Section 7
Serviceability limit states

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Chair of Timber Structures
and Building Construction

Serviceability limit states

Calculation of

- Deformations, Deflections
- Vibrations

Deflections, Deformations

$$w = \int_{System} \left(\frac{M \cdot \overline{M}}{EI} + \frac{N \cdot \overline{N}}{EA} + \frac{Q \cdot \overline{Q}}{GA^*} \right) ds + \sum_{Verbindungen} \frac{F \cdot \overline{F}}{K_{ser}}$$

7.1 Table 1

Vibrations

$$\omega = \sqrt{\frac{K}{M}} = 2\pi \cdot f$$

Section 2.2.3

(5) For structures consisting of members, components and connections with the same creep behaviour and under the assumption of a linear relationship between the actions and the corresponding deformations, as a simplification of 2.2.3(3), the final deformation, u_{fin}, may be taken as:

$$u_{\text{fin}} = u_{\text{fin,G}} + u_{\text{fin,Q}_1} + u_{\text{fin,Q}_i} \tag{2.2}$$

where:

$$u_{\text{fin,G}} = u_{\text{inst,G}} \left(1 + k_{\text{def}}\right) \quad \text{for a permanent action, } G \tag{2.3}$$

$$u_{\text{fin,Q,1}} = u_{\text{inst,Q,1}} \left(1 + \psi_{2,1} k_{\text{def}}\right) \quad \text{for the leading variable action, } Q_1 \tag{2.4}$$

$$u_{\text{fin,Q,i}} = u_{\text{inst,Q,i}} \left(\psi_{0,i} + \psi_{2,i} k_{\text{def}}\right) \quad \text{for accompanying variable actions, } Q_i \ (i > 1) \tag{2.5}$$

$u_{\text{inst,G}}$, $u_{\text{inst,Q,1}}$, $u_{\text{inst,Q,i}}$ are the instantaneous deformations for action G, Q_1, Q_i respectively;

$$E = \frac{E_{mean}}{(1 + k_{def})}$$

– w_c is the precamber (if applied);
– w_{inst} is the instantaneous deflection;
– w_{creep} is the creep deflection;
– w_{fin} is the final deflection;
– $w_{net,fin}$ is the net final deflection.

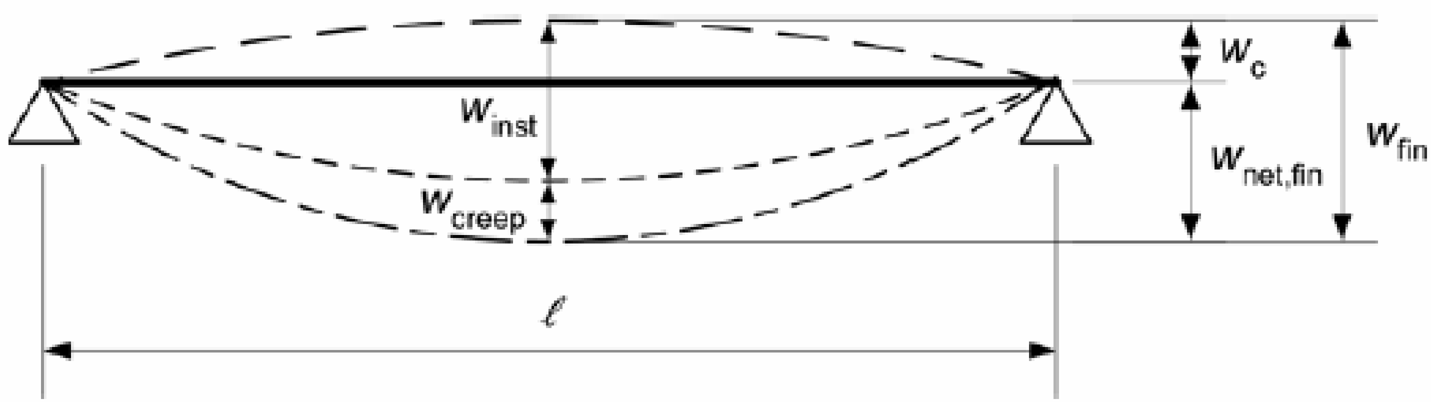


Figure 7.1 – Components of deflection

$$w_{net,fin} = w_{inst} + w_{creep} - w_c = w_{fin} - w_c$$

NOTE: The recommended range of limiting values of deflections for beams with span ℓ is given in Table 7.2 depending upon the level of deformation deemed to be acceptable. Information on National choice may be found in the National annex.

