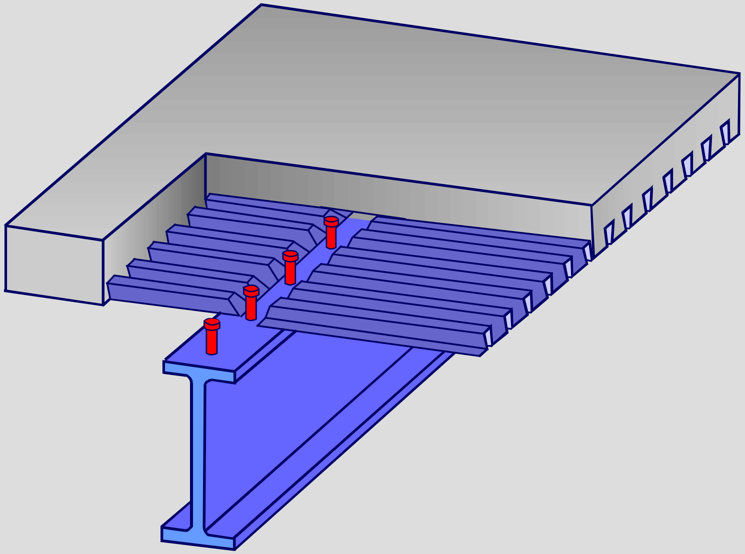
Eurocodes

Background and Applications
Dissemination of information for training
18-20 February 2008, Brussels



Eurocode 4

Serviceability limit states of composite beams

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- Limitation of stresses
- Limitation of deflections
- crack width control
- vibrations
- web breathing

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Serviceability limit states

characteristic combination: $E_d = E\left\{ \sum G_{k,j} + P_k + Q_{k,1} + \sum \psi_{0,i}\, Q_{k,i} \right\}$

frequent combination: $E_d = E\left\{ \sum G_{k,j} + P_k + \psi_{1,1} Q_{k,1} + \sum \psi_{2,i}\, Q_{k,i} \right\}$

quasi-permanent combination: $E_d = E\left\{ \sum G_{k,j} + P_k + \sum \psi_{2,i}\, Q_{k,i} \right\}$

serviceability limit states
$E_d \leq C_d$:

$C_d =$
- deformation
- crack width
- excessive compressive stresses in concrete
- excessive slip in the interface between steel and concrete
- excessive creep deformation
- web breathing
- vibrations

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Part 2:

Global analysis for serviceability limit states

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Global analysis - General

Calculation of internal forces, deformations and stresses at serviceability limit state shall take into account the following effects:

- shear lag;
- creep and shrinkage of concrete;
- cracking of concrete and tension stiffening of concrete;
- sequence of construction;
- increased flexibility resulting from significant incomplete interaction due to slip of shear connection;
- inelastic behaviour of steel and reinforcement, if any;
- torsional and distorsional warping, if any.

Shear lag- effective width

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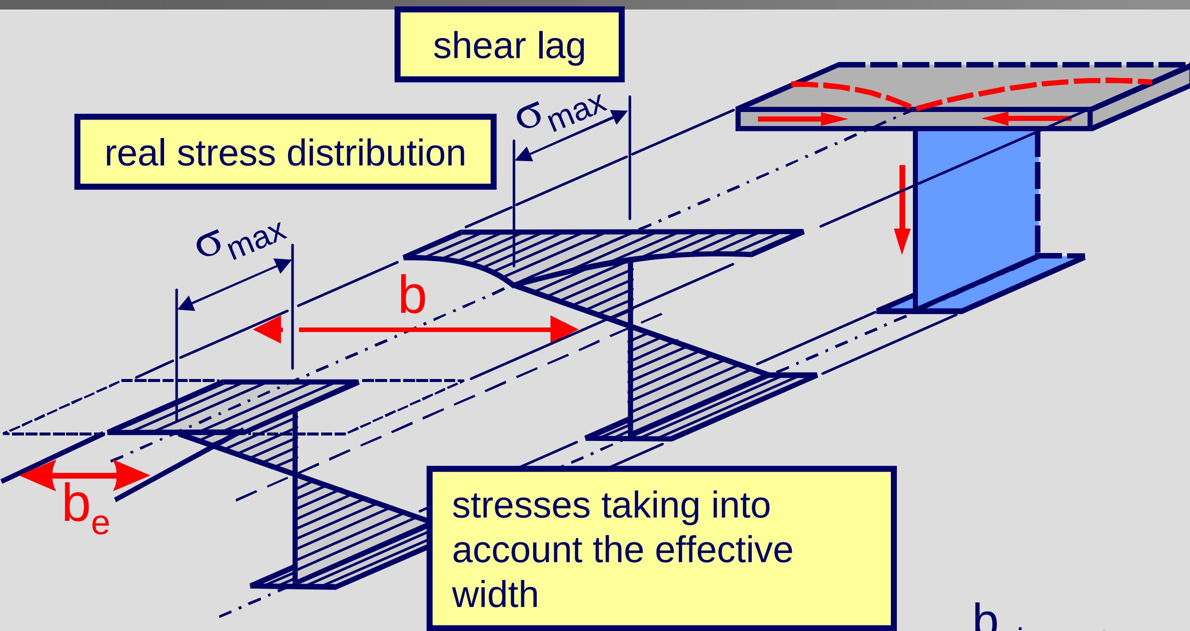
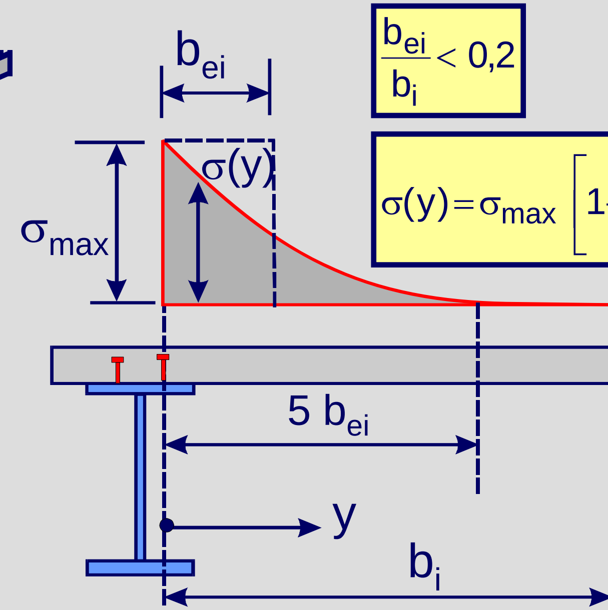
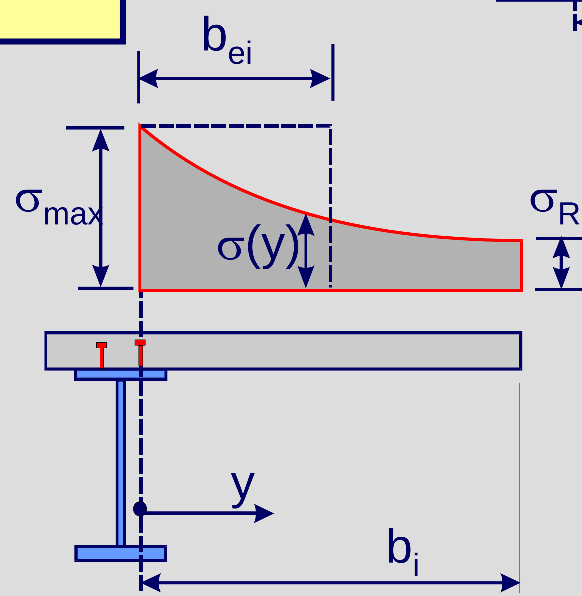




$$\frac{b_{ei}}{b_i} < 0{,}2$$

$$\sigma(y) = \sigma_{max}\left[1 - \frac{y}{b_i}\right]^4$$



$$\frac{b_{ei}}{b_i} \geq 0{,}2$$

$$\sigma_R = 1{,}25\left[\frac{b_{ei}}{b_i} - 0{,}2\right]\sigma_{max}$$

$$\sigma(y) = \sigma_R + [\sigma_{max} - \sigma_R]\left[1 - \frac{y}{b_i}\right]^4$$

The flexibility of steel or concrete flanges affected by shear in their plane (shear lag) shall be used either by rigorous analysis, or by using an effective width b_e

Effective width of concrete flanges

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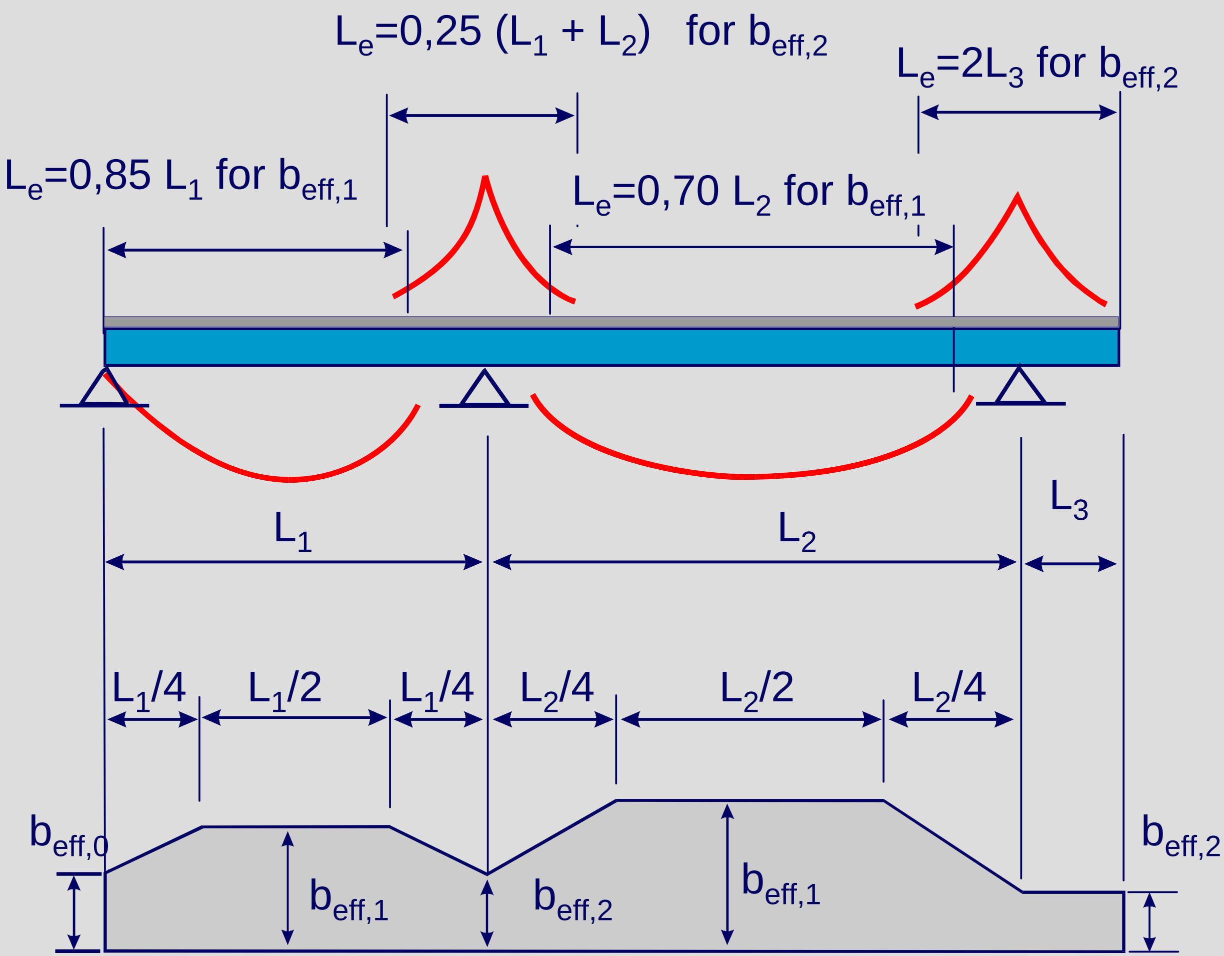

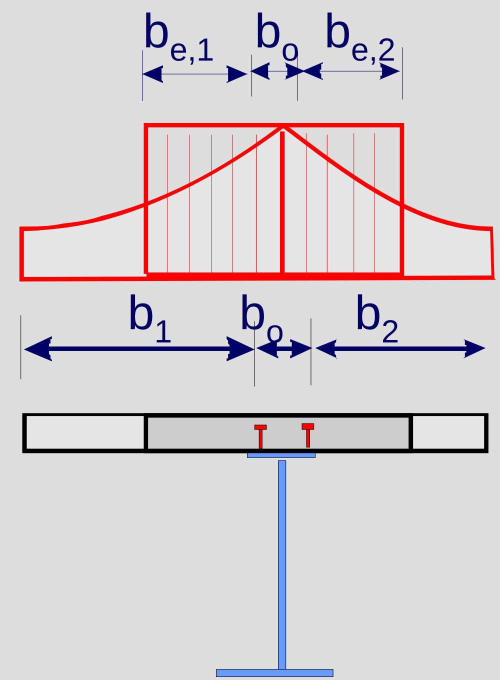

midspan regions and internal supports:

$b_{eff} = b_0 + b_{e,1} + b_{e,2}$

$b_{e,i} = L_e/8$

L_e – equivalent length

end supports: $b_{eff} = b_0 + \beta_1 b_{e,1} + \beta_2 b_{e,2}$

$\beta_i = (0{,}55 + 0{,}025\, L_e/b_i) \leq 1{,}0$

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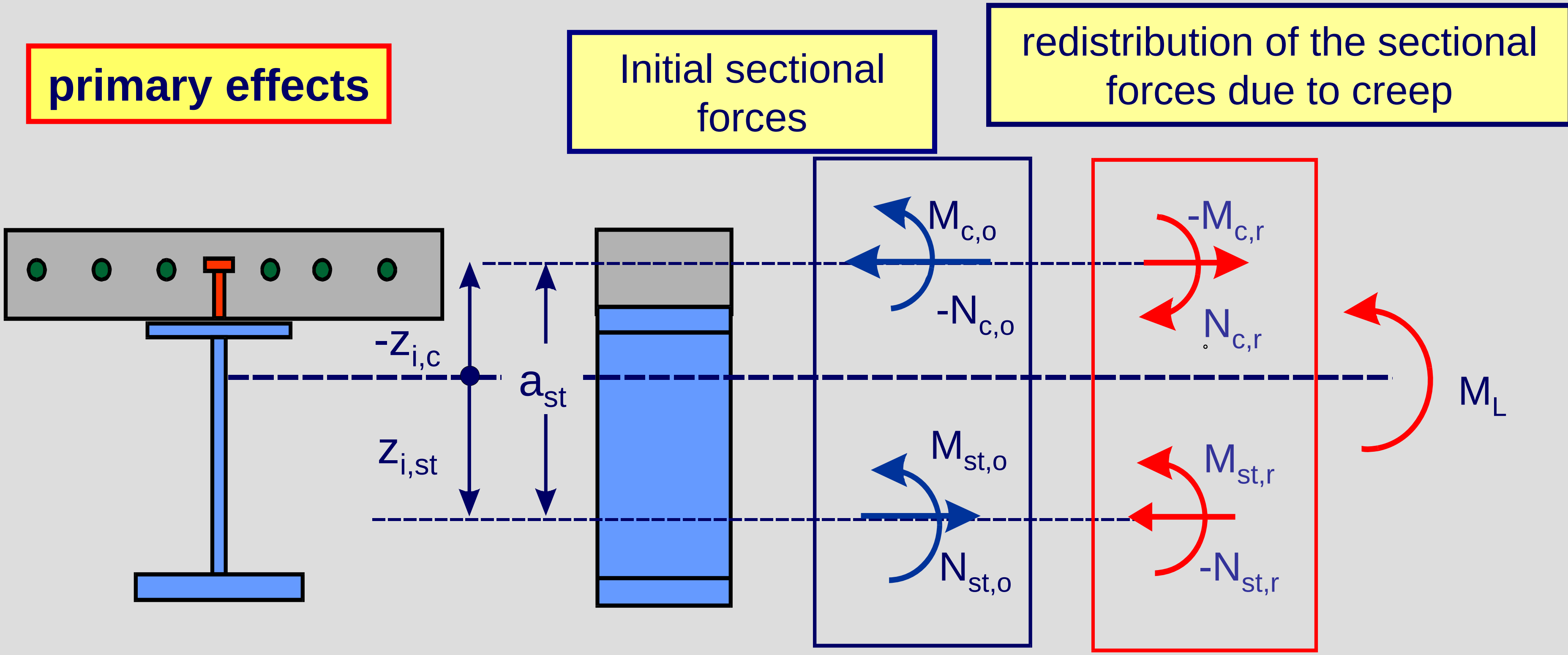
Effects of creep of concrete

primary effects



The effects of shrinkage and creep of concrete and non-uniform changes of temperature result in internal forces in cross sections, and curvatures and longitudinal strains in members; the effects that occur in statically determinate structures, and in statically indeterminate structures when compatibility of the deformations is not considered, shall be classified as primary effects.

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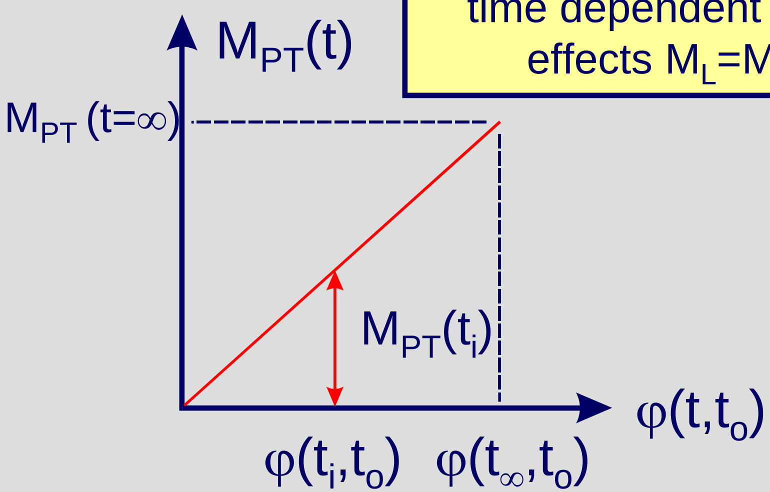
Types of loading and action effects:

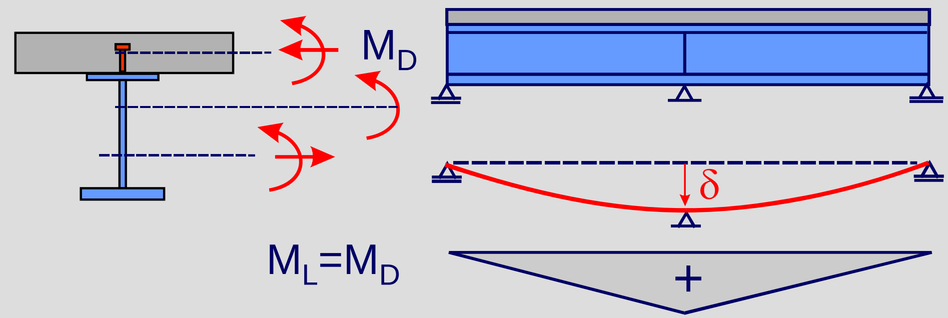
In the following the different types of loading and action effects are distinguished by a subscript L :

- **L=P** for permanent action effects not changing with time
- **L=PT** time-dependent action effects developing affine to the creep coefficient
- **L=S** action effects caused by shrinkage of concrete
- **L=D** action effects due to prestressing by imposed deformations (e.g. jacking of supports)

time dependent action effects $M_L=M_{PT}$:



action effects caused by prestressing due to imposed deformation $M_L=M_D$:



Modular ratios taking into account effects of creep

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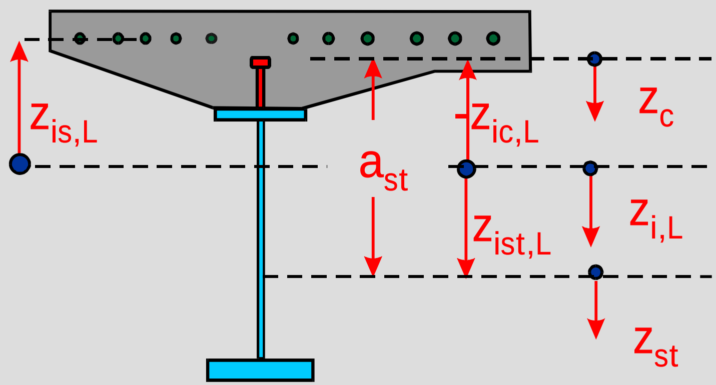

centroidal axis of the concrete section

centroidal axis of the transformed composite section

centroidal axis of the steel section (structural steel and reinforcement)

Modular ratios:

$$n_L = n_o \left[1 + \psi_L \, \varphi(t, t_o) \right] \qquad n_o = \frac{E_a}{E_{cm}}$$

action	creep multiplier
short term loading	$\Psi=0$
permanent action not changing in time	$\Psi_P=1,10$
shrinkage	$\Psi_S=0,55$
prestressing by controlled imposed deformations	$\Psi_D=1,50$
time-dependent action effects	$\Psi_{PT}=0,55$

Elastic cross-section properties of the composite section taking into account creep effects

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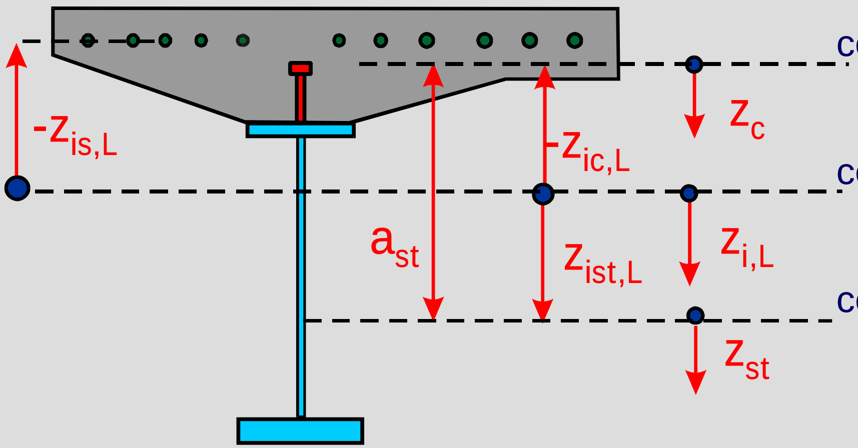

