1,0.

In the general case z_g is positive for loads acting towards the shear centre from their point of application (Figure 2.1).

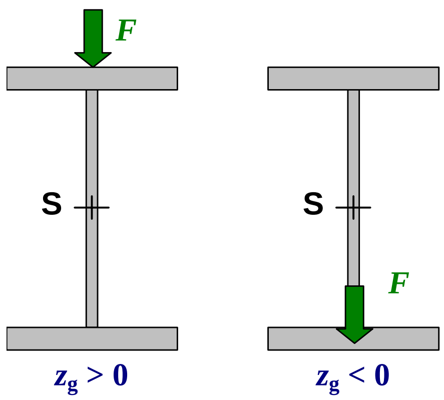


Figure 2.1 ***Point of application of the transverse load***

In the common case of normal support conditions at the ends (fork supports), k and k_w are taken equal to 1.

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{L^2} \left\{ \sqrt{\frac{I_w}{I_z} + \frac{L^2 GI_t}{\pi^2 EI_z} + (C_2 z_g)^2} - C_2 z_g \right\} \quad \textbf{(2)}$$

When the bending moment diagram is linear along a segment of a member delimited by lateral restraints, or when the transverse load is applied in the shear centre, $C_2\, z_g = 0$. The latter expression should be simplified as follows :

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{L^2} \sqrt{\frac{I_w}{I_z} + \frac{L^2 GI_t}{\pi^2 EI_z}} \quad \textbf{(3)}$$

For doubly symmetric I-profiles, the warping constant I_w may be calculated as follows :

$$I_w = \frac{I_z (h - t_f)^2}{4} \quad \textbf{(4)}$$

where

h is the total depth of the cross-section

t_f is the flange thickness

3. C_1 and C_2 factors

3.1 General

The C_1 and C_2 factors depend on various parameters :

- section properties,
- support conditions,
- moment diagram

It can be demonstrated that the C_1 and C_2 factors depend on the ratio :

$$\kappa = \frac{EI_w}{GI_t L^2} \quad \textbf{(5)}$$

The values given in this document have been calculated with the assumption that $\kappa = 0$. This assumption leads to conservative values of C_1.

3.2 Member with end moments only

The factor C_1 may be determined from Table 3.1 for a member with end moment loading.

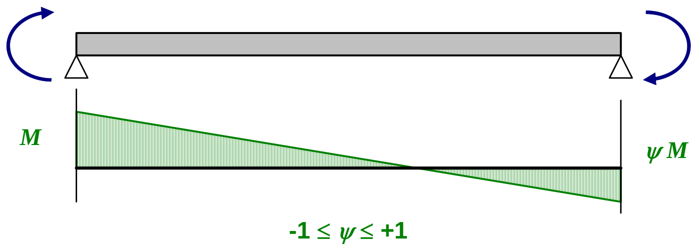


Figure 3.1 Member with end moments

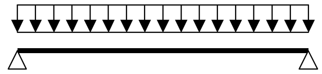
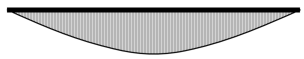
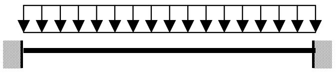
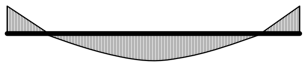
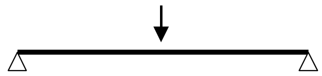
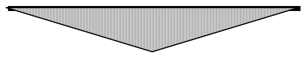
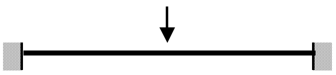
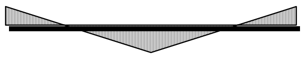
Table 3.1 Values of C_1 for end moment loading (for k = 1)

ψ	C_1
+1,00	1,00
+0,75	1,14
+0,50	1,31
+0,25	1,52
0,00	1,77
-0,25	2,05
-0,50	2,33
-0,75	2,57
-1,00	2,55

3.3 Member with transverse loading

Table 3.2 gives values of C_1 and C_2 for some cases of a member with transverse loading,

Table 3.2 Values of factors C_1 and C_2 for cases with transverse loading (for k = 1)

Loading and support conditions	Bending moment diagram	C_1	C_2
		1,127	0,454
		2,578	1,554
		1,348	0,630
		1,683	1,645

Note : the critical moment M_{cr} is calculated for the section with the maximal moment along the member