Table 7.2 – Examples of limiting values for deflections of beams

	w_{inst}	$w_{net,fin}$	w_{fin}
Beam on two supports	ℓ/300 to ℓ/500	ℓ/250 to ℓ/350	ℓ/150 to ℓ/300
Cantilevering beams	ℓ/150 to ℓ/250	ℓ/125 to ℓ/175	ℓ/75 to ℓ/150

Deflections

Vibrations

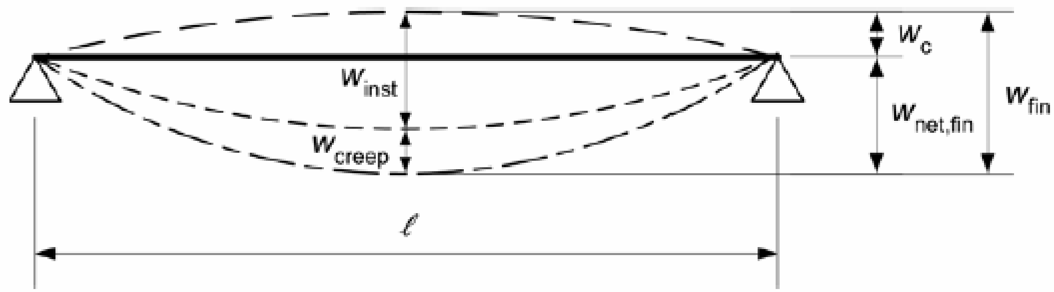


Figure 7.1 – Components of deflection

Frequency

$$f = \frac{5}{\sqrt{0{,}8 \cdot w}}$$

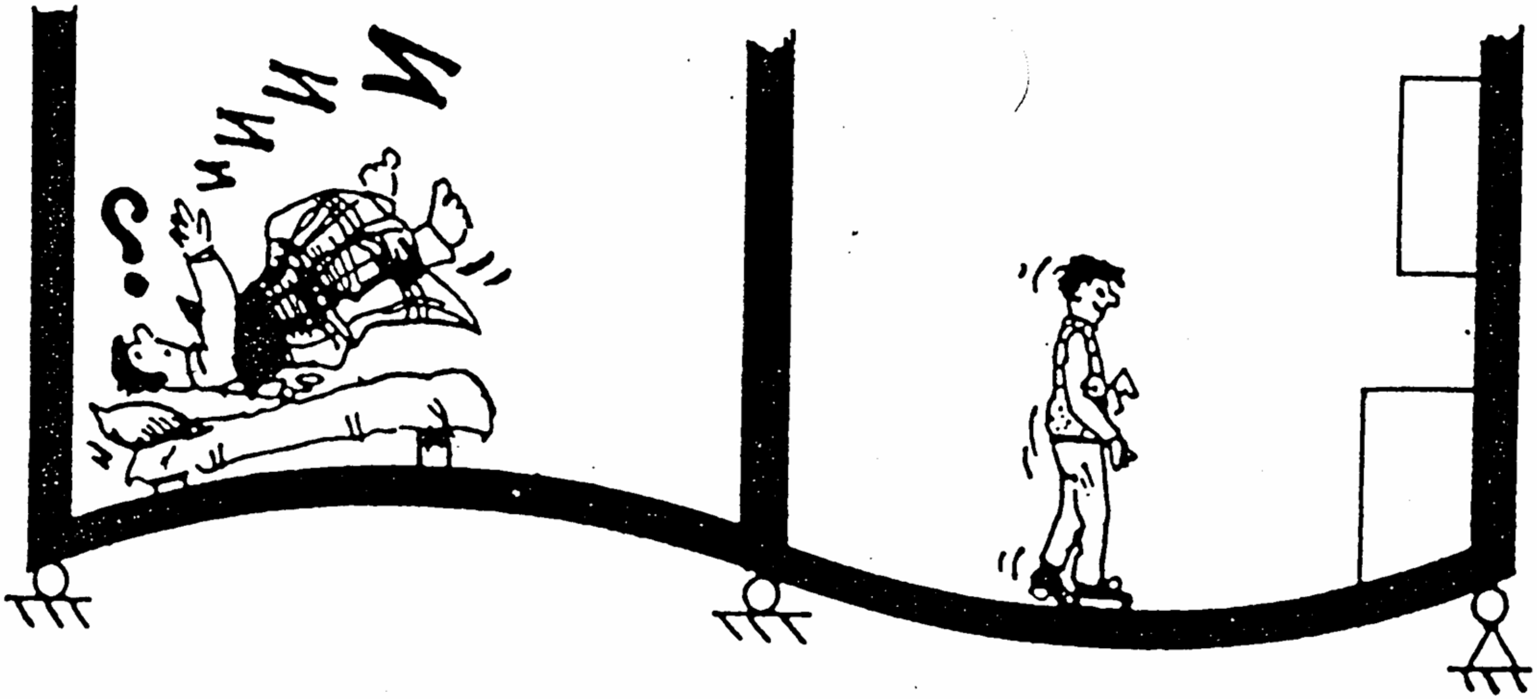
Vibrations:

Servicability limit states

If necessary

Ultimate limit states

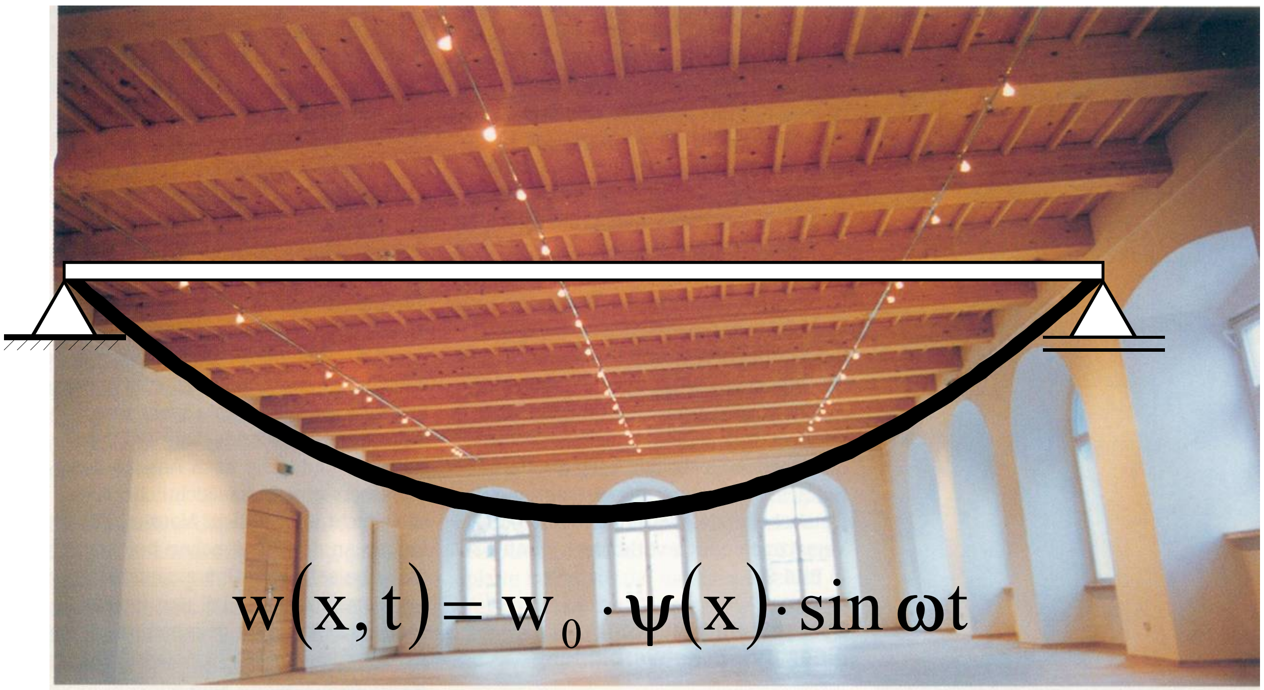
Fatigue
Durability



Serviceability (Ohlsen)