Modular ratio taking into account creep effect:

$$n_L = n_0\,(1 + \psi_L\,\varphi(t, t_0))$$

$$n_0 = \frac{E_{st}}{E_{cm}(t_0)}$$

Transformed cross-section properties of the concrete section:

$$A_{c,L} = A_c\,/\,n_L \qquad J_{c,L} = J_c\,/\,n_L$$

Transformed cross-section area of the composite section:

$$A_{i,L} = A_{St} + A_{c,L}$$

Distance between the centroidal axes of the concrete and the composite section:

$$z_{ic,L} = -A_{st}\,a_{st} / A_{i,L}$$

Second moment of area of the composite section:

$$J_{i,L} = J_{st} + J_{c,L} + A_{st}\,A_{c,L}\,a_{st}^2\,/\,A_{i,L}$$

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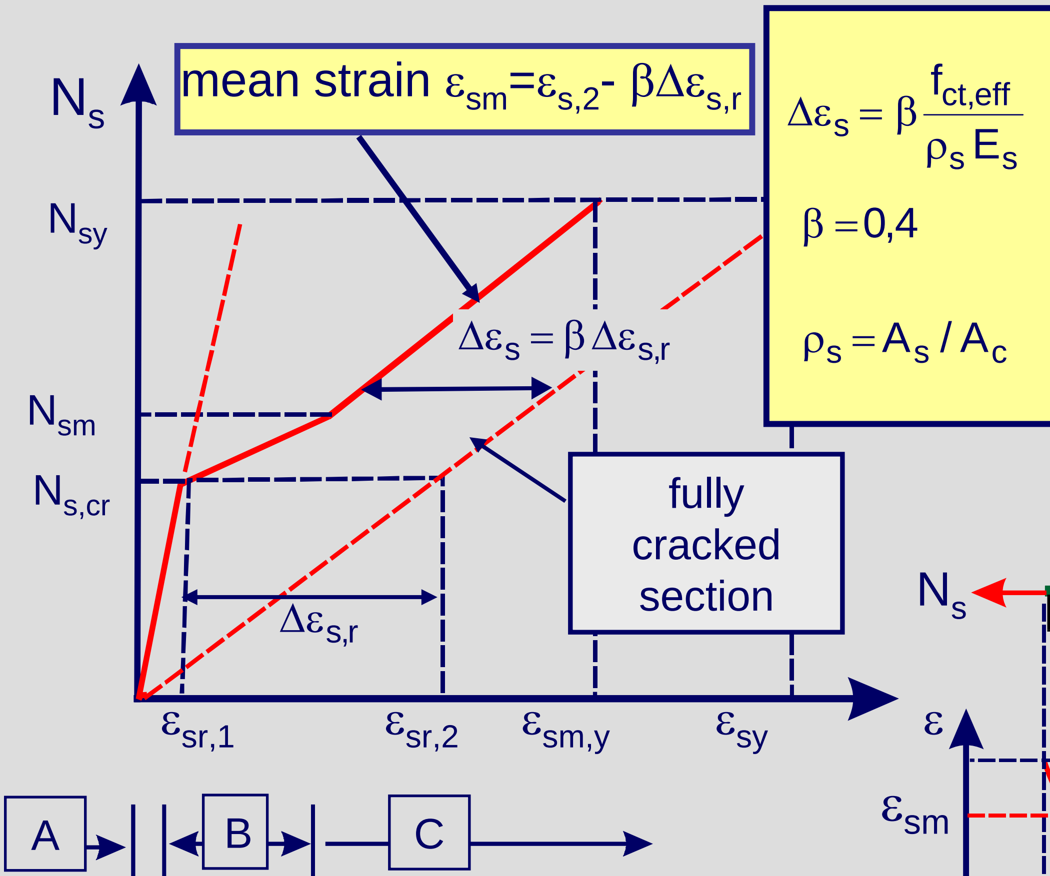
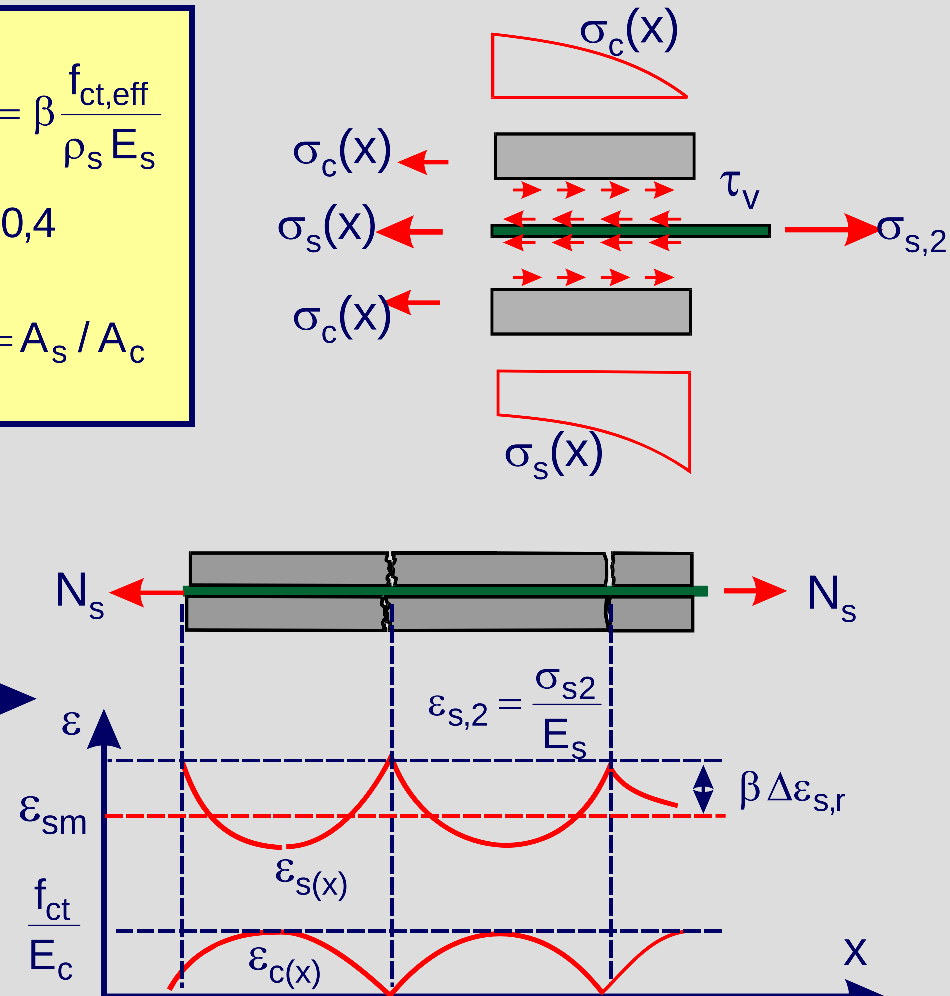
Effects of cracking of concrete and tension stiffening of concrete between cracks





stage A: uncracked section
stage B: initial crack formation
stage C: stabilised crack formation

Influence of tension stiffening of concrete on stresses in reinforcement

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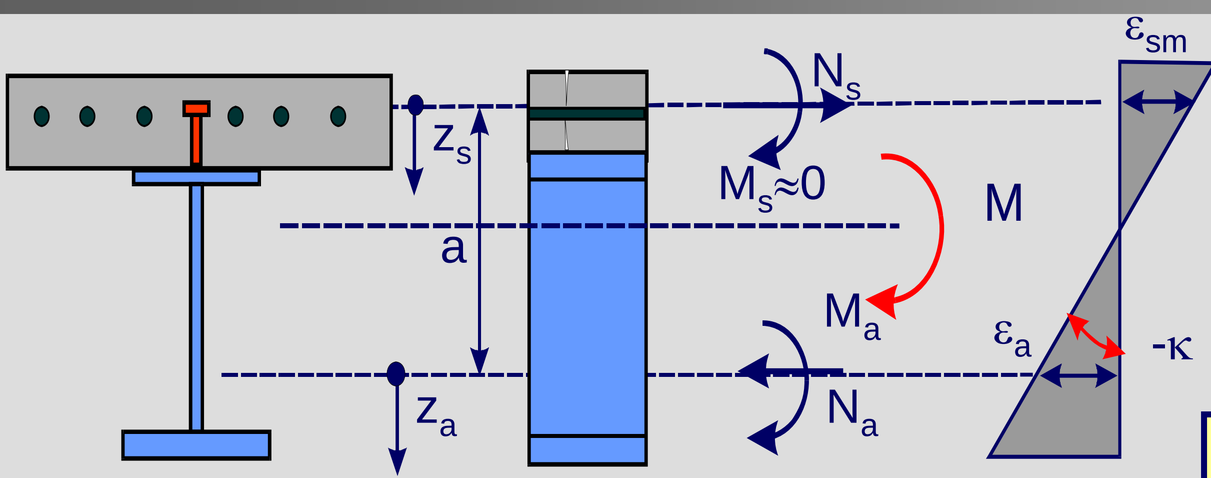


equilibrium:

$$M_a = M - N_s\, a$$

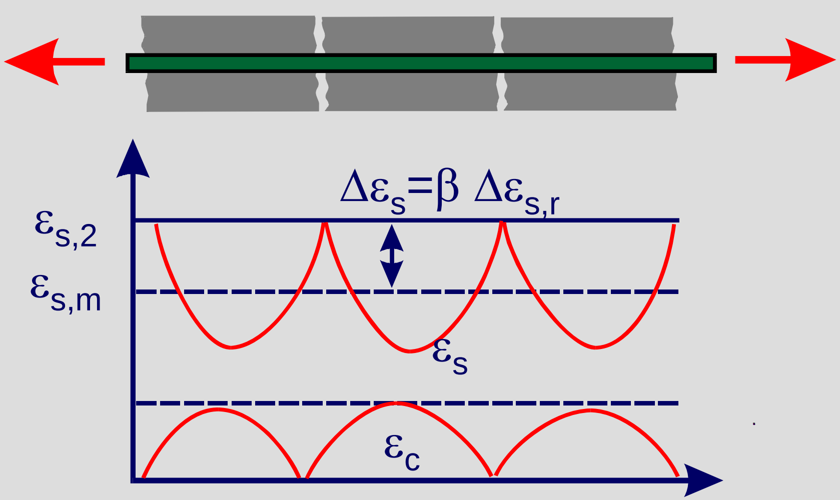
$$N_a = -N_s$$

compatibility:

$$\varepsilon_{sm} = \varepsilon_a + \kappa\, a$$

$$\varepsilon_{sm} + \frac{N_s}{E_a A_a} + \frac{N_s\, a^2}{E_a A_a} = \frac{M\ a}{E_a J_a}$$

mean strain in the concrete slab:



mean strain in the concrete slab:

$$\varepsilon_{sm} = \varepsilon_{s2} - \beta\Delta\varepsilon_{sr} = \frac{N_s}{E_s A_s} - \beta\frac{f_{ct,eff}}{\rho_s E_s}$$

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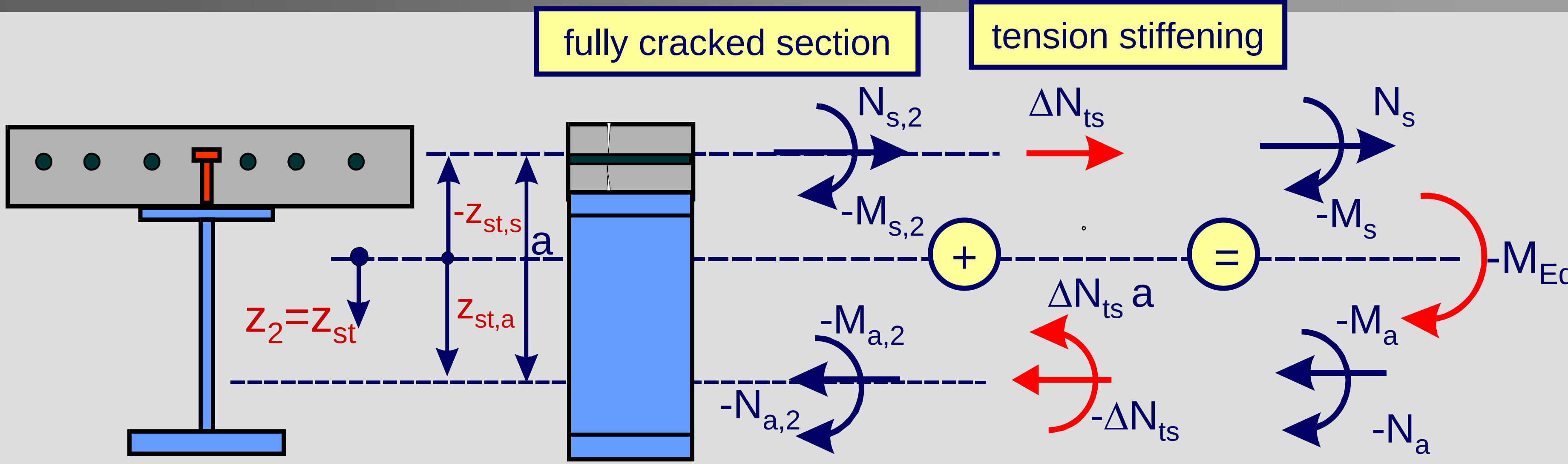
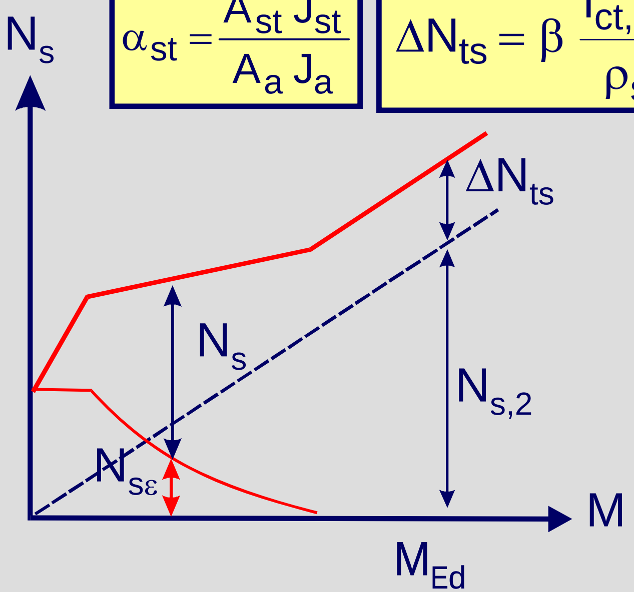
Redistribution of sectional forces due to tension stiffening



$$\alpha_{st} = \frac{A_{st}\, J_{st}}{A_a\, J_a}$$

$$\Delta N_{ts} = \beta\, \frac{f_{ct,eff}\, A_s}{\rho_s\, \alpha_{st}}$$



Sectional forces:

$$J_{st} = J_2$$

$$N_s = N_{s2} + \Delta N_{ts} = M_{Ed}\,\frac{A_s\, z_{st,s}}{J_{st}} + \Delta N_{ts}$$

$$M_s = M_{Ed}\,\frac{J_s}{J_{st}}$$

$$N_a = N_{a2} - \Delta N_{ts} = M_{Ed}\,\frac{A_a\, z_{st,a}}{J_{st}} - \Delta N_{ts}$$

$$M_a = M_{a2} + \Delta N_{ts}\, a \ = M_{Ed}\,\frac{J_a}{J_{st}} + \Delta N_{ts}\, a$$

Stresses taking into account tension stiffening of concrete

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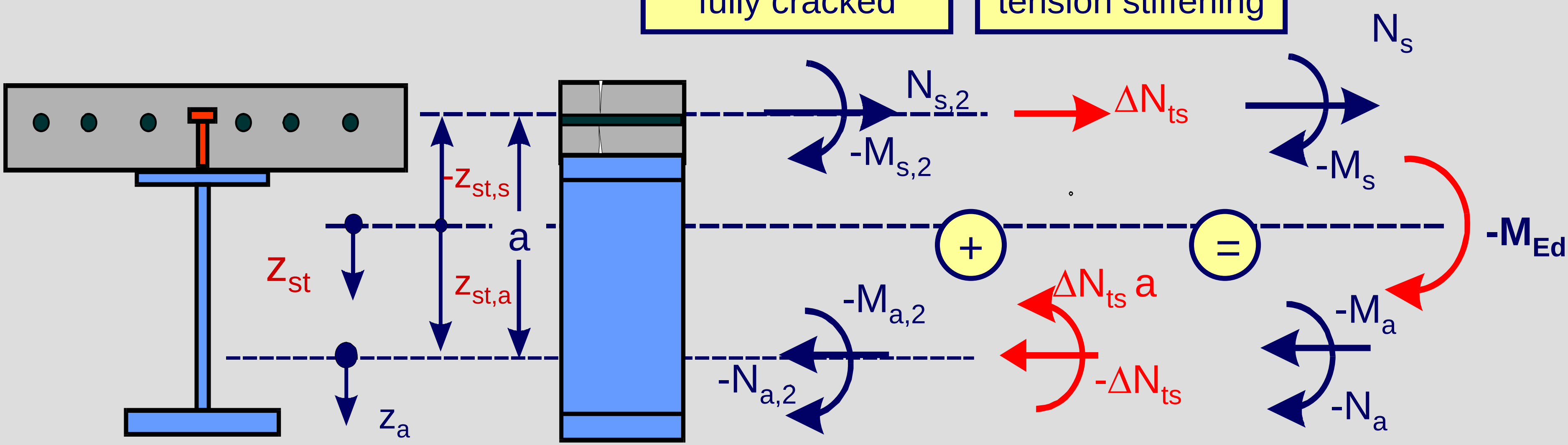

reinforcement:

$$\sigma_s = \sigma_{s,2} + \beta \frac{f_{ctm}}{\rho_s \alpha_{st}}$$

$$\sigma_s = \frac{M_{Ed}}{J_{st}} z_{st,s} + \beta \frac{f_{ctm}}{\rho_s \alpha_{st}}$$

structural steel:

$$\sigma_a = \sigma_{a,2} - \frac{\Delta N_{ts}}{A_a} + \frac{\Delta N_{ts}\, a}{J_a} z_a$$

$$\sigma_a = \frac{M_{Ed}}{J_{st}} z_{st} - \frac{\Delta N_{ts}}{A_a} + \frac{\Delta N_{ts}\, a}{J_a} z_a$$

$$\alpha_{st} = \frac{A_{st} J_{st}}{A_a J_a}$$

$$\Delta N_{ts} = \beta \frac{f_{ctm} A_s}{\rho_s \alpha_{st}}$$

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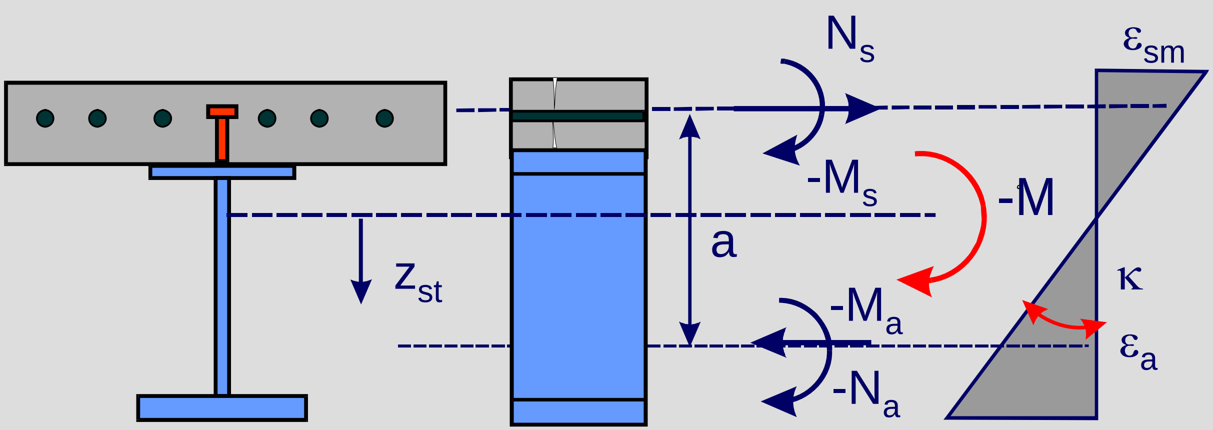
Influence of tension stiffening on flexural stiffness

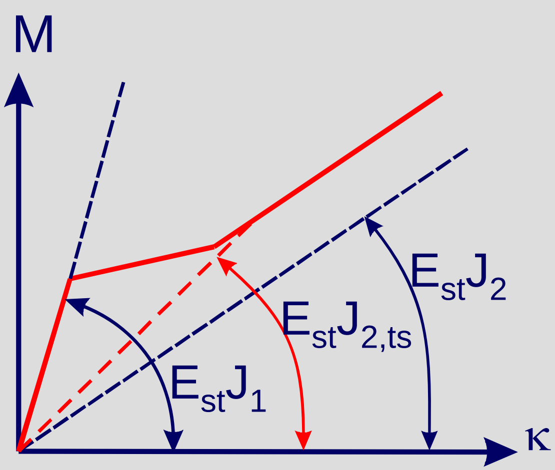


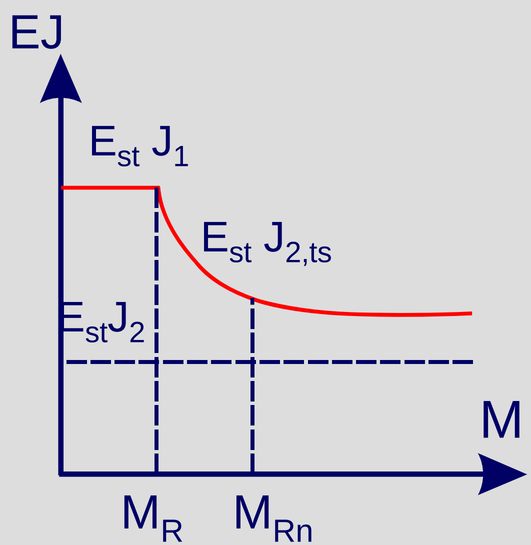
Curvature:

$$\kappa = \frac{M}{E_{st}\, I_{2,ts}} = \frac{M_a}{E_{st}\, J_a} = \frac{M - N_s\, a}{E_{st}\, J_a}$$

Effective flexural stiffness:

$$E_{st} J_{2,ts} = \frac{E_a\, J_a}{1 - \dfrac{(N_s - N_{s,\varepsilon})\, a}{M}}$$





$E_{st}J_1$ uncracked section
$E_{st}J_2$ fully cracked section
$E_{st}J_{2,ts}$ effective flexural stiffness taking into account tension stiffening of concrete

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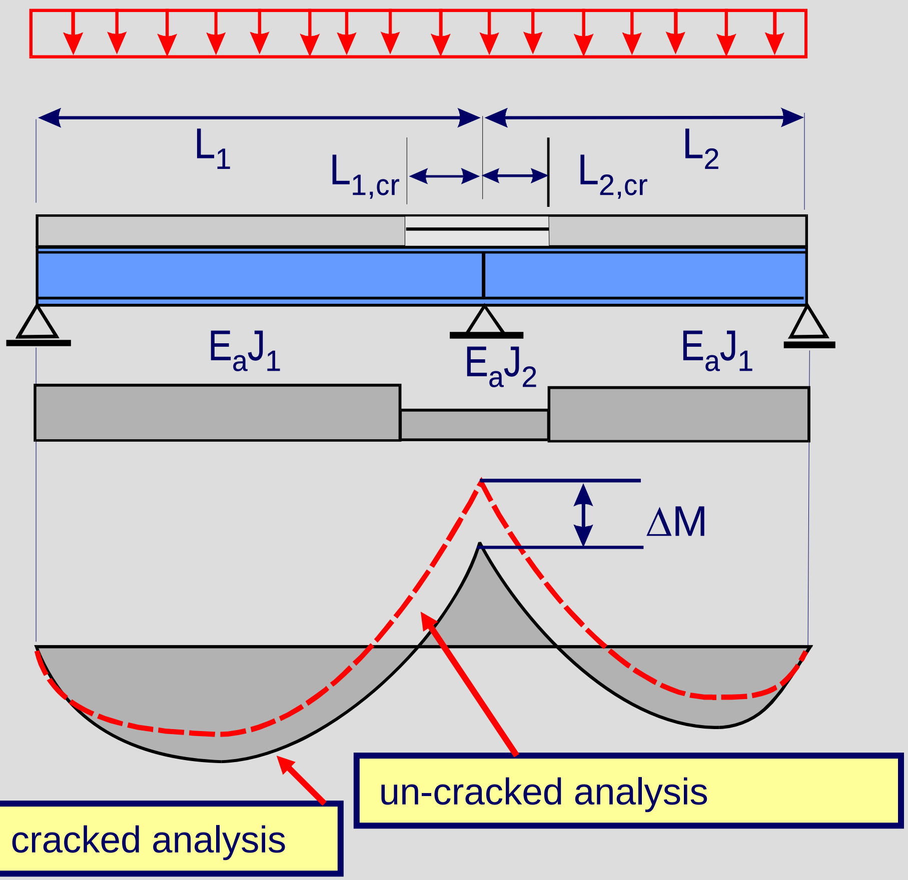
E_aJ_1 – un-cracked flexural stiffness

E_aJ_2 – cracked flexural stiffness



- Determination of internal forces by un-cracked analysis for the characteristic combination.
- Determination of the cracked regions with the extreme fibre concrete tensile stress $\sigma_{c,max} = 2{,}0\ f_{ct,m}$.
- Reduction of flexural stiffness to E_aJ_2 in the cracked regions.
- New structural analysis for the new distribution of flexural stiffness.