3.4 Member with end moments and transverse loading

For combined loading of end moments and transverse loads as shown in Figure 3.2, values of C_1 and C_2 may be obtained from the curves given hereafter. Two cases are considered:

Case a) end moments with a uniformly distributed load

Case b) end moments with a concentrated load at mid-span

The moment distribution may be defined using two parameters :

ψ is the ratio of end moments. By definition, M is the maximum end moment, and so :

$-1 \leq \psi \leq 1$ ($\psi = 1$ for a uniform moment)

μ is the ratio of the moment due to transverse load to the maximum end moment M

Case a) $\mu = \dfrac{qL^2}{8M}$

Case b) $\mu = \dfrac{FL}{4M}$

Sign convention for μ:

$\mu > 0$ if M and the transverse load (q or F), each supposed acting alone, bend the beam in the same direction (e.g. as shown in the figure below)

$\mu < 0$ otherwise

The values of C_1 and C_2 have been determined for $k = 1$ and $k_w = 1$.

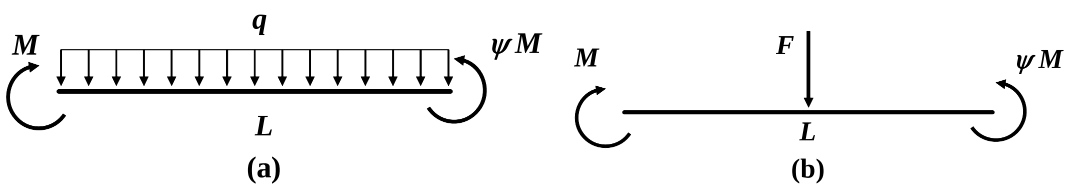
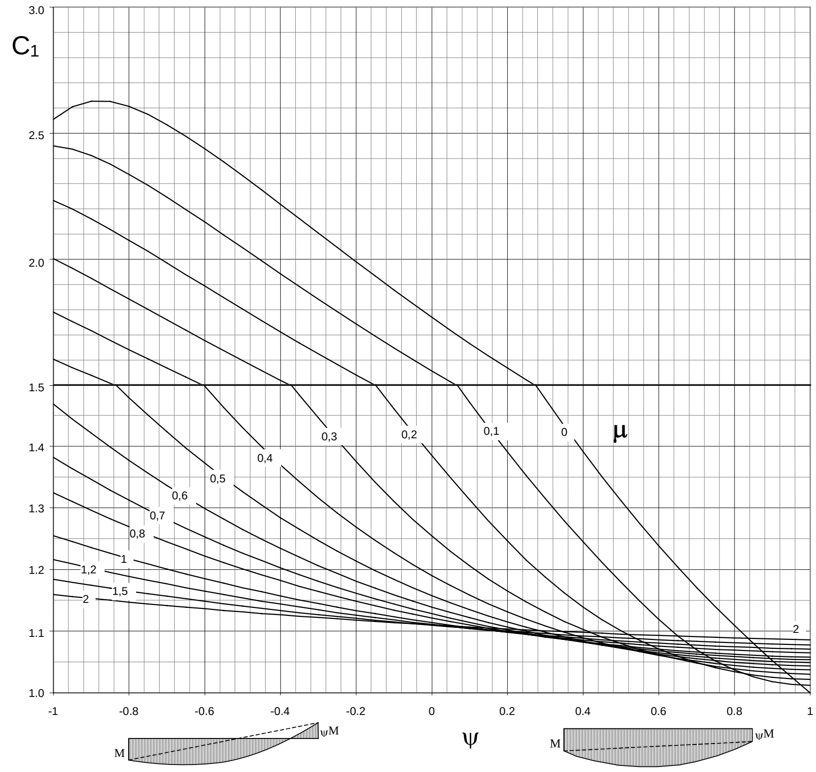
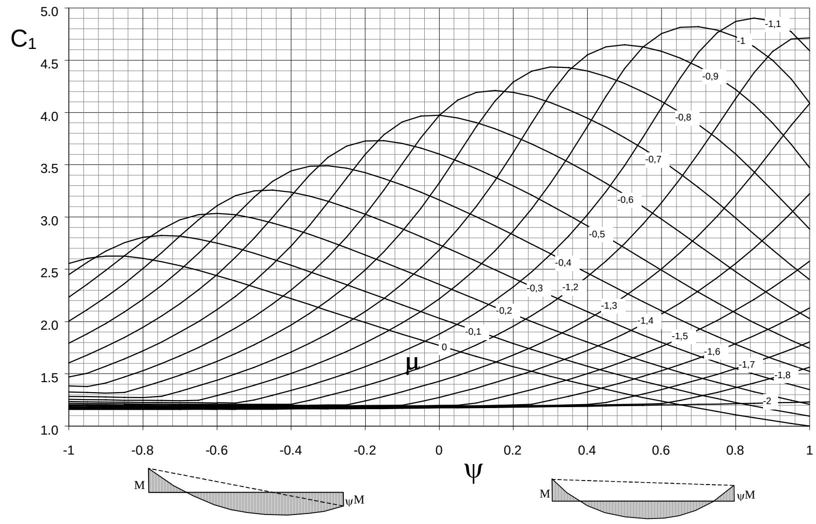