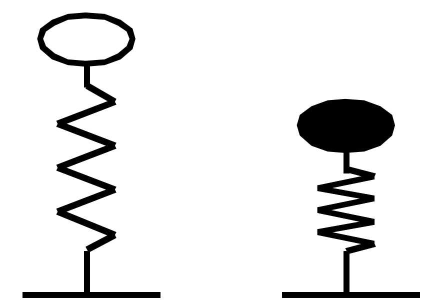
Single degree of freedom System

floor, beam



$$w(x,t) = w_0 \cdot \psi(x) \cdot \sin \omega t$$

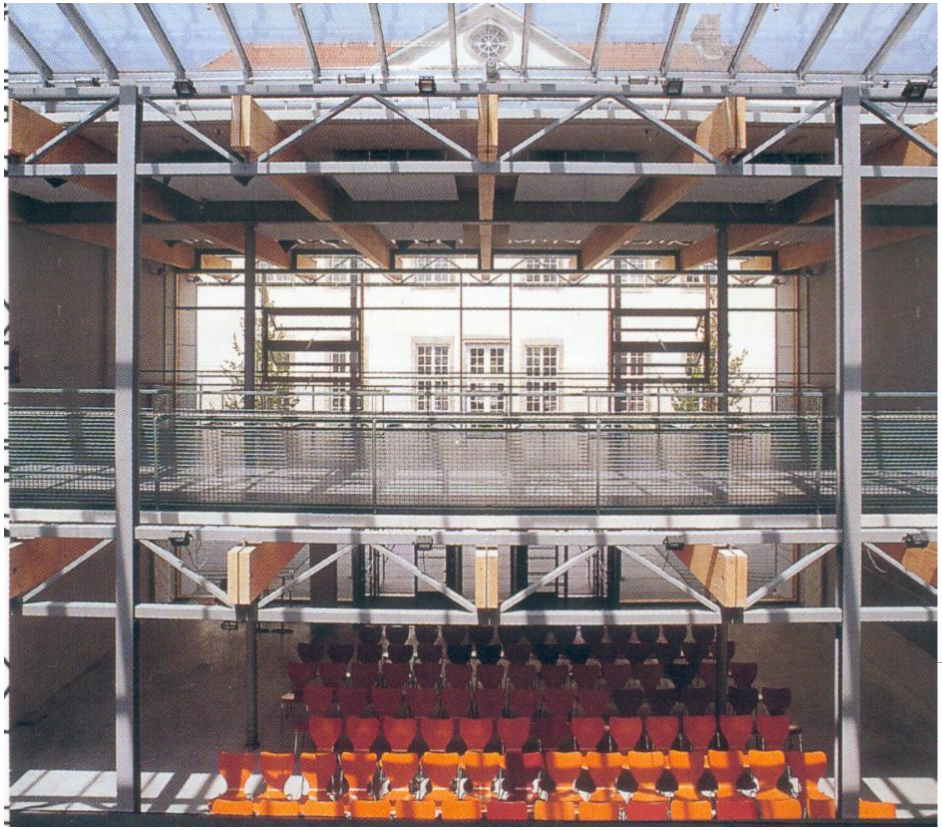
Bild 1: Untersicht der Holzbalken-Stahlbetonplatten-Verbunddecke des Alten Rathauses der Stadt Viechtach