ΔM Redistribution of bending moments due to cracking of concrete

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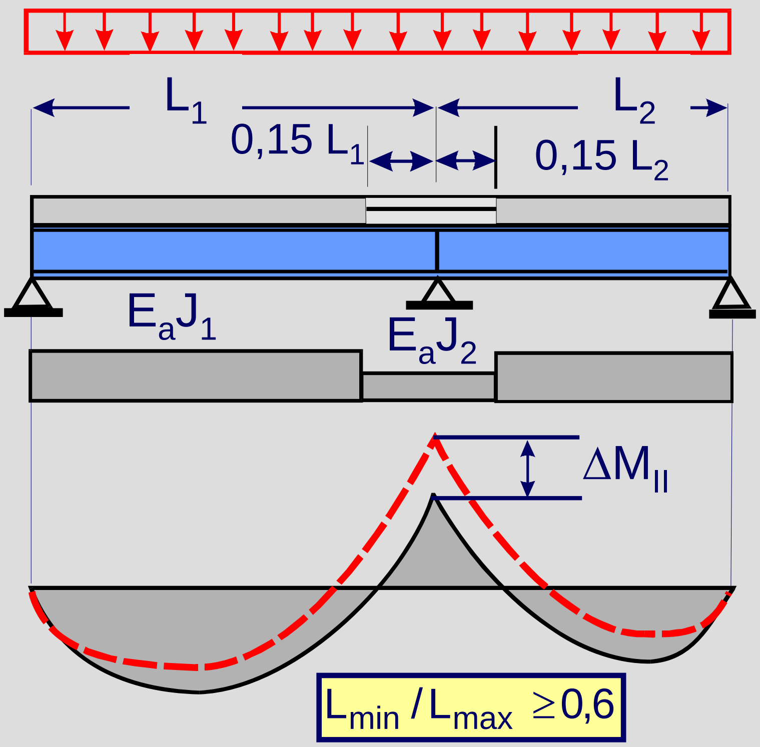
Effects of cracking of concrete – simplified method



For continuous composite beams with the concrete flanges above the steel section and not pre-stressed, including beams in frames that resist horizontal forces by bracing, a simplified method may be used. Where all the ratios of the length of adjacent continuous spans (shorter/longer) between supports are at least 0,6, the effect of cracking may be taken into account by using the flexural stiffness $E_a\ J_2$ over 15% of the span on each side of each internal support, and as the un-cracked values $E_a\ J_1$ elsewhere.

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Part 3:

Limitation of crack width

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General considerations

minimum reinforcement

If crack width control is required, a minimum amount of bonded reinforcement is required to control cracking in areas where tension due to restraint and or direct loading is expected. The amount may be estimated from equilibrium between the tensile force in concrete just before cracking and the tensile force in the reinforcement at yielding or at a lower stress if necessary to limit the crack width. According to Eurocode 4-1-1 the minimum reinforcement should be placed, where under the characteristic combination of actions, stresses in concrete are tensile.

control of cracking due to direct loading

Where at least the minimum reinforcement is provided, the limitation of crack width for direct loading may generally be achieved by limiting bar spacing or bar diameters. Maximum bar spacing and maximum bar diameter depend on the stress σ_s in the reinforcement and the design crack width.

Recommended values for w_{max}

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Exposure class	reinforced members, prestressed members with unbonded tendons and members prestressed by controlled imposed deformations	prestressed members with bonded tendons
	quasi - permanent load combination	frequent load combination
XO, XC1	0,4 mm (1)	0,2 mm
XC2, XC3,XC4	0,3 mm	0,2 mm (2)
XD1,XD2,XS1, XS2,XS3		decompression

(1) For XO and XC1 exposure classes, crack width has no influence on durability and this limit is set to guarantee acceptable appearance. In absence of appearance conditions this limit may be relaxed.

(2) For these exposure classes, in addition, decompression should be checked under the quasi-permanent combination of loads.

Exposure classes according to EN 1992-1-1 (risk of corrosion of reinforcement)

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Class	Description of environment	Examples
no risk of corrosion or attack		
XO	for concrete without reinforcement, for concrete with reinforcement : very dry	concrete inside buildings with very low air humidity
Corrosion induced by carbonation		
XC1	dry or permanently wet	concrete inside buildings with low air humidity
XC2	wet, rarely dry	concrete surfaces subjected to long term water contact, foundations
XC3	moderate humidity	external concrete sheltered from rain
XC4	cyclic wet and dry	concrete surfaces subject to water contact not within class XC2
Corrosion induced by chlorides		
XD1	moderate humidity	concrete surfaces exposed to airborne chlorides
XD2	wet, rarely dry	swimming pools, members exposed to industrial waters containing chlorides
XD3	cyclic wet and dry	car park slabs, pavements, parts of bridges exposed to spray containing
Corrosion induced by chlorides from sea water		
XS1	exposed to airborne salt	structures near to or on the coast
XS2	permanently submerged	parts of marine structures
XS3	tidal, splash and spray zones	parts of marine structures

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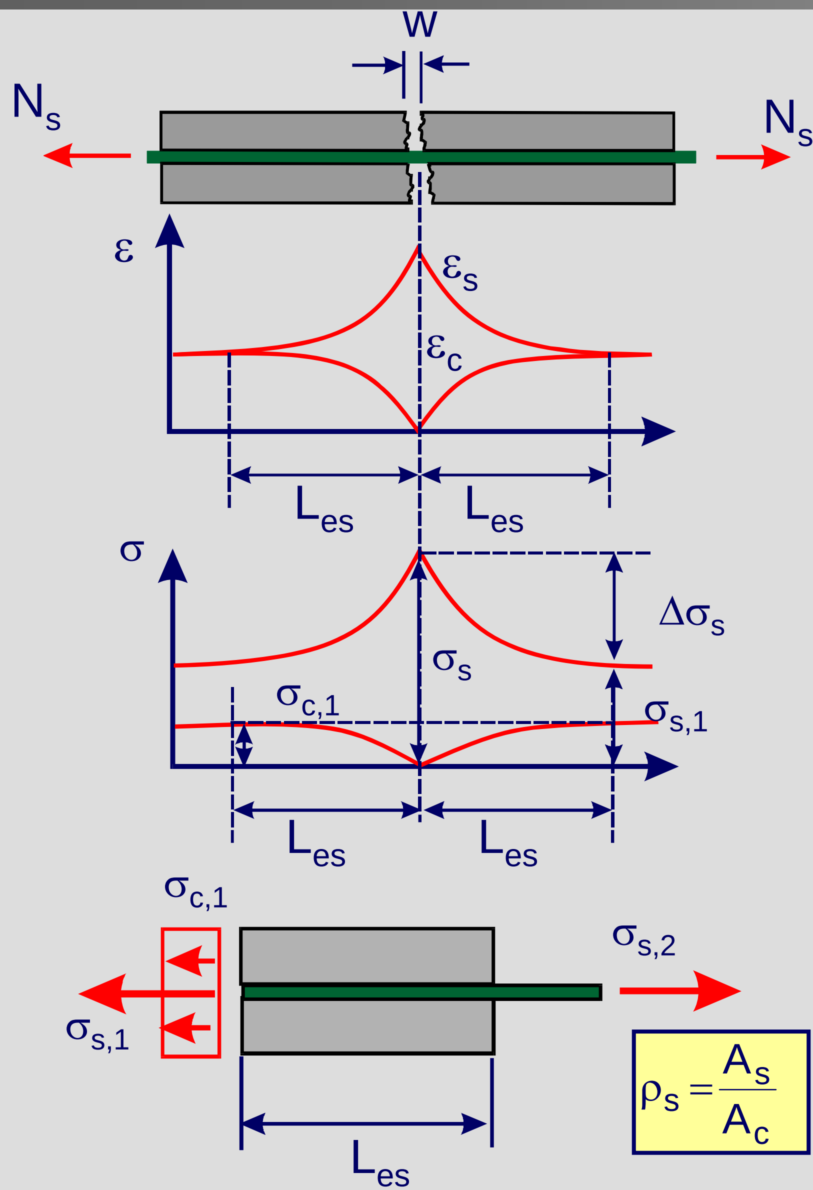
Cracking of concrete (initial crack formation)



$$\rho_s = \frac{A_s}{A_c}$$

Equilibrium in longitudinal direction:

$$\sigma_s A_s = \sigma_{s,1} A_s + \sigma_{c,1} A_c$$

Compatibility at the end of the introduction length:

$$\varepsilon_{s,1} = \varepsilon_{c,1} \;\Rightarrow\; \frac{\sigma_{s,1}}{E_s} = \frac{\sigma_{c,1}}{E_c}$$

$$\sigma_{s,1} = \sigma_s \left[\frac{\rho_s n_o}{1 + \rho_s n_o} \right] \qquad n_o = \frac{E_s}{E_c}$$

Change of stresses in reinforcement due to cracking:

$$\Delta\sigma_s = \sigma_s - \sigma_{s,1} = \frac{\sigma_s}{1 + \rho_s n_o}$$

$$N_{s,r} = f_{ctm} A_c \left(1 + \rho_s n_o\right)$$

- A_s cross-section area of reinforcement
- ρ_s reinforcement ratio
- f_{ctm} mean value of tensile strength of concrete

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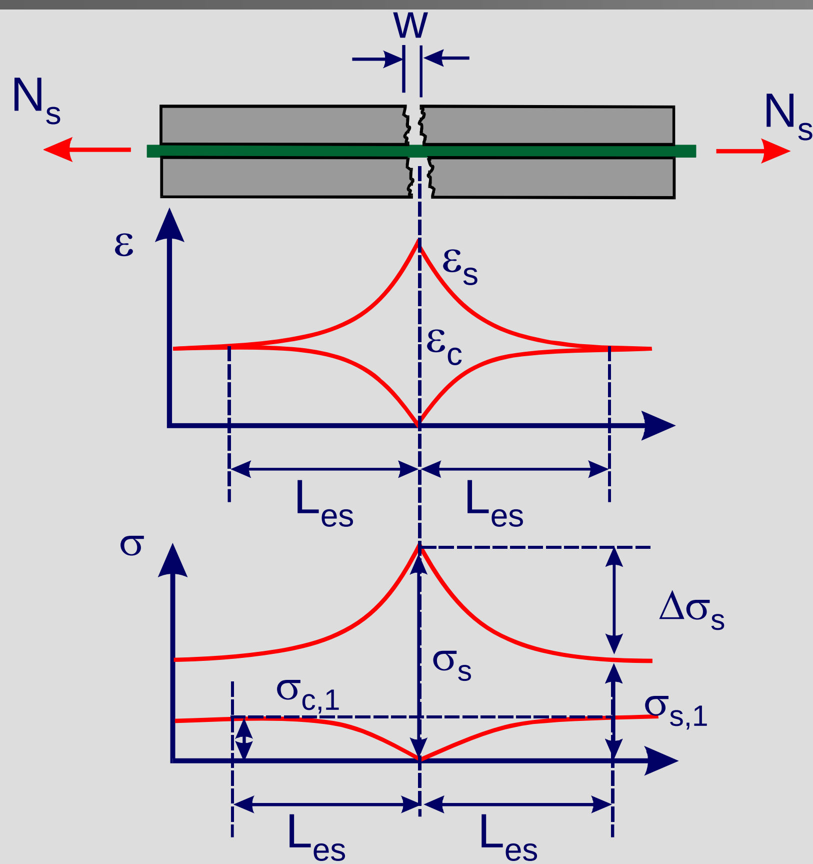


Change of stresses in reinforcement due to cracking:

$$\Delta\sigma_s = \sigma_s - \sigma_{s,1} = \frac{\sigma_s}{1+\rho_s n_o}$$

Equilibrium in longitudinal direction

$$L_{es} U_s \; \tau_{sm} = \Delta\sigma_s \; A_s$$

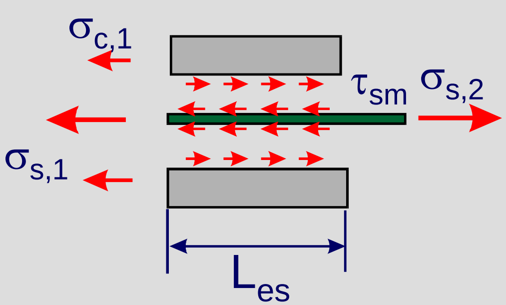
$$L_{es} \pi \; d_s \; \tau_{sm} = \Delta\sigma_s \frac{\pi d_s^2}{4}$$

introduction length L_{Es}

$$\rho_s = \frac{A_s}{A_c}$$

$$L_{es} = \frac{\sigma_s d_s}{4 \tau_{sm}} \frac{1}{1+n_o \rho_s}$$

$$n_o = \frac{E_s}{E_c}$$

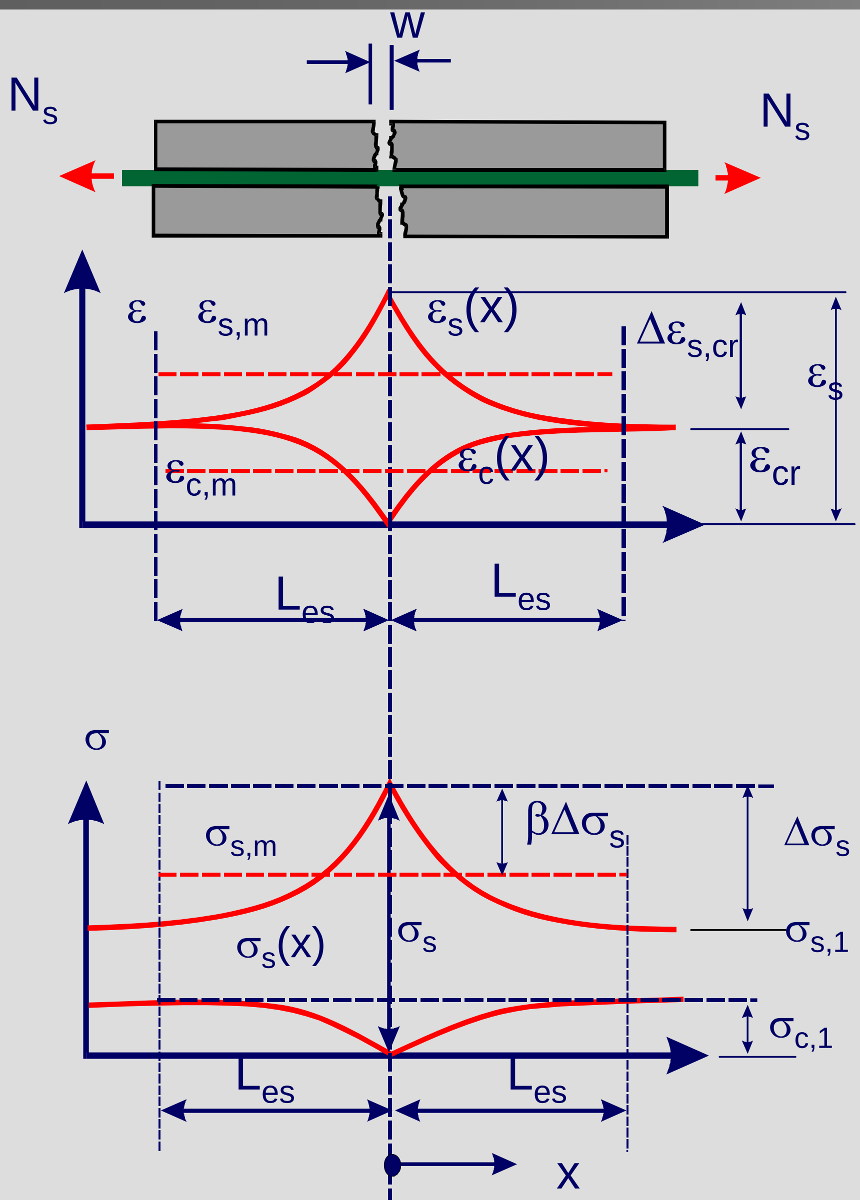


- U_s -perimeter of the bar
- A_s -cross-section area
- ρ_s -reinforcement ratio
- τ_{sm} -mean bond strength

crack width

$$w = 2 L_{es} (\varepsilon_{sm} - \varepsilon_{cm})$$

Determination of the mean strains of reinforcement and concrete in the stage of initial crack formation



Mean bond strength:

$$\tau_{s,m} = \frac{1}{L_{es}} \int_0^{L_{Es}} \tau_s(x)\,dx \approx 1{,}8\ f_{ctm}$$

Mean stress in the reinforcement:

$$\sigma_{s,m} = \sigma_s - \beta\ \Delta\sigma_s \ \Rightarrow \beta = \frac{\sigma_s - \Delta\sigma_{sm}}{\Delta\sigma_s}$$

$$\Delta\sigma_{sm} = \frac{1}{L_{es}} \int_0^{L_{es}} \Delta\sigma_s(x)\,dx \qquad \Delta\sigma_s(x) = \frac{4}{U_s} \int_0^{x} \tau_s(x)\,dx$$

Mean strains in reinforcement and concrete:

$$\varepsilon_{s,m} = \varepsilon_{s,2} - \beta\ \Delta\varepsilon_{s,cr}$$

$$\varepsilon_{c,m} = \beta\ \varepsilon_{cr}$$

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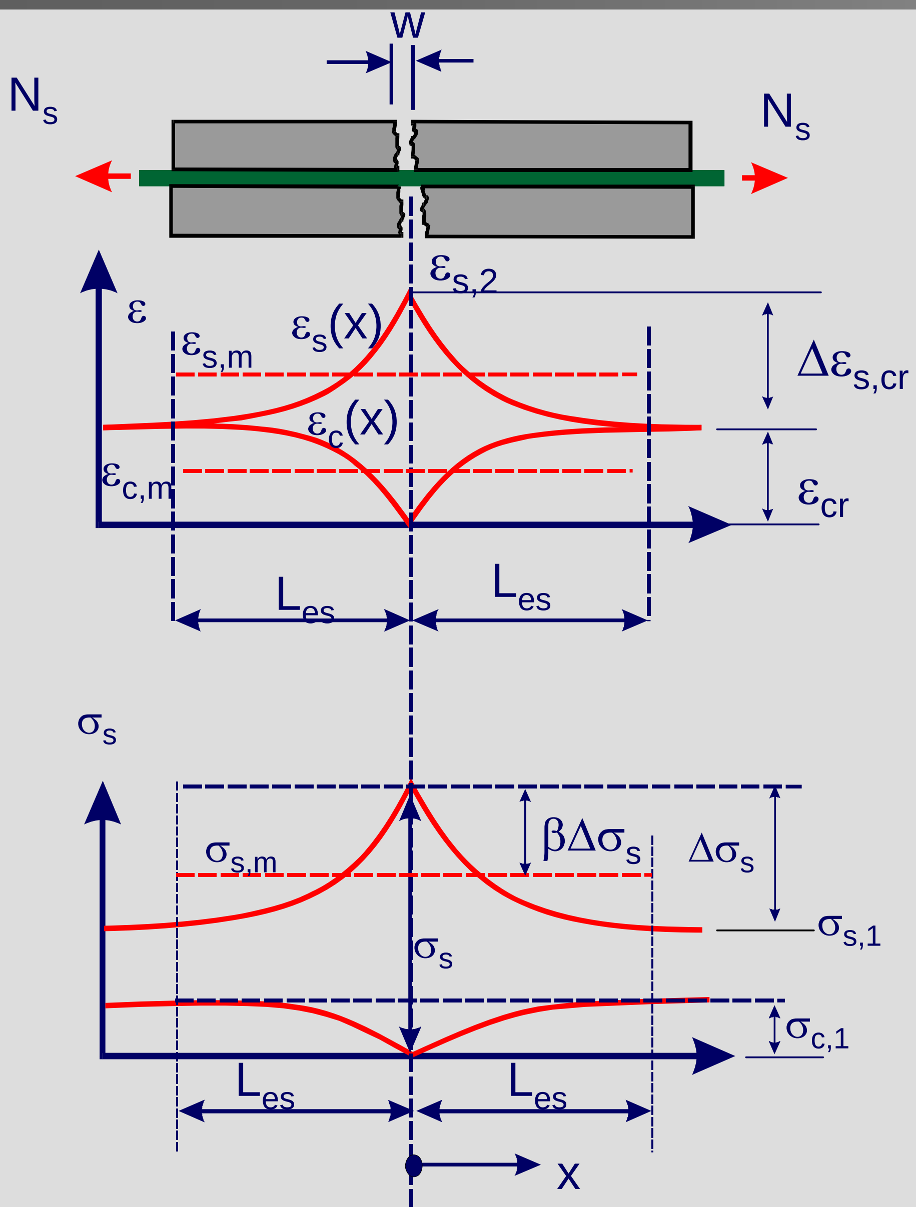
Determination of initial crack width



crack width

$$w = 2L_{es}(\varepsilon_{sm} - \varepsilon_{cm})$$

$$\varepsilon_{s,m} - \varepsilon_{cm} = (1-\beta)\,\varepsilon_{s,2}$$

$$L_{es} = \frac{\sigma_s\, d_s}{4\,\tau_{sm}} \frac{1}{1 + n_o\,\rho_s}$$

$$\tau_{sm} \approx 1{,}8\ f_{ctm}$$

$$w = \frac{(1-\beta)\,\sigma_s^2\, d_s}{2\,\tau_{sm}\ E_s} \frac{1}{1 + n_o\,\rho_s}$$

with β= 0,6 for short term loading und
β= 0,4 for long term loading

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Maximum bar diameters acc. to EC4

σ_s [N/mm²]	maximum bar diameter d_s^* for		
	w_k= 0,4	w_k= 0,3	w_k= 0,2
160	40	32	25
200	32	25	16
240	20	16	12
280	16	12	8
320	12	10	6
360	10	8	5
400	8	6	4
450	6	5	-

β= 0,4 for long term loading and repeated loading

Crack width w:

$$w = \frac{(1-\beta)\,\sigma_s^2\, d_s}{2\,\tau_{sm}\;E_s}\;\frac{1}{1+n_o\,\rho_s} \approx \frac{\sigma_s^2\, d_s}{6\; f_{ct,m}\;E_s}$$