Figure 3.2 End moments with a transverse load

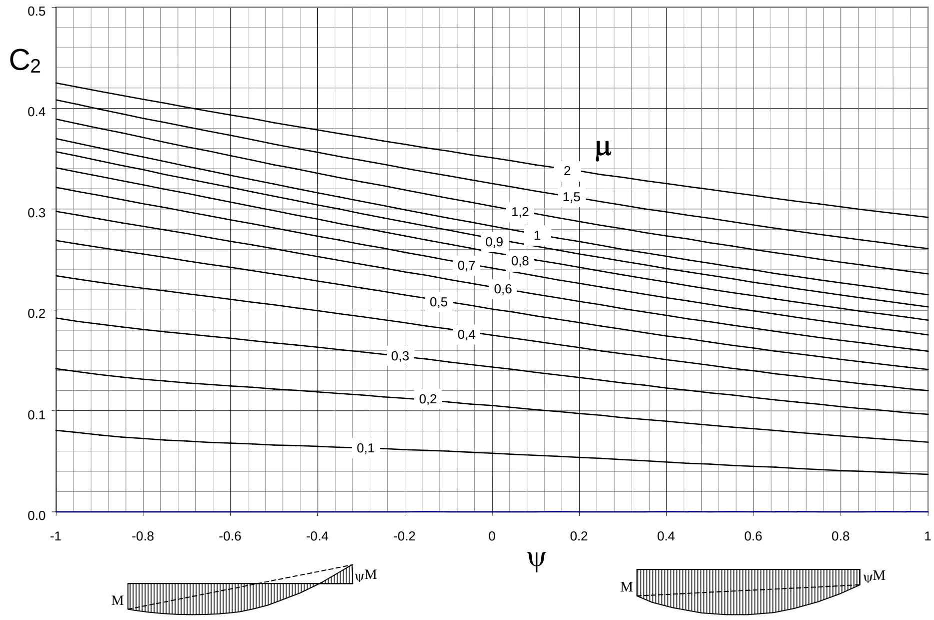


$\mu > 0$

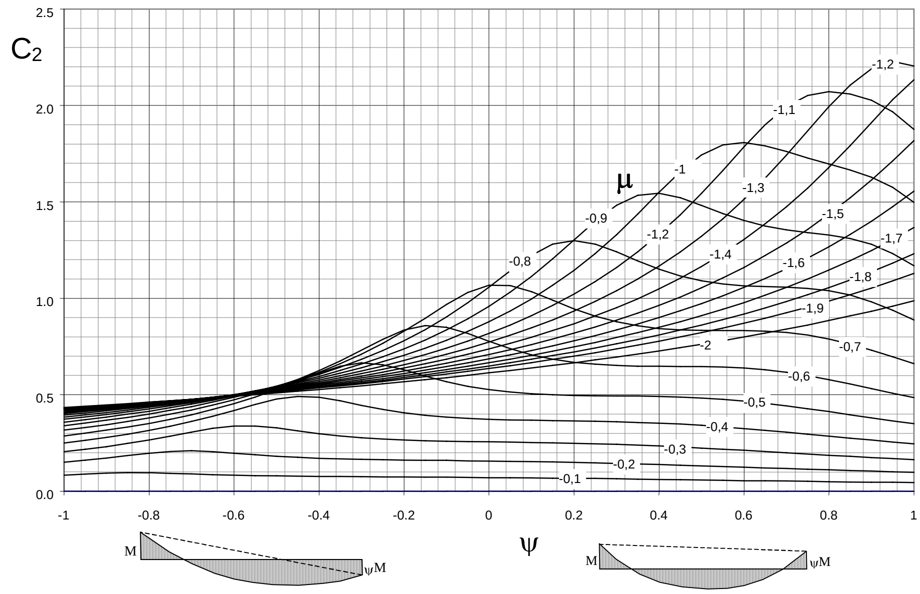


$\mu < 0$

Figure 3.3 End moments and uniformly distributed load – Factor C_1