Maximum bar diameter for a required crack width w:

$$d_s = w\;\frac{2\,\tau_{sm}\;E_s\,(1+n_o\,\rho_s)}{\sigma_s^2\;(1-\beta)}$$

With τ_{sm}= 1,8 $f_{ct,mo}$ and the reference value for the mean tensile strength of concrete $f_{ctm,o}$= 2,9 N/mm² follows:

$$d_s^* = w_k\;\frac{3{,}6\;f_{ctm,o}\;\;E_s\,(1+n_o\,\rho_s)}{\sigma_s^2\;(1-\beta)}$$

$$d_s^* \approx 6\,\frac{w_k\;\,f_{ctm,o}\;\,E_s}{\sigma_s^2}$$

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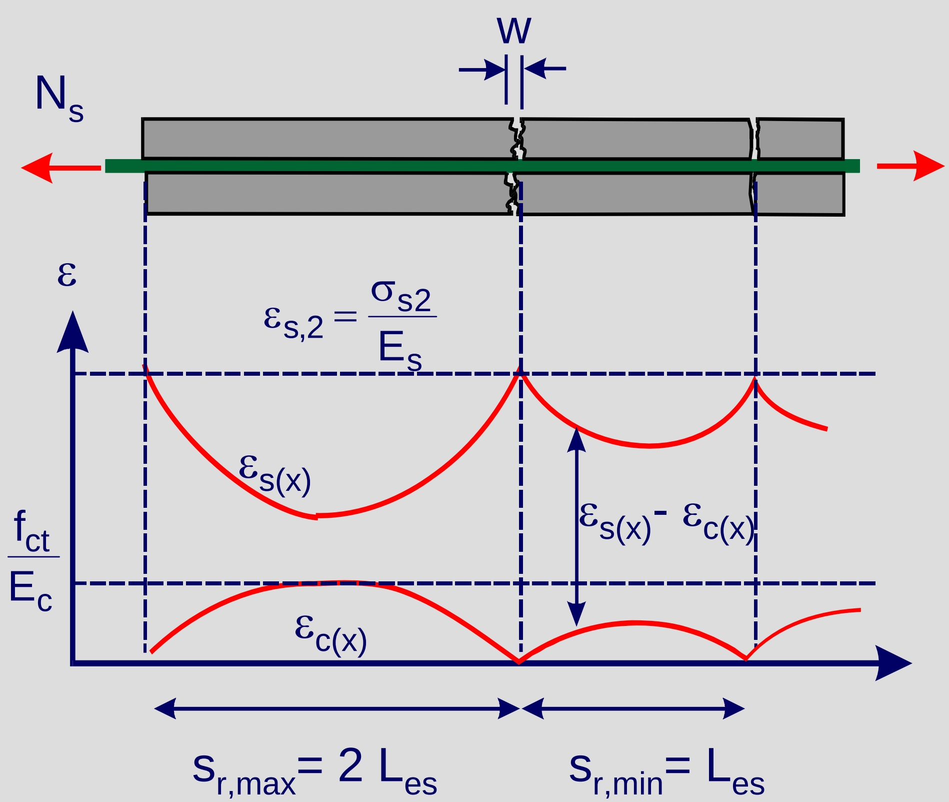
Crack width for stabilised crack formation



β= 0,6 for short term loading

β= 0,4 for long term loading and repeated loading

Crack width for high bond bars

$$w = s_{r,max}\left(\varepsilon_{sm} - \varepsilon_{cm}\right)$$

Mean strain of reinforcement and concrete:

$$\varepsilon_{s,m} = \varepsilon_{s,2} - \beta\,\Delta\,\varepsilon_s$$

$$\varepsilon_{s,m} = \varepsilon_{s,2} - \beta\,\frac{A_c\,f_{ctm}}{E_s\,A_s} = \varepsilon_{s,2} - \beta\,\frac{f_{ctm}}{E_s\,\rho_s}$$

$$\varepsilon_{cm} = \beta\,\frac{f_{ctm}}{E_c}$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s}{E_s} - \beta\,\frac{f_{ctm}}{E_s\,\rho_s}\,(1 + n_o\,\rho_s)$$

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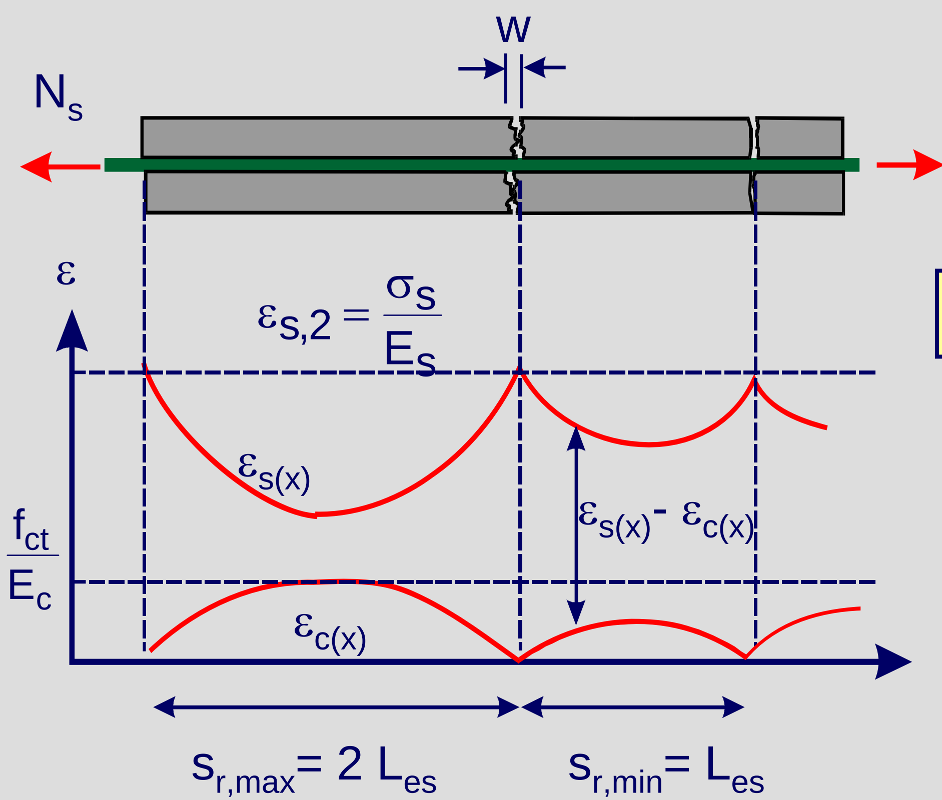
Crack width for stabilised crack formation



The maximum crack spacing $s_{r,max}$ in the stage of stabilised crack formation is twice the introduction length L_{es}.

$$w = s_{r,max}\left(\varepsilon_{sm} - \varepsilon_{cm}\right)$$

$$L_{es} = \frac{f_{ctm}\ A_c}{U_s\ \ \tau_{sm}} = \frac{f_{ctm}\ d_s}{\rho_s\ 4\ \tau_{sm}}$$

maximum crack width for $s_r = s_{r,max}$

$$w = \frac{f_{ctm}\, d_s}{2\ \tau_{sm}\, \rho_s}\left(\frac{\sigma_s}{E_s} - \beta\ \frac{f_{ctm}}{\rho_s\, E_s}\ (1 + n_o\, \rho_s)\right)$$

$\beta = 0{,}6$ for short term loading

$\beta = 0{,}4$ for long term loading and repeated loading

Crack width and crack spacing according Eurocode 2

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Crack width

$$w = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm})$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s}{E_s} - \beta \frac{f_{ctm}}{E_s\ \rho_s}(1 + n_o\ \rho_s) \geq 0{,}6\ \frac{\sigma_s}{E_s}$$

β= 0,6 for short term loading
β= 0,4 for long term loading and repeated loading

Crack spacing

In Eurocode 2 for the maximum crack spacing a semi-empirical equation based on test results is given

$$s_{r,max} = 3{,}4\,c + k_1 \cdot k_2 \cdot 0{,}425\ \frac{d_s}{\rho_s}$$

d_s-diameter of the bar

c- concrete cover

k_1 coefficient taking into account bond properties of the reinforcement with k_1=o,8 for high bond bars

k_2 coefficient which takes into account the distribution of strains (1,0 for pur tension and 0,5 for bending)

Determination of the cracking moment M_{cr} and the normal force of the concrete slab in the stage of initial cracking

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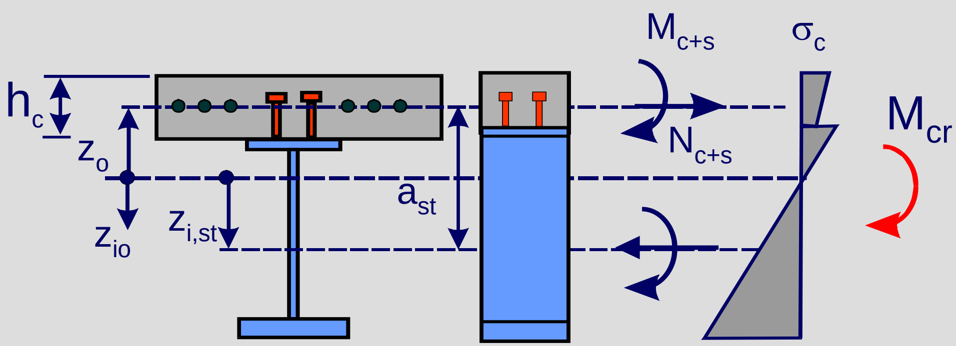
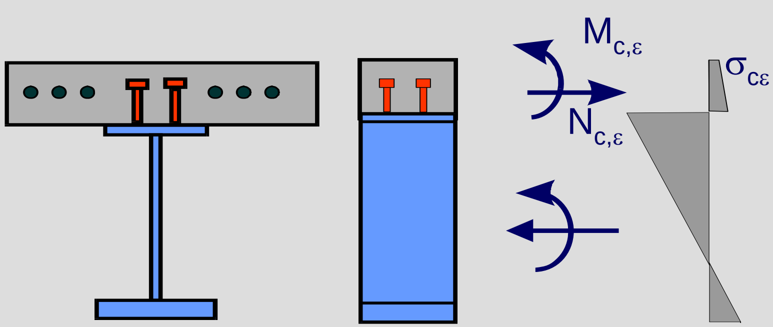
cracking moment M_{cr}



primary effects due to shrinkage



cracking moment M_{cr}:

$$\sigma_c + \sigma_{c,\varepsilon} = f_{ct,eff} = k_1 f_{ctm}$$

$$M_{cr} = \left[f_{ct,eff} - \sigma_{c,\varepsilon}\right] \frac{n_o \; J_{io}}{z_o + h_c / 2}$$

$$M_{cr} = \left[f_{ct,eff} - \sigma_{c,\varepsilon}\right] \frac{n_o \; J_{io}}{z_{ic,o} \, (1 + h_c / (2 \; z_o))}$$

sectional normal force of the concrete slab:

$$N_{cr} = M_{cr} \frac{A_{co} \, z_o + A_s \, z_{is}}{J_{io}} + N_{c+s,\varepsilon}$$

$$N_{cr} = \frac{A_c \, (f_{ct,eff} - \sigma_{c,\varepsilon})(1 + \rho_s \, n_o)}{1 + h_c / (2 \; z_o)} + N_{c+s,\varepsilon}$$

$$N_{cr} = A_c \; f_{ct,eff} \; (1 + \rho_s \, n_0) \left[\underbrace{\frac{1}{1 + h_c / (2 \; z_o)}}_{k_c} + \underbrace{\frac{N_{c+s,\varepsilon} - \dfrac{A_c \sigma_{c,\varepsilon} (1 + \rho_s \, n_o)}{1 + h_c / (2 \; z_o)}}{A_c \; f_{ct,eff} \; (1 + \rho_s \, n_0)}}_{k_{c,\varepsilon} \approx 0{,}3} \right]$$

Simplified solution for the cracking moment and the normal force in the concrete slab

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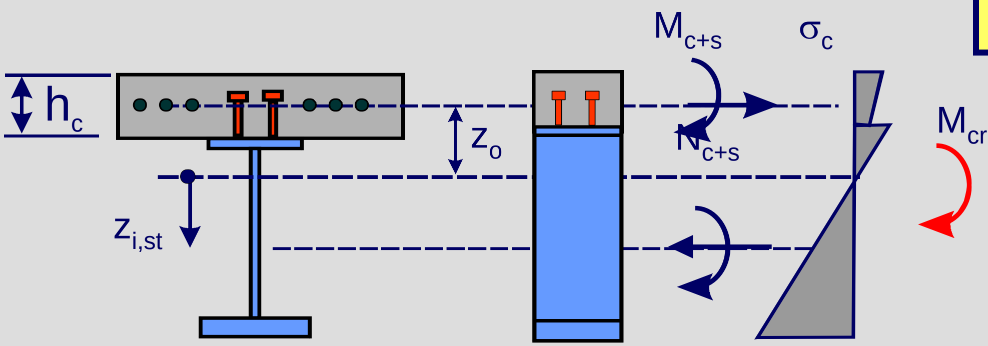
cracking moment M_{cr}



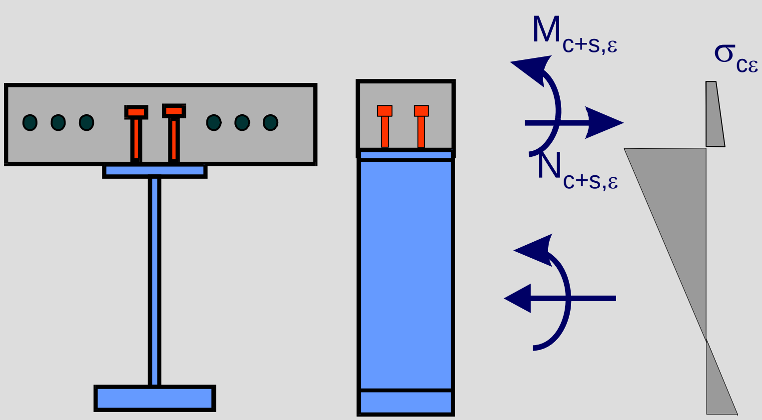
simplified solution for the normal force in the concrete slab:

$$N_{cr} \approx A_c \, f_{ctm} \, k_s \cdot k \cdot k_c$$

k = 0,8 coefficient taking into account the effect of non-uniform self-equilibrating stresses

k_s= 0,9 coefficient taking into account the slip effects of shear connection

primary effects due to shrinkage



$$k_c = \frac{1}{1+\dfrac{h_c}{2\,z_o}} + 0{,}3 \le 1{,}0$$

cracking moment

shrinkage

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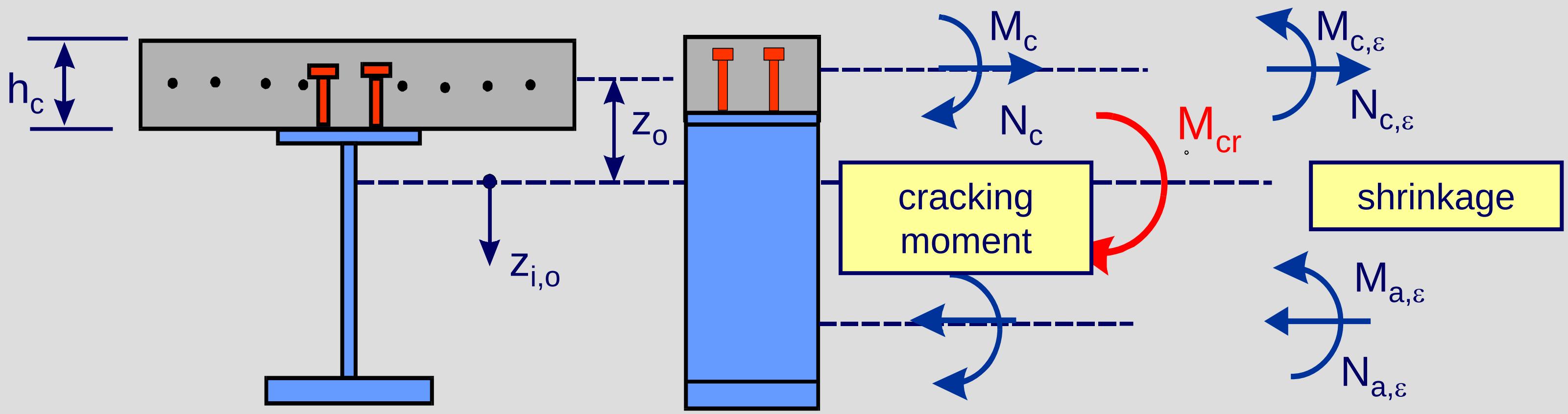
Determination of minimum reinforcement



$$A_s \geq \frac{A_c\, f_{ct,eff}}{\sigma_s}\, k \;\; k_s \; k_c$$

$$k_c = \frac{1}{1 + h_c / z_o} + 0{,}3 \leq 1{,}0$$

$$d_s = d_s^* \frac{f_{ct,eff}}{f_{ct,o}}$$

$f_{cto} = 2{,}9\ N/mm^2$

$k = 0{,}8$	Influence of non linear residual stresses due to shrinkage and temperature effects
$k_s = 0{,}9$	flexibility of shear connection
k_c	Influence of distribution of tensile stresses in concrete immediately prior to cracking
d_s^*	maximum bar diameter
d_s	modified bar diameter for other concrete strength classes
σ_s	stress in reinforcement acc. to Table 1
$f_{ct,eff}$	effective concrete tensile strength

Control of cracking due to direct loading – Verification by limiting bar spacing or bar diameter

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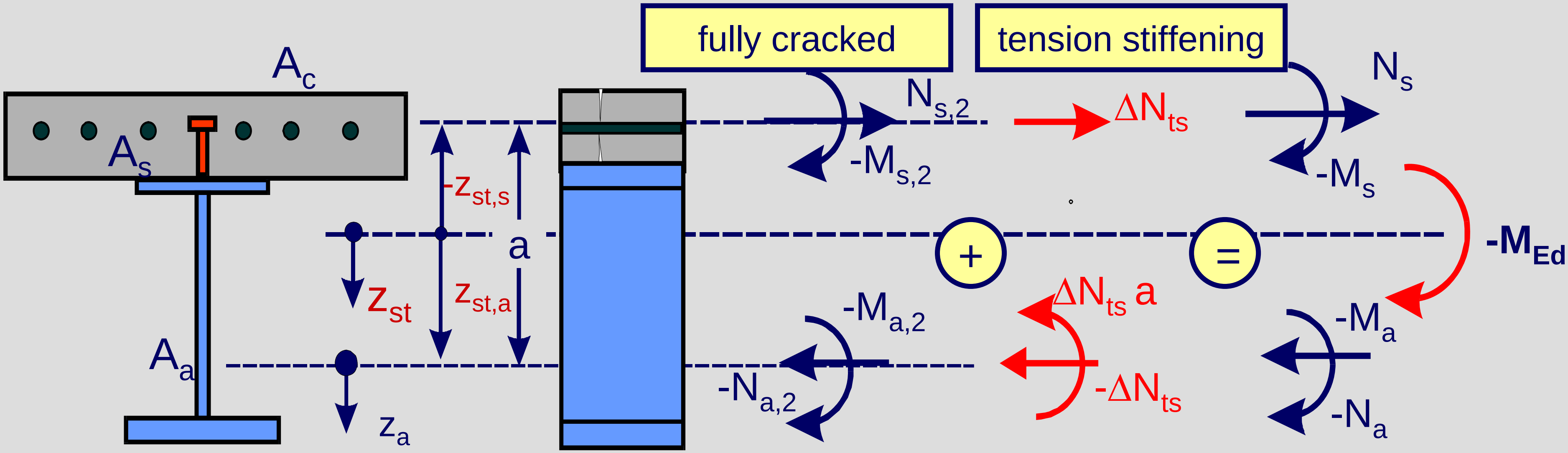

The calculation of stresses is based on the mean strain in the concrete slab. The factor β results from the mean value of crack spacing. With $s_{rm} \approx 2/3\, s_{r,max}$ results $\beta \approx 2/3 \cdot 0{,}6 = 0{,}4$

stresses in reinforcement taking into account tension stiffening for the bending moment M_{Ed} of the quasi permanent combination:

$$\sigma_s = \sigma_{s,2} + \Delta\sigma_{ts}$$

$$\sigma_s = \frac{M_{Ed}}{J_2} z_{st,s} + \beta \frac{f_{ct,eff}}{\rho_s \alpha_{st}}$$

$$\rho_s = \frac{A_s}{A_c} \qquad \beta = 0{,}4 \qquad \alpha_{st} = \frac{A_2 J_2}{A_a J_a}$$

The bar diameter <u>or</u> the bar spacing has to be limited

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Table 1: Maximum bar diameter

σ_s [N/mm²]	**maximum bar diameter** d_s^* for		
	w_k= 0,4	w_k= 0,3	w_k= 0,2
160	40	32	25
200	32	25	16
240	20	16	12
280	16	12	8
320	12	10	6
360	10	8	5
400	8	6	4
450	6	5	-

Table 2: Maximum bar spacing

σ_s [N/mm²]	**maximum bar spacing** in [mm] for		
	w_k= 0,4	w_k= 0,3	w_k= 0,2
160	300	300	200
200	300	250	150
240	250	200	100
280	200	150	50
320	150	100	-
360	100	50	-

Direct calculation of crack width w for composite sections based on EN 1992-2

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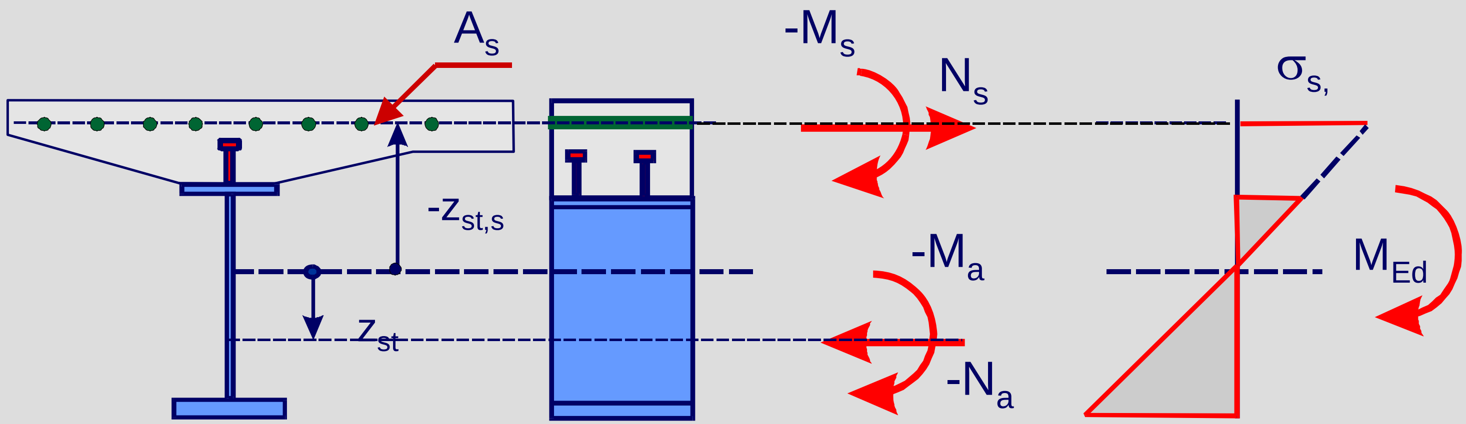

$$\sigma_s = \frac{M_{Ed}}{J_{st}} z_{st,s} + \beta \frac{f_{ct,eff}}{\rho_s \, \alpha_{st}}$$

$$\alpha_{st} = \frac{A_{st} J_{st}}{A_a J_a} \qquad \rho_s = \frac{A_s}{A_c} \qquad \beta = 0{,}4$$

crack width for high bond bars:

$$w = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm})$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s}{E_s} - \beta \frac{f_{ctm}}{E_s \, \rho_s}(1 + n_o \, \rho_s) \geq 0{,}6 \frac{\sigma_s}{E_s}$$

$$s_{r,max} = 3{,}4\,c + 0{,}34 \frac{d_s}{\rho_s}$$

c - concrete cover of reinforcement

Stresses in reinforcement in case of bonded tendons – initial crack formation

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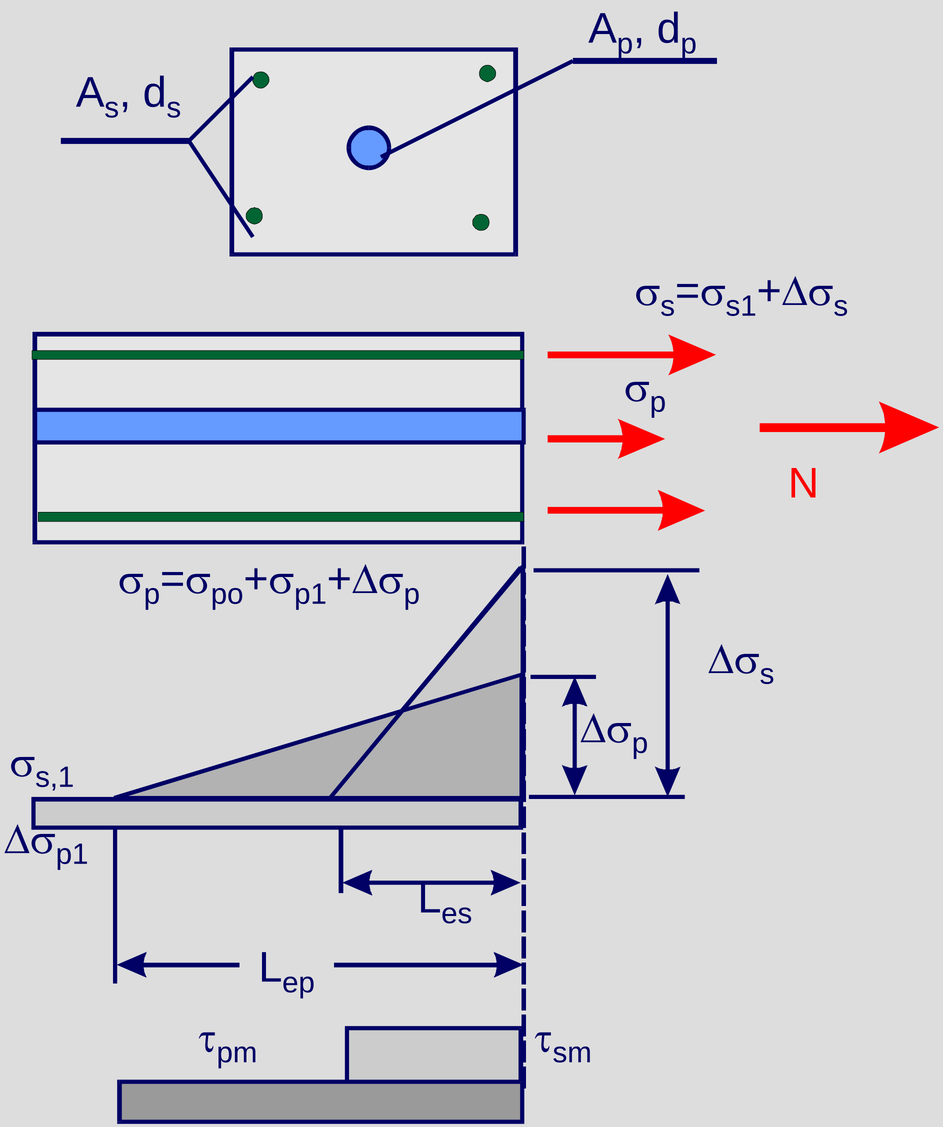

Equilibrium at the crack:

$$\sigma_s\ A_s + \Delta\sigma_p\ A_p = N = f_{ct,eff}\ A_c\,(1+n_o\,\rho_{tot})$$

Equilibrium in longitudinal direction:

$$\sigma_s\ A_s = \pi\ d_s\ \tau_{sm}\ L_{e,s}$$

$$\Delta\sigma_p\ A_p = \pi\ d_p\ \tau_{pm}\ L_{ep}$$

Compatibility at the crack:

$$\delta_s = \delta_p \Rightarrow \frac{\sigma_s-\sigma_{s1}}{E_s}L_{es} = \frac{\Delta\sigma_p-\Delta\sigma_{p1}}{E_p}L_{ep}$$

With $E_s \approx E_p$ and $\sigma_{s1}=\Delta\sigma_{p1}=0$ results:

Stresses:

$$\sigma_s = \frac{N}{A_s+\xi_1\,A_p} \qquad \Delta\sigma_p = \frac{\xi_1\ N}{A_s+\xi_1\,A_p}$$

$$\xi_1 = \sqrt{\frac{\tau_{pm}}{\tau_{sm}}\,\frac{d_s}{d_v}}$$

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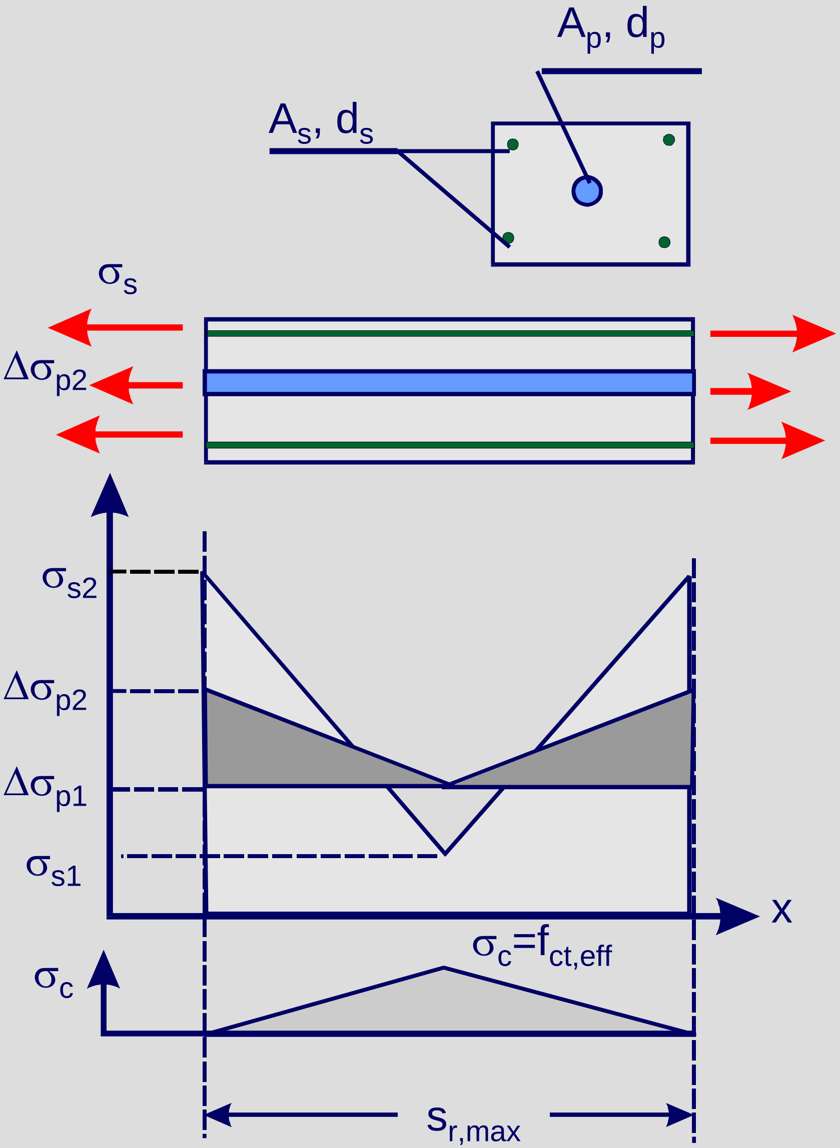
Stresses in reinforcement for final crack formation



Equilibrium at the crack:

$$N - P_o = \sigma_{s2} A_s + \Delta\sigma_{p2} A_p$$

Maximum crack spacing:

$$f_{ct} A_c = \frac{s_{r,max}}{2} \left[\tau_{sm} n_s d_s \pi + \tau_{pm} n_p d_p \pi \right]$$

$$s_{r,max} = \frac{d_s f_{ct,eff} A_c}{2 \tau_{sm} (A_s + \xi^2 A_p)}$$

Equilibrium in longitudinal direction:

$$\sigma_{s2} - \sigma_{s1} = \frac{s_{r,max}}{2} \frac{U_s}{A_s} \tau_{sm} \qquad \sigma_{p2} - \sigma_{p1} = \frac{s_{r,max}}{2} \frac{U_p}{A_p} \tau_{pm}$$

Compatibility at the crack:

$$\delta_s = \delta_p = \frac{\sigma_{s2} - \beta(\sigma_{s2} - \sigma_{s1})}{E_s} = \frac{\Delta\sigma_{p,2} - \beta(\Delta\sigma_{p2} - \Delta\sigma_{p1})}{E_p}$$

mean crack spacing: $s_{r,m} \approx 2/3\ s_{r,max}$

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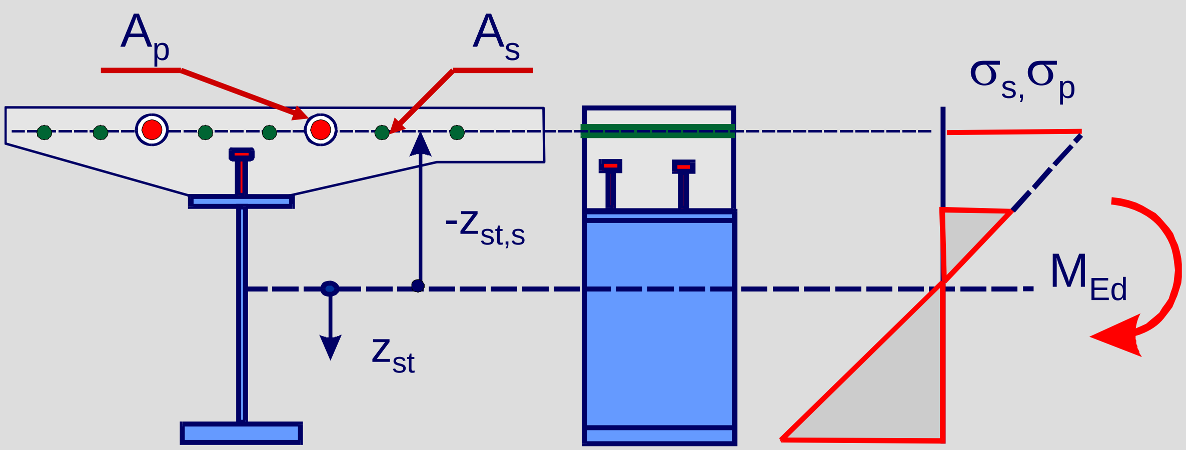

Stresses σ^*_s in reinforcement at the crack location neglecting different bond behaviour of reinforcement and tendons:

$$\sigma^*_s = \frac{M_{Ed}}{J_{st}} z_{st,s} + \beta \frac{f_{ctm}}{\rho_{tot}\,\alpha_{st}}$$

$$\alpha_{st} = \frac{A_{st}\,J_{st}}{A_a\,J_a} \qquad \beta = 0{,}4$$

Stresses in reinforcement taking into account the different bond behaviour:

$$\sigma_s = \sigma^*_s + 0{,}4\,f_{ct,eff}\left[\frac{A_c}{A_s + \xi_1^2 A_p} - \frac{A_c}{A_s + A_p}\right] = \sigma^*_s + 0{,}4\,f_{ct,eff}\left[\frac{1}{\rho_{eff}} - \frac{1}{\rho_{tot}}\right]$$

$$\Delta\sigma_p = \sigma^*_s - 0{,}4\,f_{ct,eff}\left[\frac{A_c}{A_s + A_p} - \frac{\xi_1^2 A_c}{A_s + \xi_1^2 A_p}\right] = \sigma^*_s - 0{,}4\,f_{ct,eff}\left[\frac{1}{\rho_{tot}} - \frac{\xi_1^2}{\rho_{eff}}\right]$$

$$\rho_{tot} = \frac{A_s + A_p}{A_c}$$

$$\rho_{eff} = \frac{A_s + \xi_1^2 A_p}{A_c}$$

Part 4:

Deformations

Deflections

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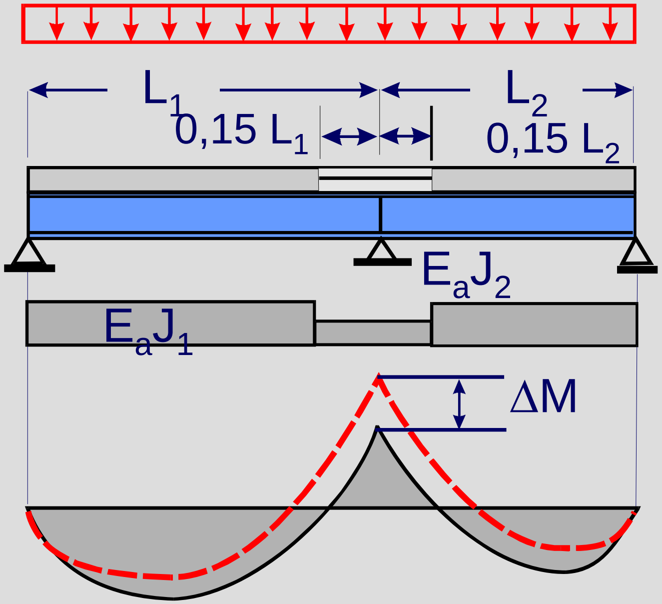
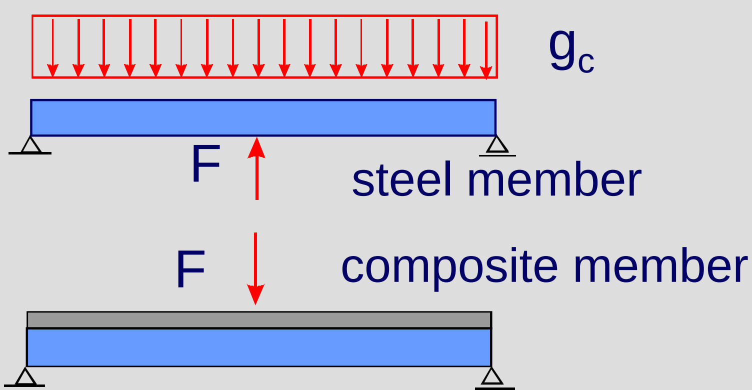
Effects of cracking of concrete



Sequence of construction



Deflections due to loading applied to the composite member should be calculated using elastic analysis taking into account effects from

- **cracking of concrete,**
- **creep and shrinkage,**
- **sequence of construction,**
- **influence of local yielding of structural steel at internal supports,**
- **influence of incomplete interaction.**

Deformations and pre-cambering

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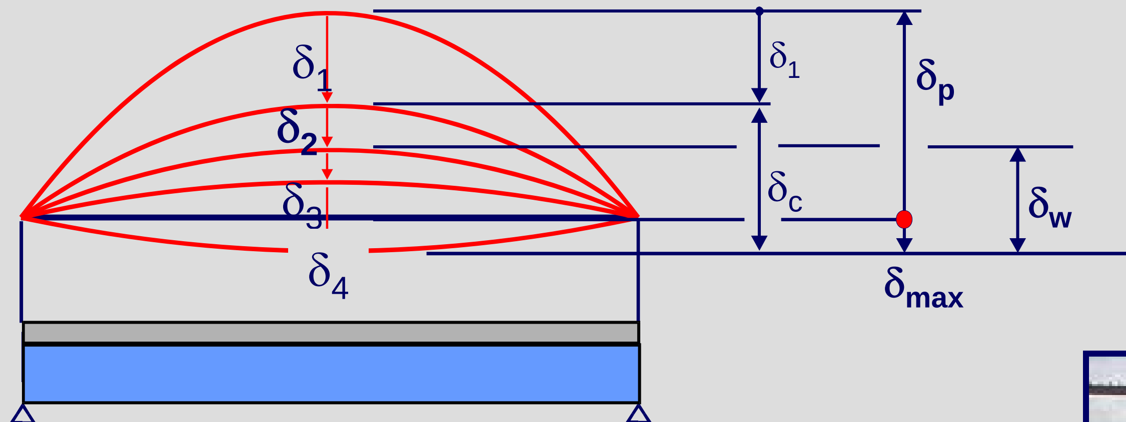
	combination	limitation
general	quasi - permanent	$\delta_{max} \leq L/250$
risk of damage of adjacent parts of the structure (e.g. finish or service work)	quasi – permanent (better frequent)	$\delta_w \leq L/500$



δ_1 – self weight of the structure
δ_2 – loads from finish and service work
δ_3 – creep and shrinkage
δ_4 – variable loads and temperature effects

δ_1 deflection of the steel girder

δ_c deflection of the composite girder

Pre-cambering of the steel girder:

$$\delta_p = \delta_1 + \delta_2 + \delta_3 + \psi_2\, \delta_4$$

δ_{max} maximum deflection

δ_w effective deflection for finish and service work

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For the calculation of deflection of un-propped beams, account may be taken of the influence of local yielding of structural steel over a support.

For beams with critical sections in Classes 1 and 2 the effect may be taken into account by multiplying the bending moment at the support with an additional reduction factor f_2 and corresponding increases are made to the bending moments in adjacent spans.

$f_2 = 0,5$ if f_y is reached before the concrete slab has hardened;

$f_2 = 0,7$ if f_y is reached after concrete has hardened.

This applies for the determination of the maximum deflection but not for pre-camber.

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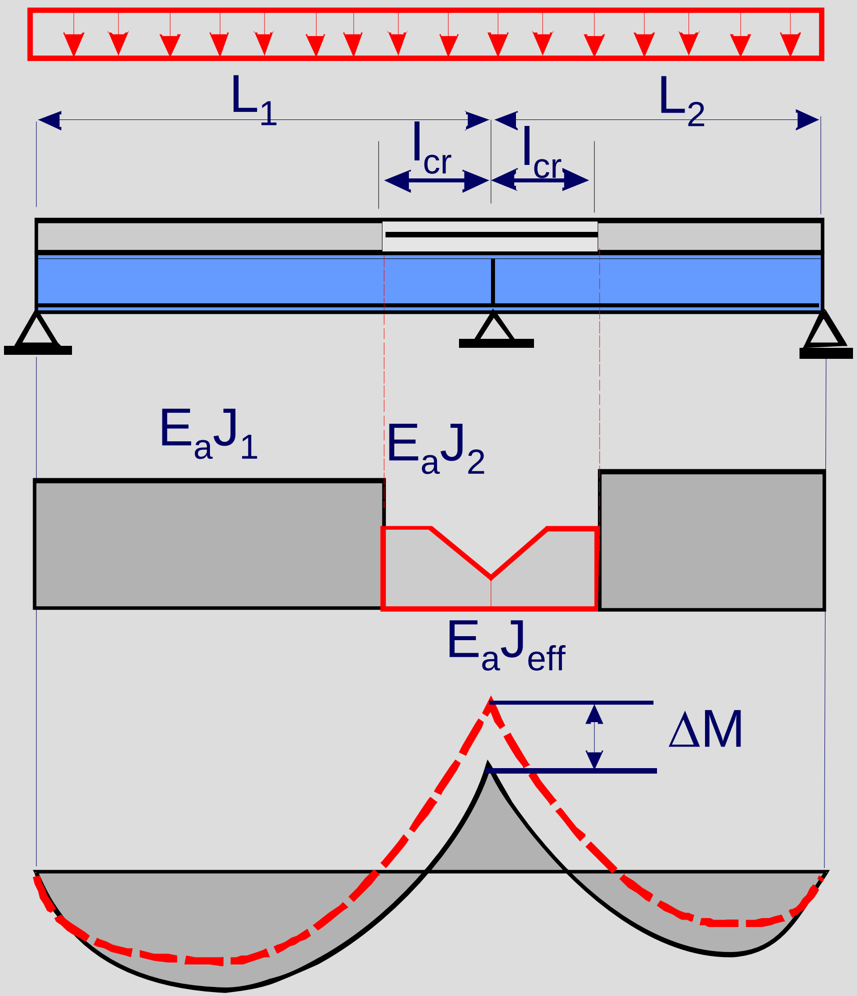
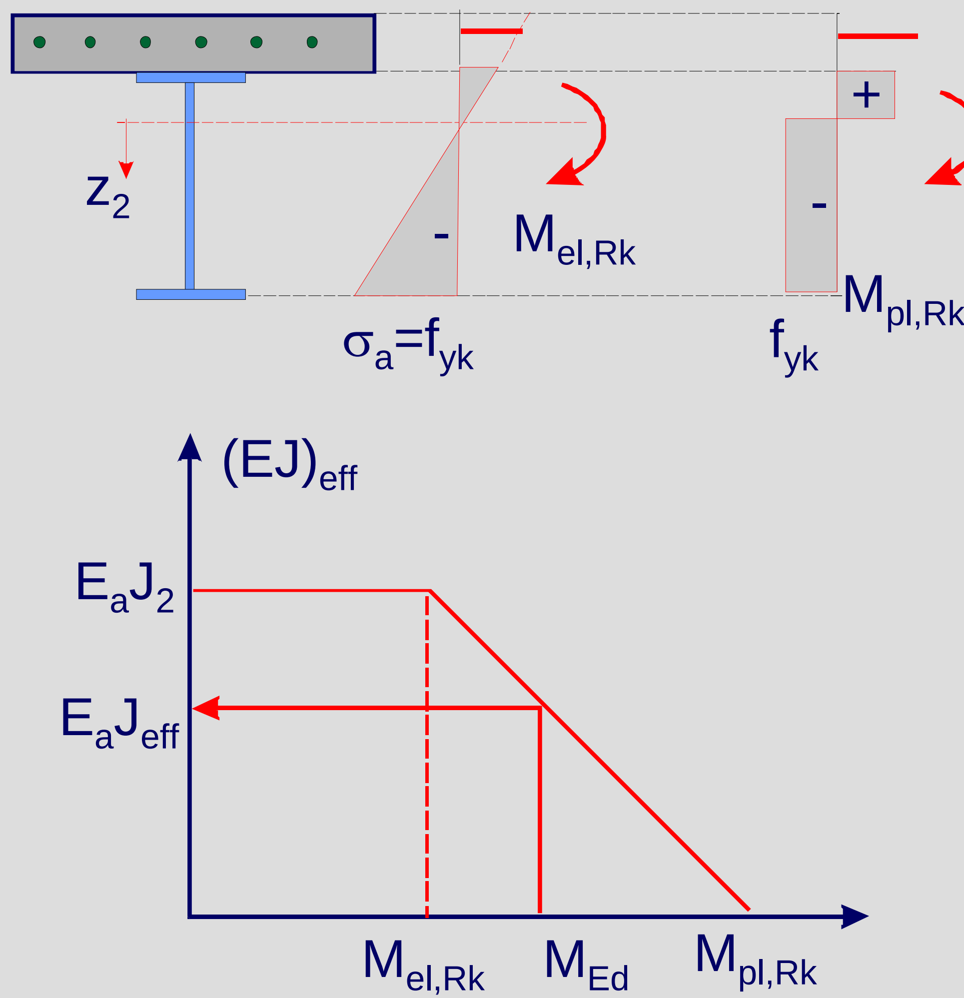
More accurate method for the determination of the effects of local yielding on deflections





Effects of incomplete interaction on deformations

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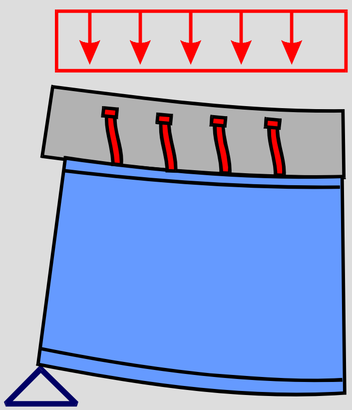

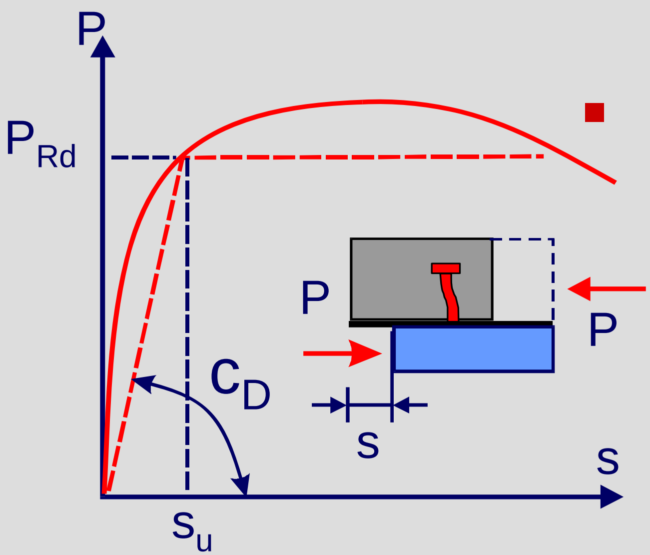

The effects of incomplete interaction may be ignored provided that:

- The design of the shear connection is in accordance with clause 6.6 of Eurocode 4,
- either not less shear connectors are used than **half the number for full shear connection**, or the forces resulting from an elastic behaviour and which act on the shear connectors in the serviceability limit state do not exceed P_{Rd} and
- in case of a ribbed slab with ribs transverse to the beam, the height of the ribs does not exceed 80 mm.