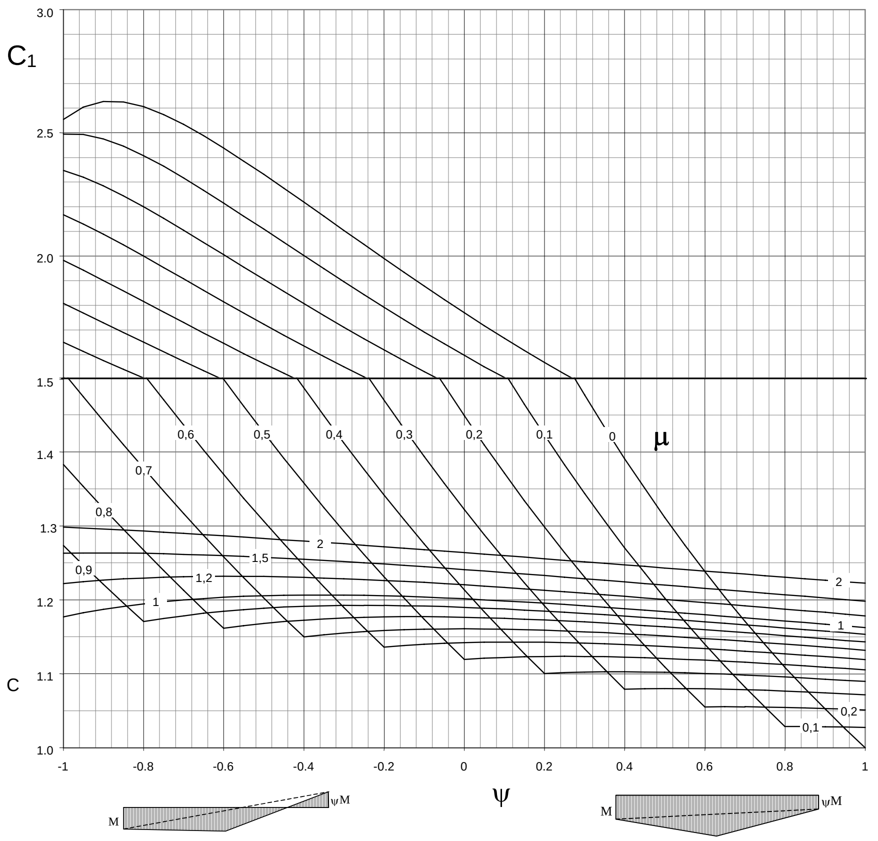


$\mu > 0$

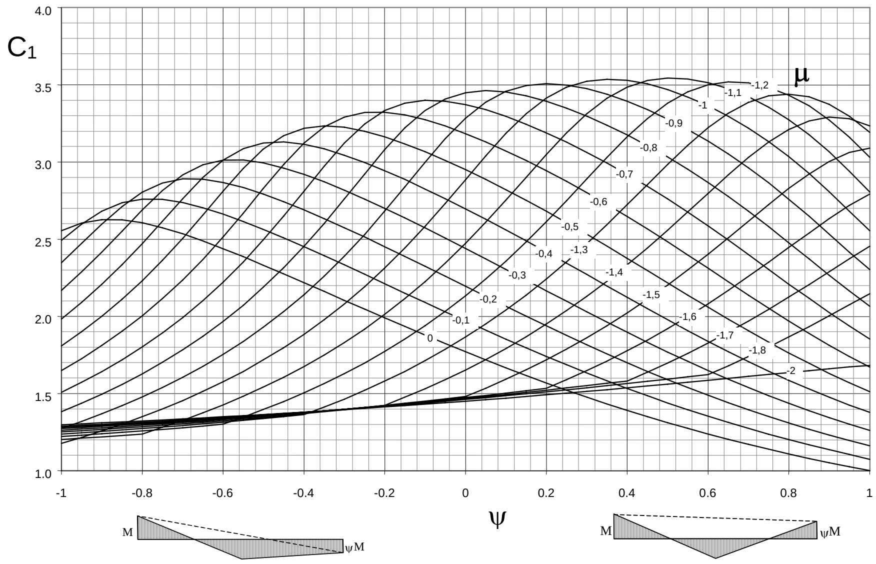


$\mu < 0$

Figure 3.4 End moments and uniformly distributed load – Factor C_2

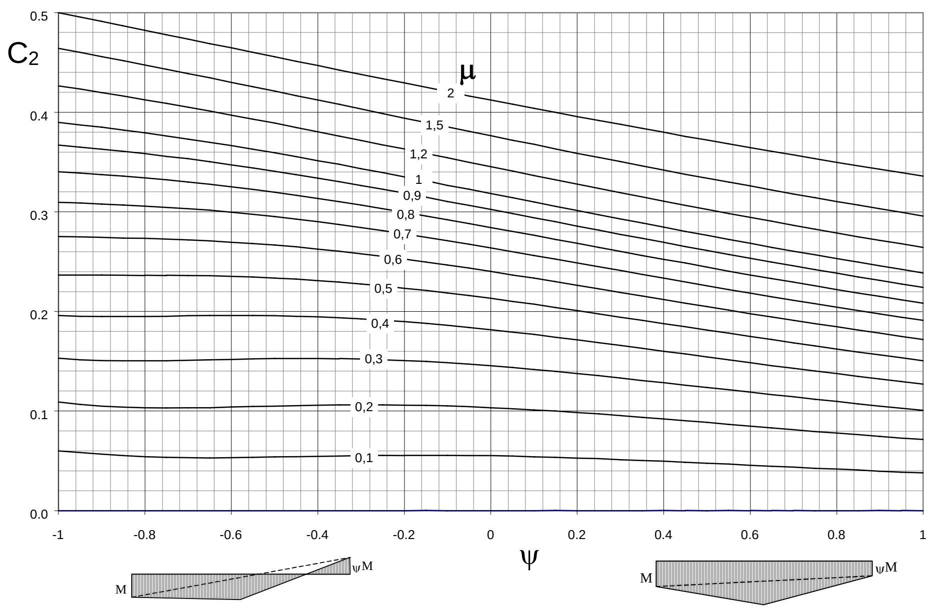


$\mu > 0$

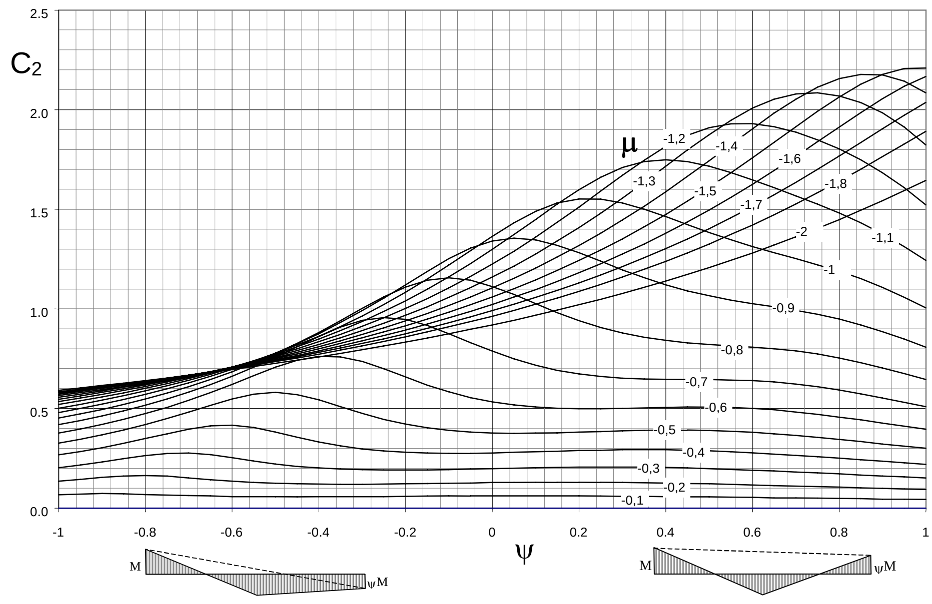


$\mu < 0$

Figure 3.5 End moments and point load at mid-span – Factor C_1



$\mu > 0$



$\mu < 0$

Figure 3.6 End moments and point load at mid-span – Factor C_2

4. References

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