Differential equations in case of incomplete interaction

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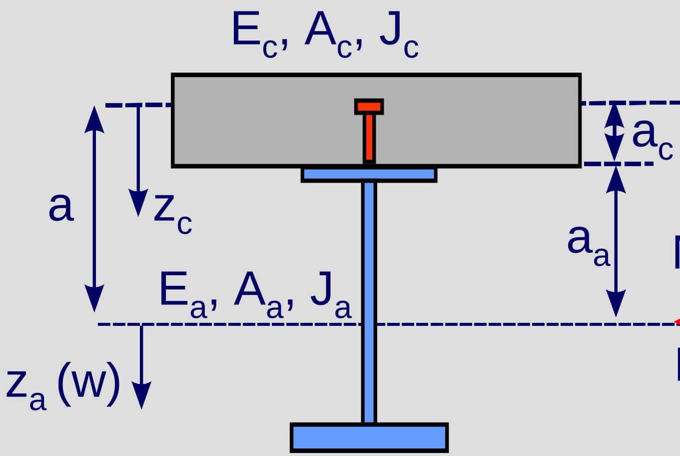
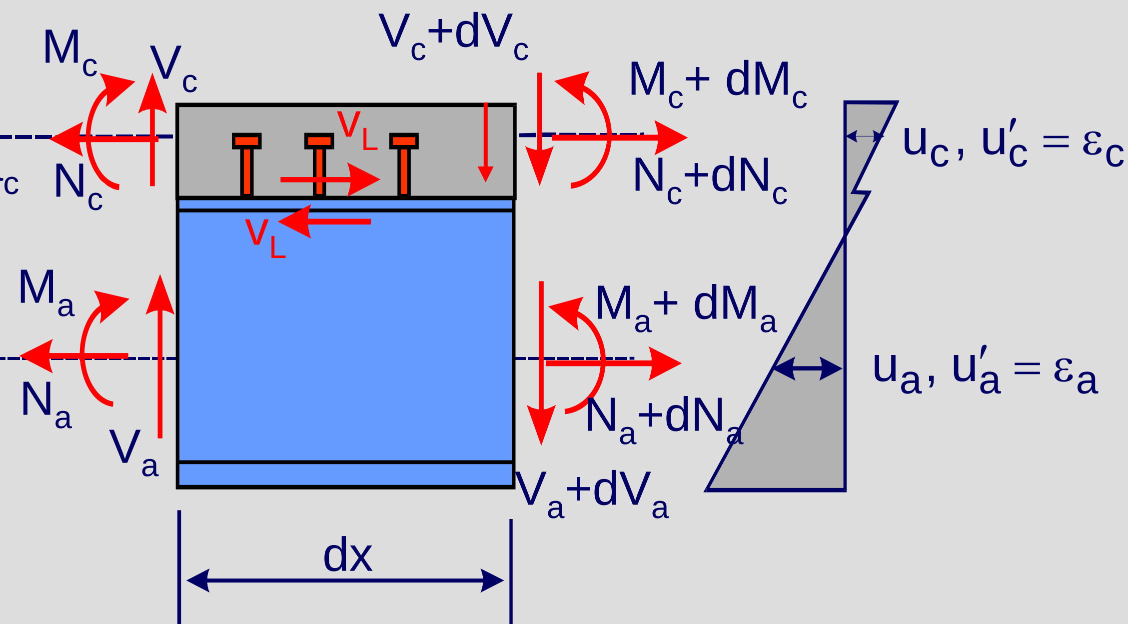
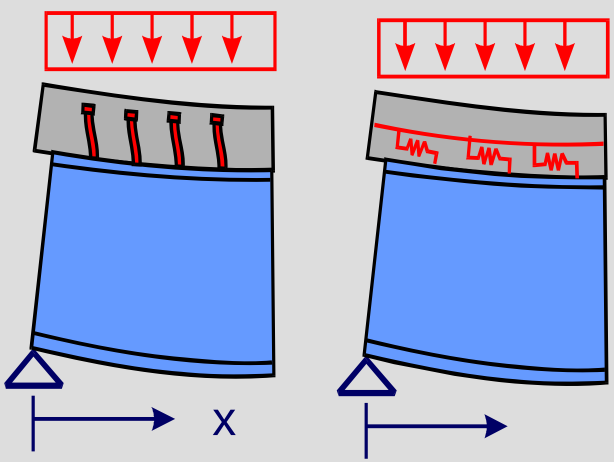




Slip: $s_v = u_a - u_c + w' a$



$$E_c\,A_c\,u_c'' + c_s\,(u_a - u_c + w'\,a) = 0$$

$$E_a\,A_a\,u_c'' - c_s\,(u_a - u_c + w'\,a) = 0$$

$$(E_c\,J_c + E_a\,J_a)\,w'''' - c_s\,a\,(u_a' - u_c' + w''\,a) = q$$

$$N_c = E_c\,A_c\,u_c' \qquad M_c = -E_c\,J_c\,w'' \qquad V_c = -E_c J_c\,w''' \approx 0$$

$$N_a = E_a\,A_a\,u_a' \qquad M_a = -E_a\,J_a\,w'' \qquad V_a = -E_a J_a\,w'''$$

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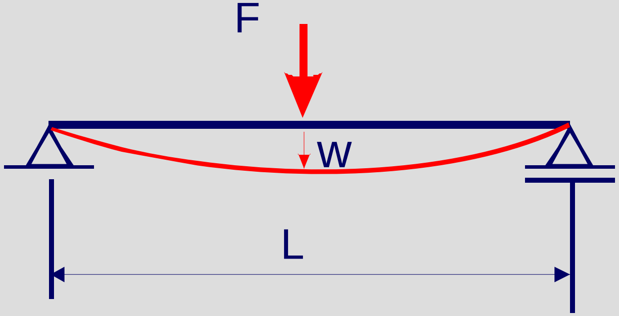
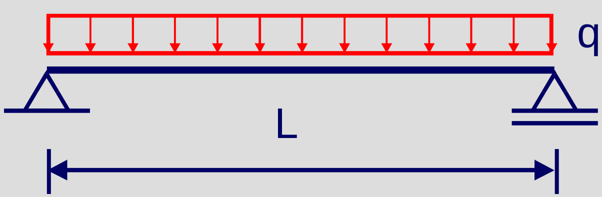
Deflection in case of incomplete interaction for single span beams



$$w = \frac{F\,L^3}{48\,E_a\,I_{i,o}}\left[1 + \frac{12}{\alpha\,\lambda^2} - \frac{48}{\alpha\,\lambda^3}\,\frac{\sinh^2(\frac{\lambda}{2})}{\sinh(\lambda)}\right]$$



$$w = \frac{5}{384}\,\frac{q\;L^4}{E_a\;J_{i,o}}\left[1 + \frac{48}{5}\,\frac{1}{\alpha\,\lambda^2} - \frac{384}{5}\,\frac{1}{\alpha\,\lambda^4}\,\frac{\cosh(\frac{\lambda}{2}) - 1}{\cosh(\frac{\lambda}{2})}\right]$$

concrete section

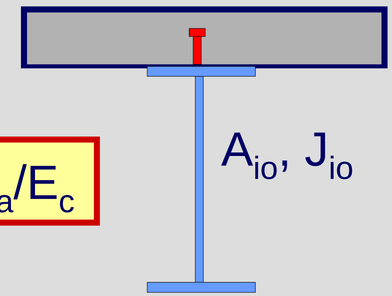
$A_{co}=A_c/n_o$, $J_{co}= J_c/n_o$

steel section

A_a, J_a

$n_o=E_a/E_c$

composite section



A_{io}, J_{io}

$$\lambda^2 = \frac{1+\alpha}{\alpha\,\beta}$$

$$\beta = \frac{E_a\;A_{c,o}\;A_a}{A_{i,o}\;c_s\;L^2}$$

$$\alpha = \frac{1}{\frac{J_{i,o}}{J_a + J_{c,o}} - 1}$$

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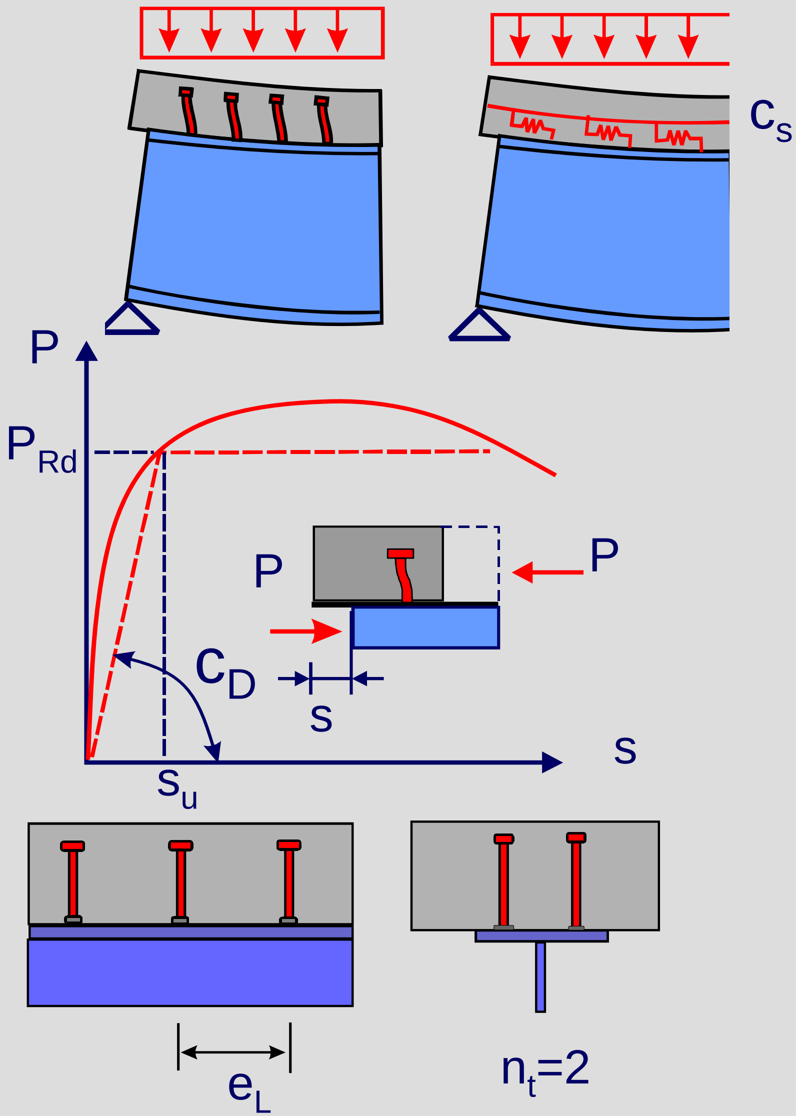
Mean values of stiffness of headed studs



spring constant per stud: $C_D = \frac{s_u}{P_{Rd}}$

spring constant of the shear connection: $c_s = \frac{C_D \, n_t}{e_L}$

type of shear connection	C_D [kN/cm]
headed stud ∅ 19mm in solid slabs	2500
headed stud ∅ 22mm in solid slabs	3000
headed studs ∅ 25mm in solid slab	3500
headed stud ∅ 19mm with Holorib-sheeting and one stud per rib	1250
headed stud ∅ 22mm with Holorib-sheeting and one stud per rib	1500

Simplified solution for the calculation of deflections in case of incomplete interaction

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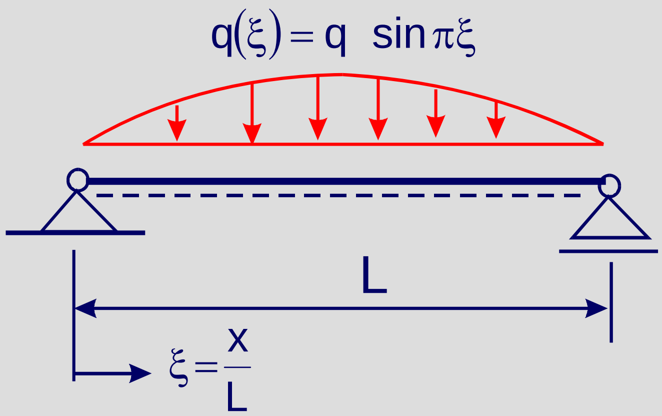
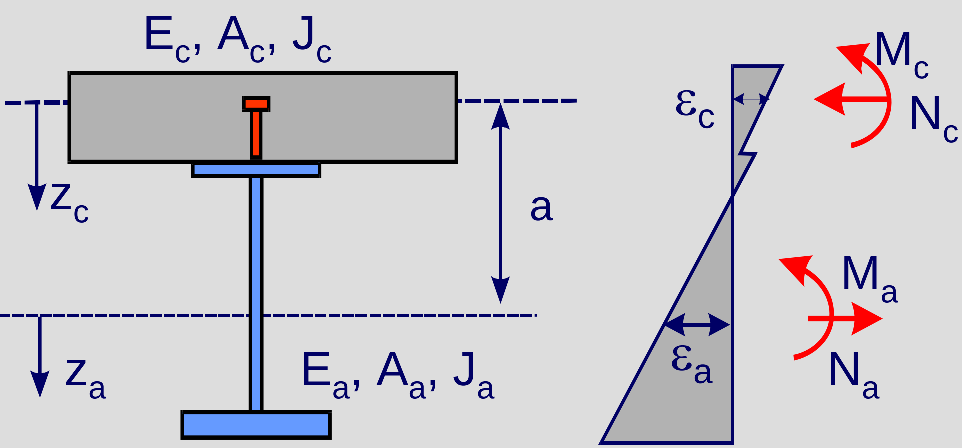


The influence of the flexibility of the shear connection is taken into account by a reduced value for the modular ratio.

$$w_o = q \frac{L^4}{\pi^4} \frac{1}{E_{cm} J_c + E_a J_a + \dfrac{\beta_o E_{cm} A_c E_a A_a}{E_a A_a + \beta_o E_{cm} A_c} a^2} = q \frac{L^4}{\pi^4} \frac{1}{E_a J_{io,eff}}$$



$$J_{io,eff} = J_{c,o} + J_a + \frac{A_{c,eff} A_a}{A_{c,eff} + A_a} a^2$$

$$A_{c,eff} = \frac{A_c}{n_{o,eff}}$$

effective modular ratio for the concrete slab

$$n_{o,eff} = n_o (1 + \beta_s)$$

$$\beta_s = \frac{\pi^2 E_{cm} A_c}{L^2 c_s}$$

Comparison of the exact method with the simplified method

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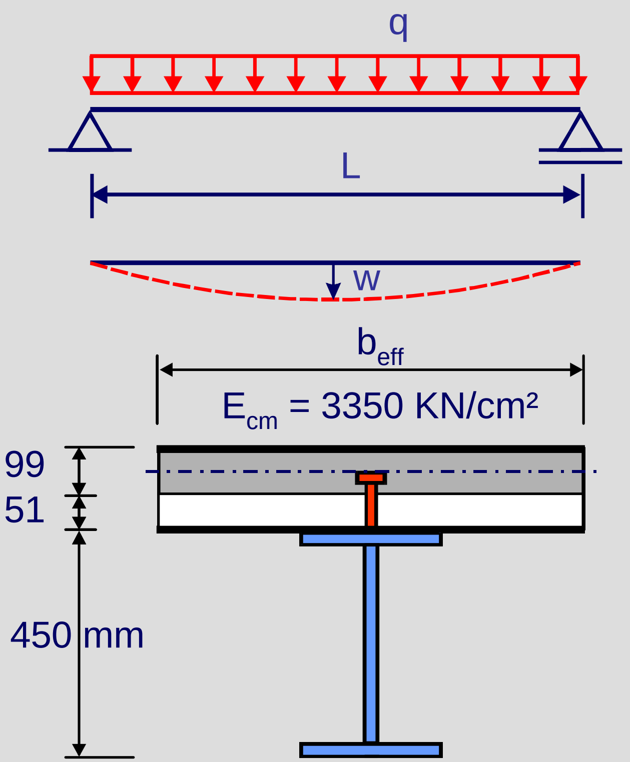

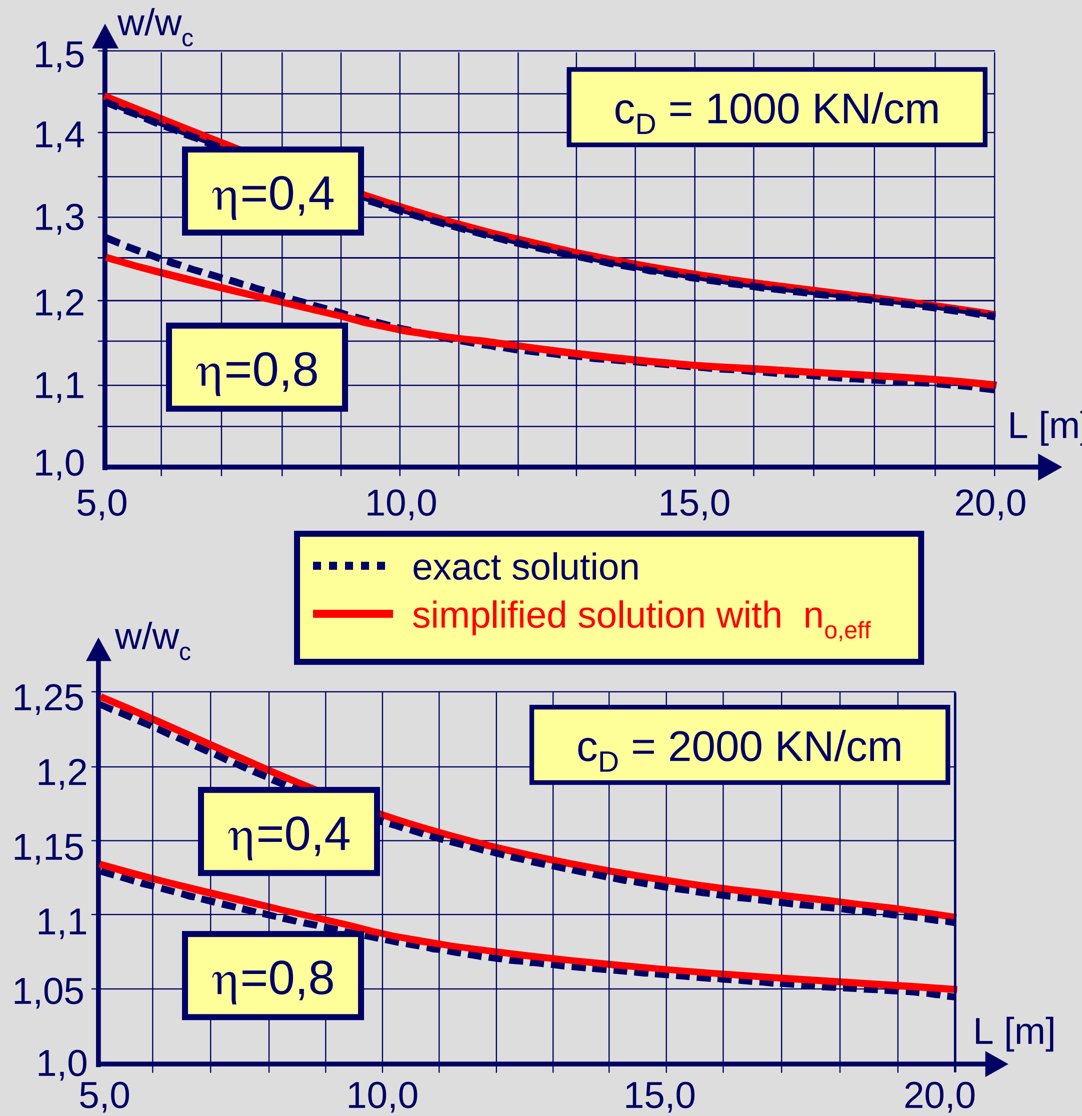

w_o- deflection in case of neglecting effects from slip of shear connection

η degree of shear connection

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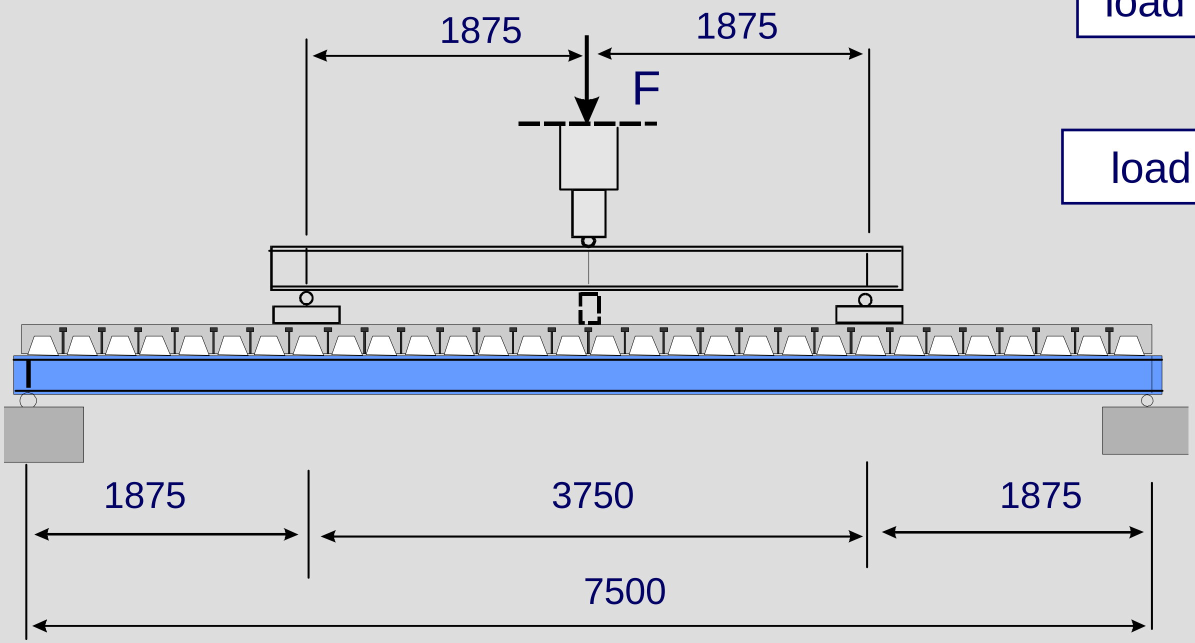

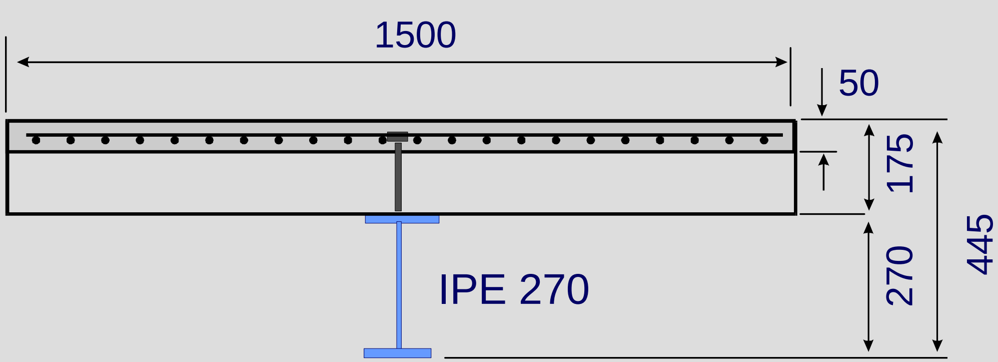

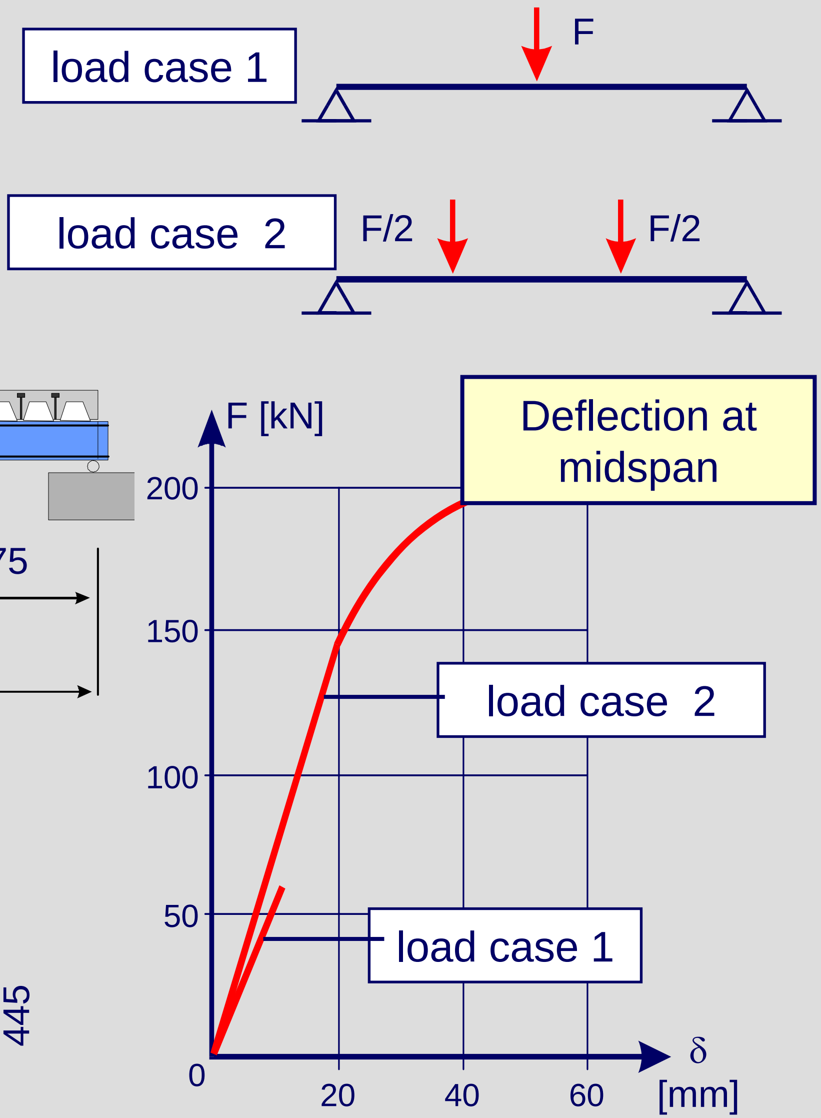

Deflection in case of incomplete interaction- Comparison with test results

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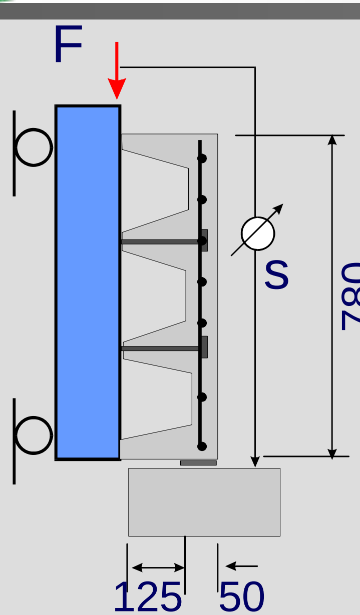

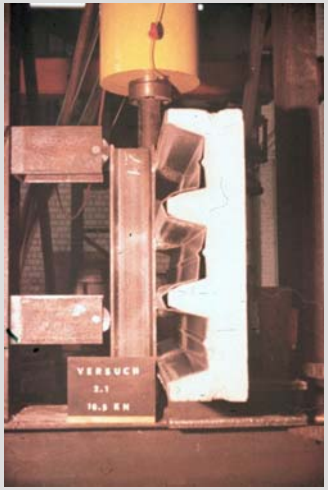

	second moment of area cm^4	Load case 1 F= 60 kN	Load case 2 F=145 kN
		Deflection at midspan in mm	
Test	-	11,0 (100%)	20,0 (100 %)
Theoretical value, neglecting flexibility of shear connection	J_{io}= 32.387,0	7,8 (71%)	12,9 (65%)
Theoretical value, taking into account flexibility of shear connection	$J_{io,eff}$= 21.486,0	11,7 (106%)	19,4 (97%)

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Part 5:

Limitation of stresses

Limitation of Stresses

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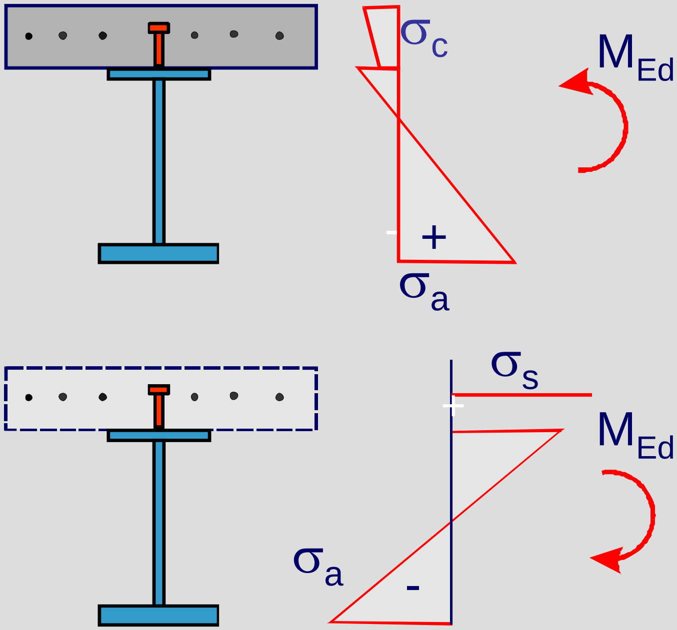

Stress limitation is not required for beams if in the ultimate limit state,

- no verification of fatigue is required and
- no prestressing by tendons and /or
- no prestressing by controlled imposed deformations is provided.

	combination	stress limit	recommended values k_i
structural steel	characteristic	$\sigma_{Ed} \leq k_a f_{yk}$	$k_a = 1{,}00$
reinforcement	characteristic	$\sigma_{Ed} \leq k_s f_{sk}$	$k_s = 0{,}80$
concrete	characteristic	$\sigma_{Ed} \leq k_c f_{ck}$	$k_c = 0{,}60$
headed studs	characteristic	$P_{Ed} \leq k_s P_{Rd}$	$k_s = 0{,}75$

Local effects of concentrated longitudinal shear forces

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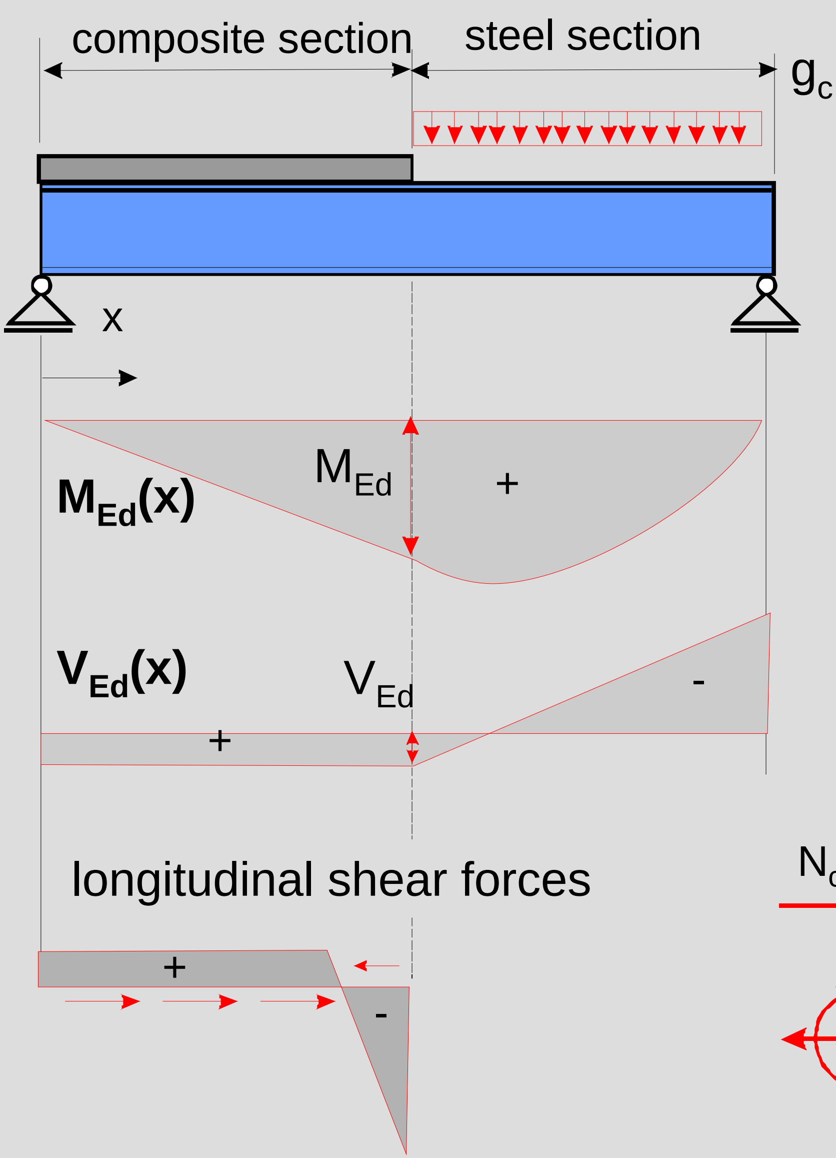
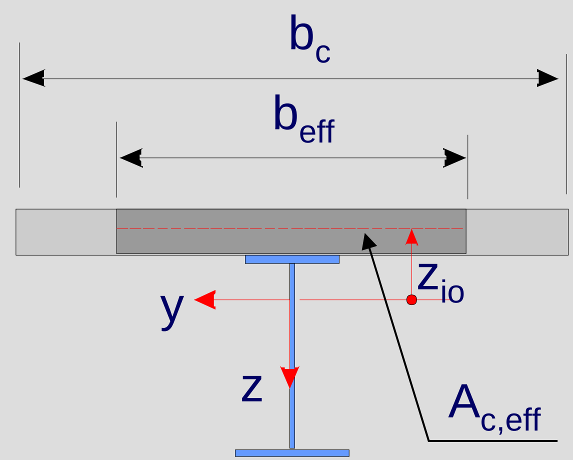
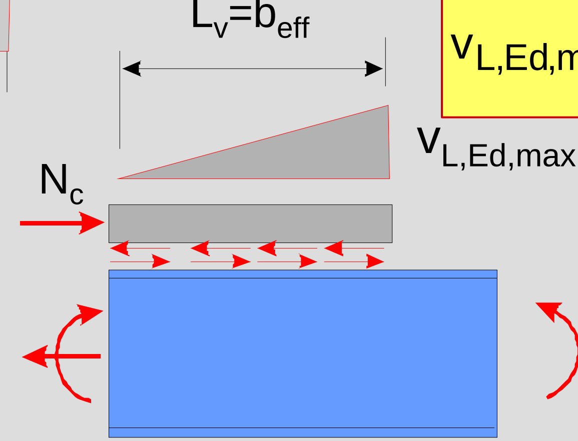




Concentrated longitudinal shear force at sudden change of cross-section



$$v_{L,Ed,max} = \frac{2\, M_{Ed}\; A_{c,eff}\; z_{io}}{E_a / E_c\; J_{io}\; b_{eff}}$$

Local effects of concentrated longitudinal shear forces

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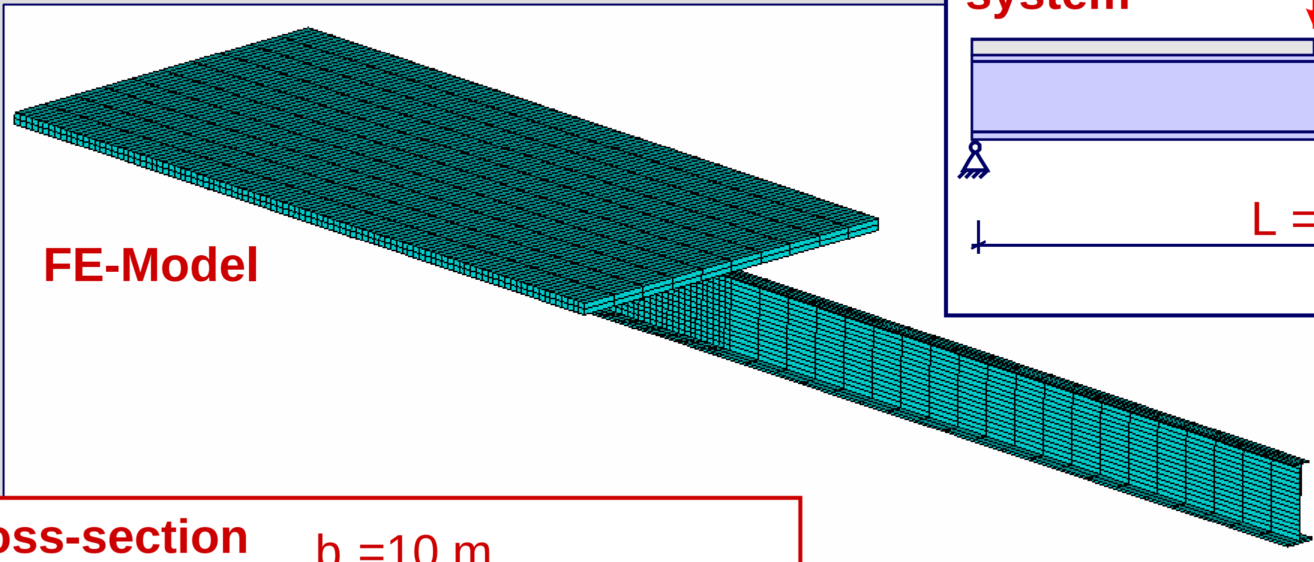

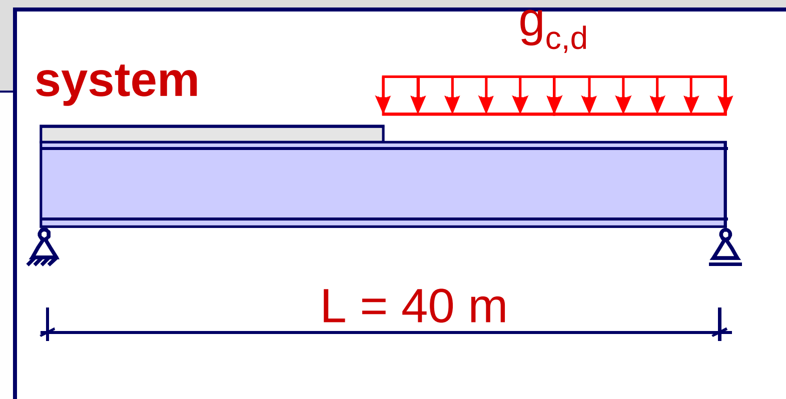

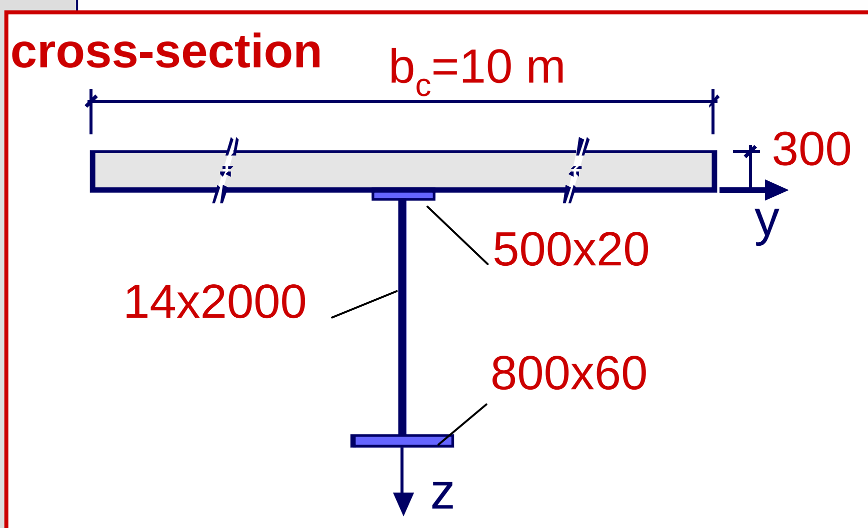

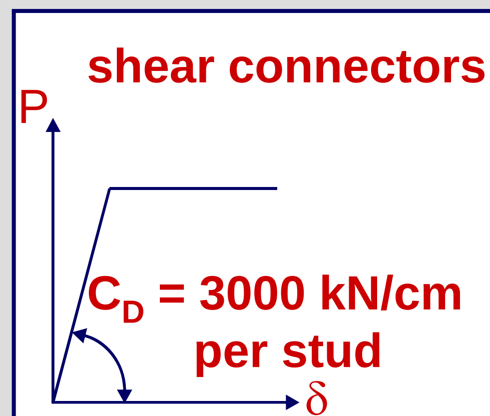

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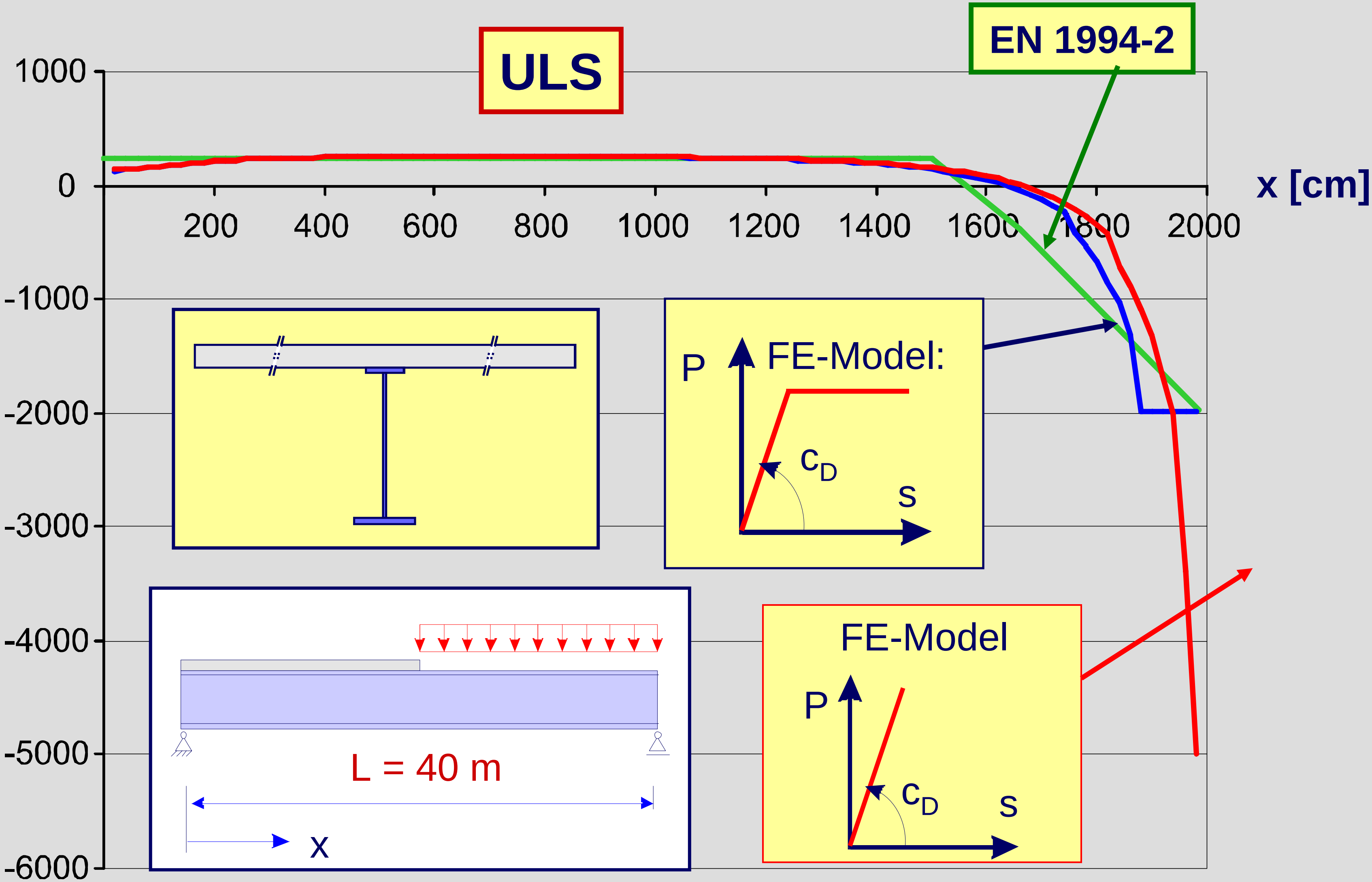

Serviceability limit state - longitudinal shear forces

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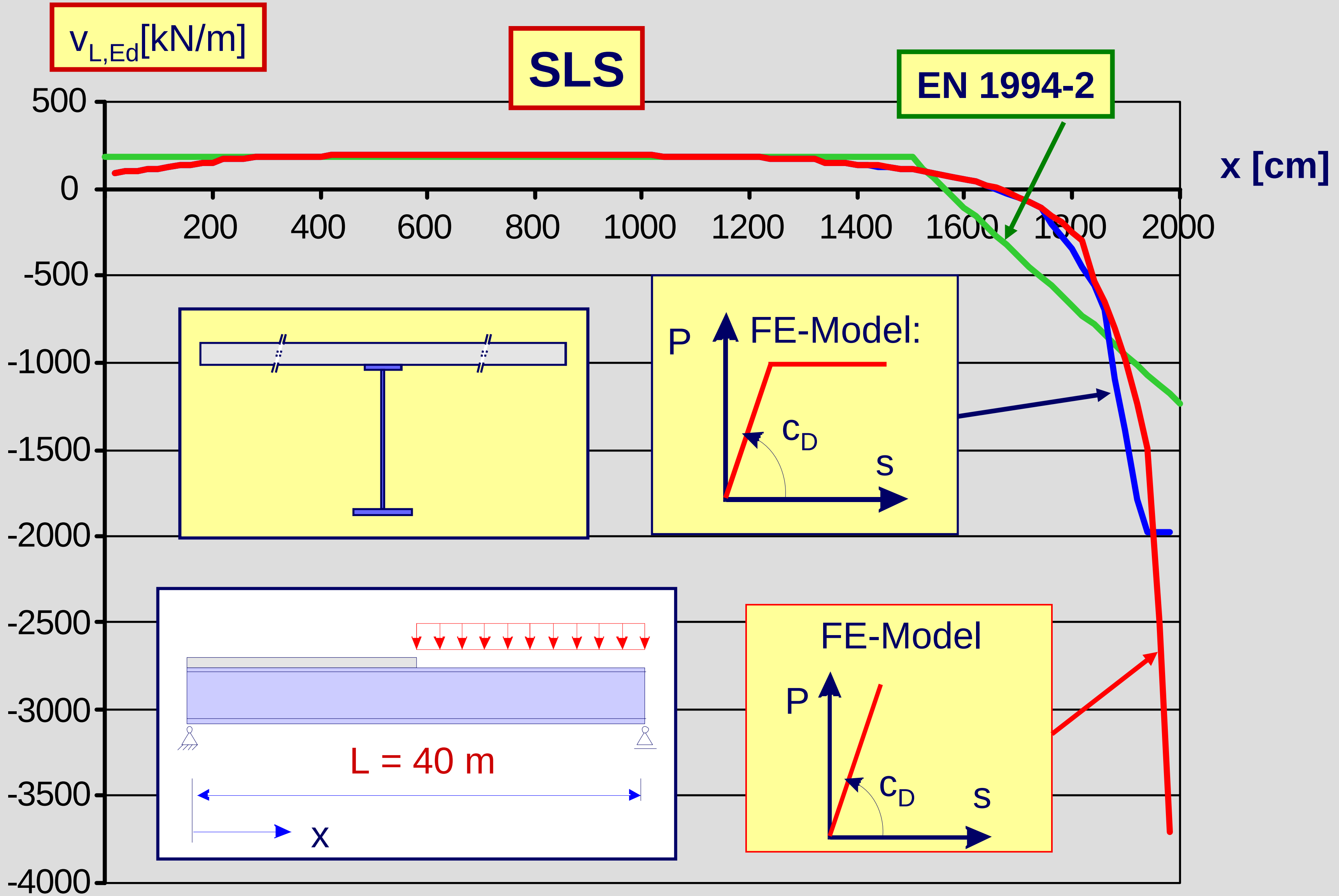

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Part 6:

Vibrations

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EN 1994-1-1: The dynamic properties of floor beams should satisfy the criteria in EN 1990,A.1.4.4

EN 1990, A1.4.4: To achieve satisfactory vibration behaviour of buildings and their structural members under serviceability conditions, the following aspects, among others, should be considered:

- the comfort of the user
- the functioning of the structure or its structural members

Other aspects should be considered for each project and agreed with the client

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EN 1990-A1.4.4:

For serviceability limit state of a structure or a structural member not to be exceeded when subjected to vibrations, **the natural frequency of vibrations of the structure or structural member should be kept above appropriate values** which depend upon the function of the building and the source of the vibration, and agreed with the client and/or the relevant authority.

Possible sources of vibration that should be considered include walking, synchronised movements of people, machinery, ground borne vibrations from traffic and wind actions. **These, and other sources, should be specified for each project and agreed with the client**.

Note in EN 1990-A.1.4.4: Further information is given in ISO 10137.

Vibration – Example vertical vibration due to walking persons

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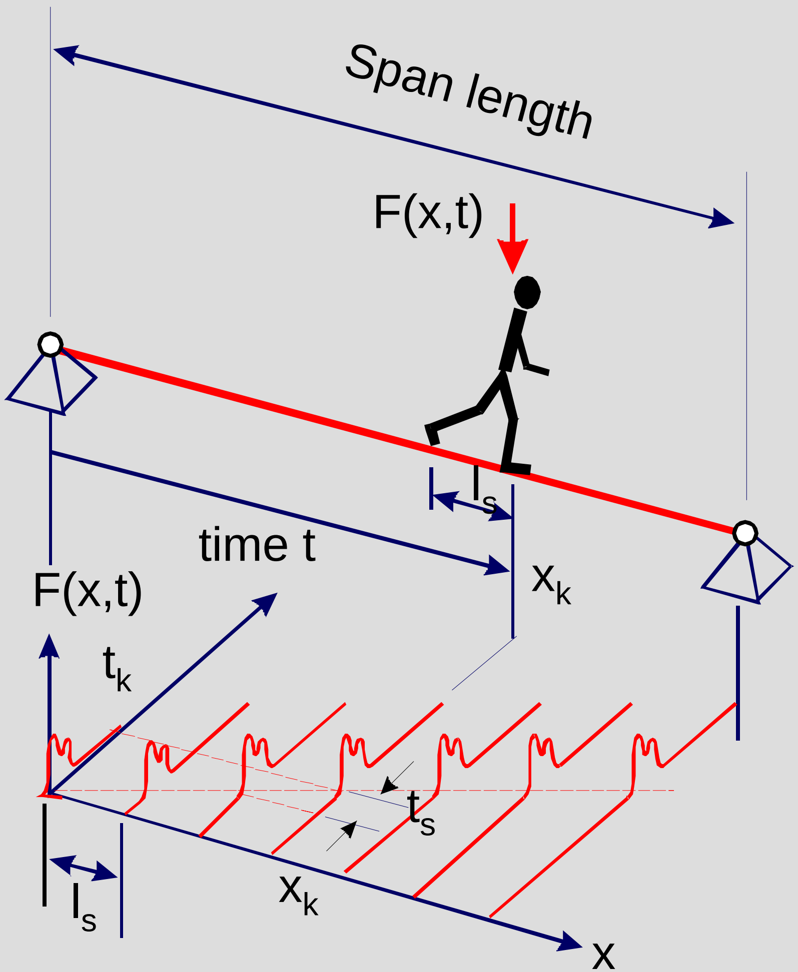

The pacing rate f_s dominates the dynamic effects and the resulting dynamic loads. The speed of pedestrian propagation v_s is a function of the pacing rate f_s and the stride length l_s.

	pacing rate f_s [Hz]	forward speed $v_s = f_s\, l_s$ [m/s]	stride length l_s [m]
slow walk	~1,7	1,1	0,6
normal walk	~2,0	1,5	0,75
fast walk	~2,3	2,2	1,00
slow running (jog)	~2,5	3,3	1,30
fast running (sprint)	> 3,2	5,5	1,75

Vibration –vertical vibrations due to walking of one person

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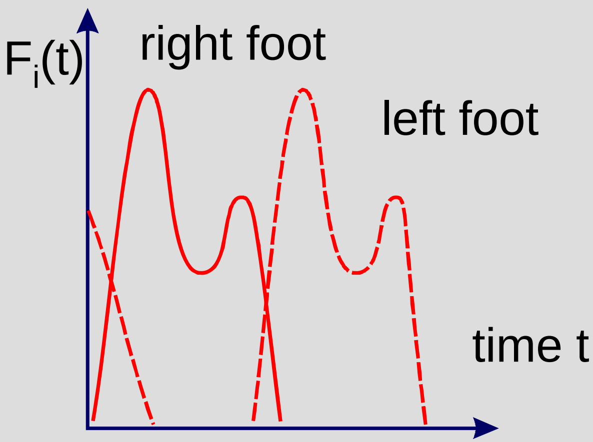

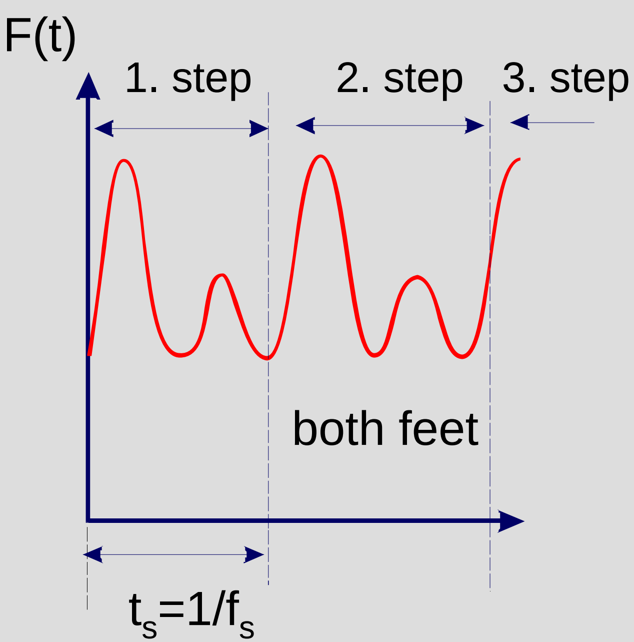

During walking, one of the feet is always in contact with the ground. The load-time function can be described by a Fourier series taking into account the 1st, 2nd and 3rd harmonic.

$$F(t) = G_o \left[1 + \sum_{n=1}^{3} \alpha_n \sin\left(2\, n\, \pi\, f_s\, t - \Phi_n\right) \right]$$

- G_o weight of the person (800 N)
- α_n coefficient for the load component of n-th harmonic
- n number of the n-th harmonic
- f_s pacing rate
- Φ_n phase angle oh the n-th harmonic

Fourier-coefficients and phase angles:

α_1=0,4-0,5 $\Phi_1=0$

α_2=0,1-0,25 $\Phi_2=\pi/2$

α_3=0,1-0,15 $\Phi_3=\pi/2$

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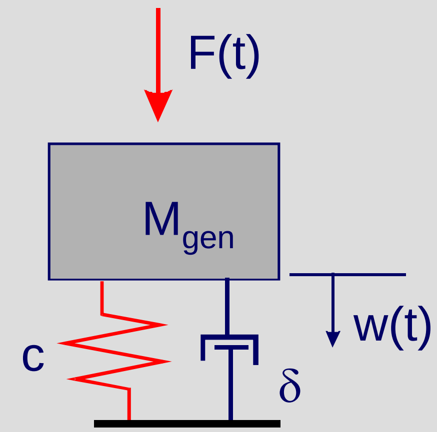
Vibration – vertical vibrations due to walking of persons



acceleration

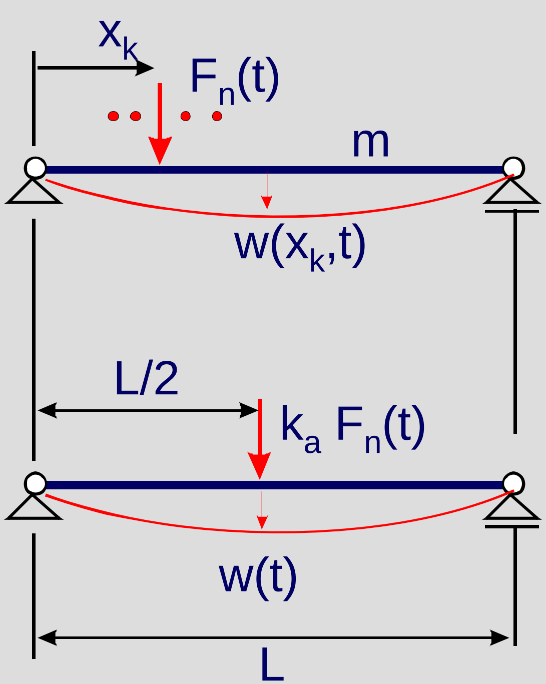
$$\ddot{w}(t) = k_a \frac{F_n}{M_{gen}} \frac{\pi}{\delta} \sin(2\pi f_E t)\left(1 - e^{-\delta f_E t}\right) \qquad t = \frac{L}{v_s}$$

maximum acceleration a, vertical deflection w and maximum velocity v

$$a_{max} = k_a \frac{F_n}{M_{gen}} \frac{\pi}{\delta}\left(1 - e^{-f_E \delta L / v_s}\right)$$

$$w_{max} = \frac{a}{(2\pi f_E)^2}$$

$$v_{max} = \frac{a}{2\pi f_E}$$



f_E	natural frequency
F_n	load component of n-th harmonic
δ	logarithmic damping decrement
v_s	forward speed of the person
k_a	factor taking into account the different positions x_k during walking along the beam
M_{gen}	generated mass of the system (single span beam: M_{gen}=0,5 m L)

Logarithmic damping decrement

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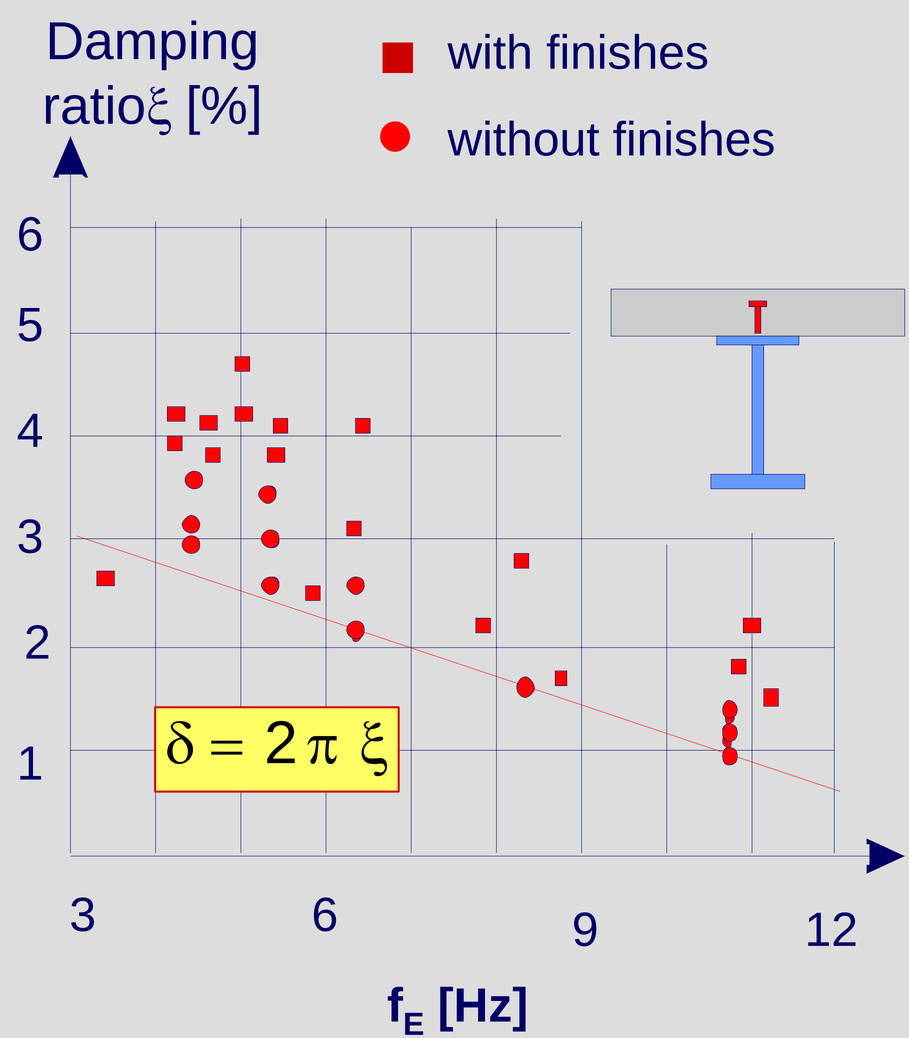
results of measurements in buildings



For the determination of the maximum acceleration the damping coefficient ζ or the logarithmic damping decrement δ must be determined. Values for composite beams are given in the literature. The logarithmic damping decrement is a function of the used materials, the damping of joints and bearings or support conditions and the natural frequency.

For typical composite floor beams in buildings with natural frequencies between 3 and 6 Hz the following values for the logarithmic damping decrement can be assumed:

δ=0,10	floor beams without not load-bearing inner walls
δ=0,15	floor beams with not load-bearing inner walls

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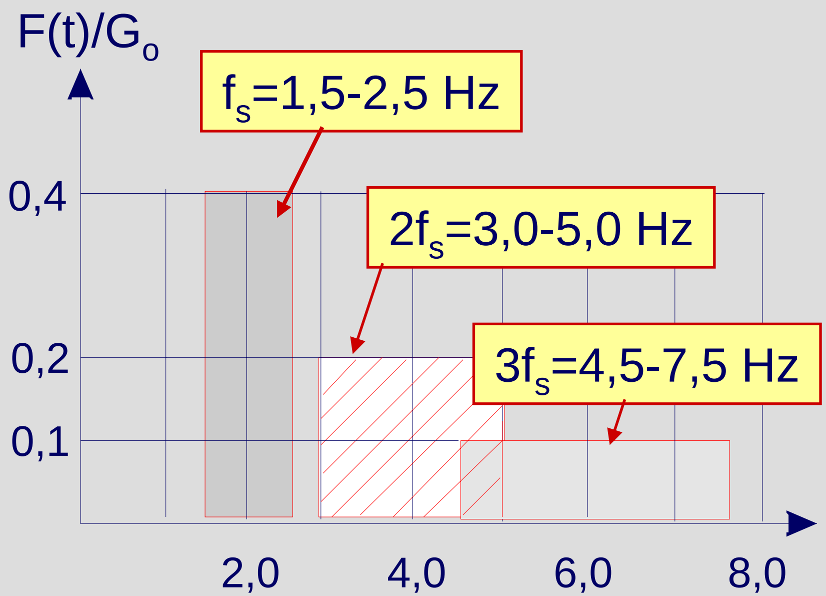
$$F(t) = G_o + \sum_{n=1}^{3} F_n \sin\left(2\,n\,\pi\,f_s\,t - \Phi_n\right)$$



People in office buildings sitting or standing many hours are very sensitive to building vibrations. Therefore the effects of the second and third harmonic of dynamic load-time function should be considered, especially for structure with small mass and damping. In case of walking the pacing rate is in the rage of 1.7 to 2.4 Hz. The verification can be performed by frequency tuning or by limiting the maximum acceleration.

In case of **frequency tuning** for composite structures in office buildings the natural frequency normally should exceed **7,5 Hz** if the first, second and third harmonic of the dynamic load-time function can cause significant acceleration.

Otherwise the **maximum acceleration or velocity should be determined and limited** to acceptable values in accordance with ISO 10137

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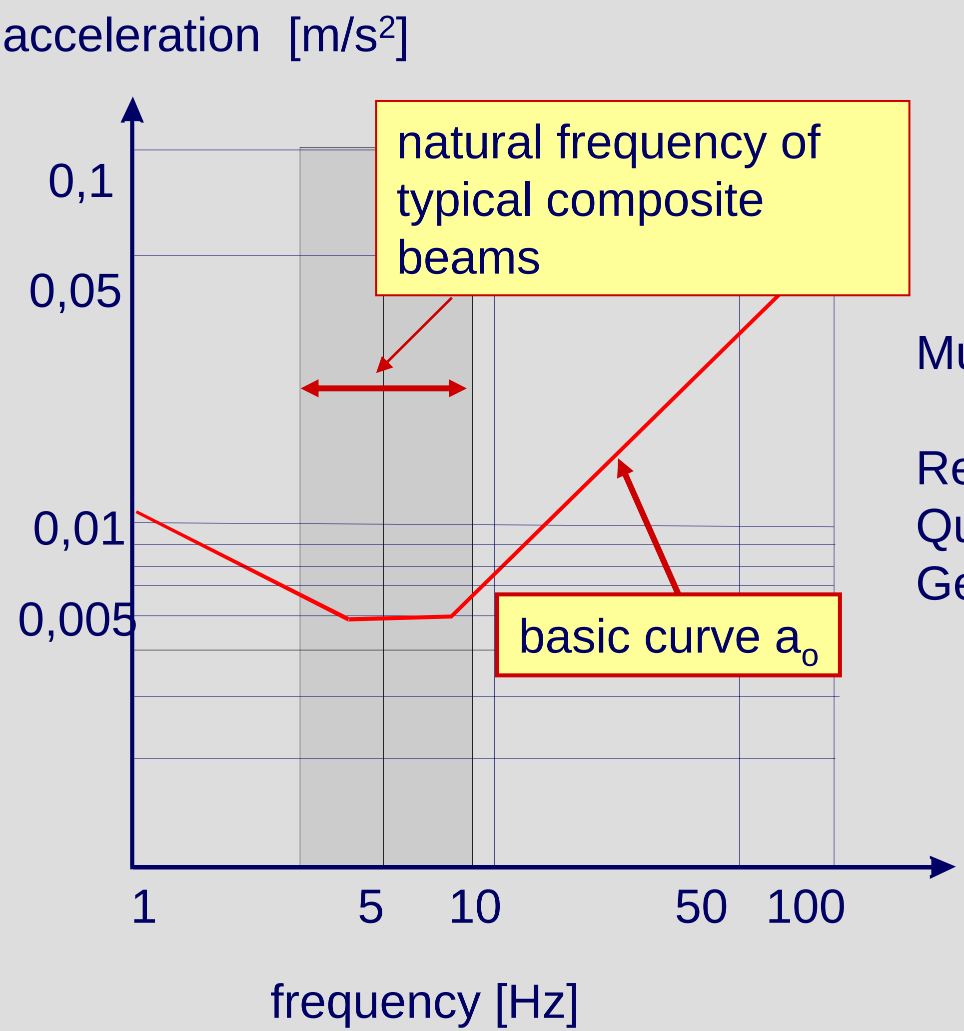
Limitation of acceleration-recommended values acc. to ISO 10137



Multiplying factors K_a for the basic curve

Residential (flats, hospitals)	K_a=1,0
Quiet office	K_a=2-4
General office (e. g. schools)	K_a=4

$$a \leq a_o\ K_a$$

Thank you very much for your kind attention

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Thank you very much for your kind attention