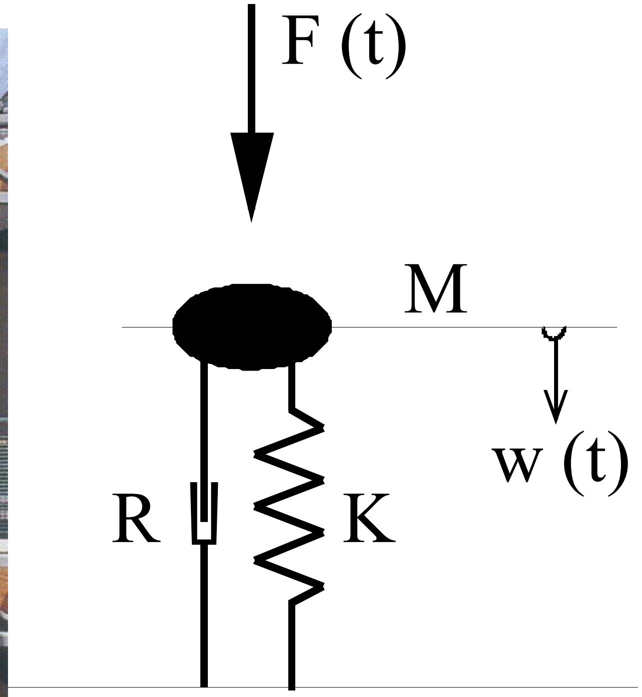


$$w(t) = w_0 \cdot \sin \omega t$$

Viechtach, Bertsche

Single degree of freedom System

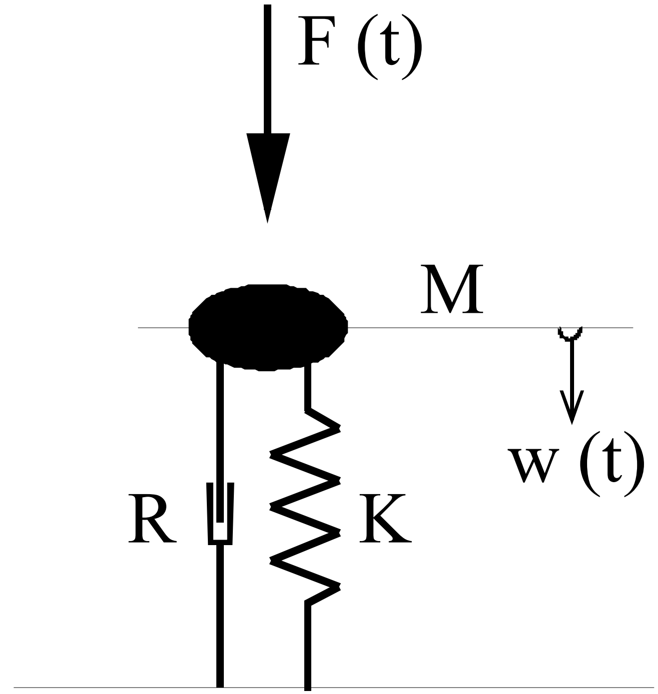




Lemgo, Mayer/Ludscher, SFS

Single degree of freedom System

M Mass t
K Stiffness kN/m
R Damping kN/(m/s)
R=2 M D ω
D Damping ratio



$$\omega = \sqrt{\frac{K}{M}} = 2 \cdot \pi \cdot f$$

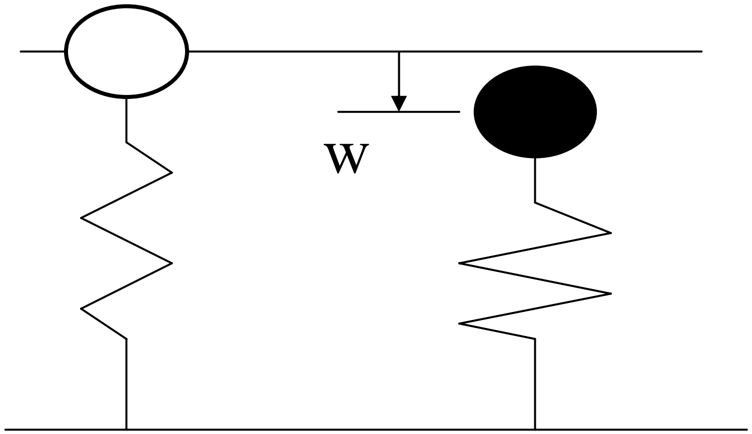
$$w_g = \frac{G}{K} = \frac{M \cdot g}{K} = \frac{g}{\omega^2} = \frac{g}{(2 \cdot \pi \cdot f)^2}$$

$$f = \frac{5}{\sqrt{w_g}} \qquad w_g \ in\ cm!$$

Single degree of freedom System

Frequency - Deformation

$$f = \frac{5}{\sqrt{w_g}} \qquad w_g \ in\ cm!$$

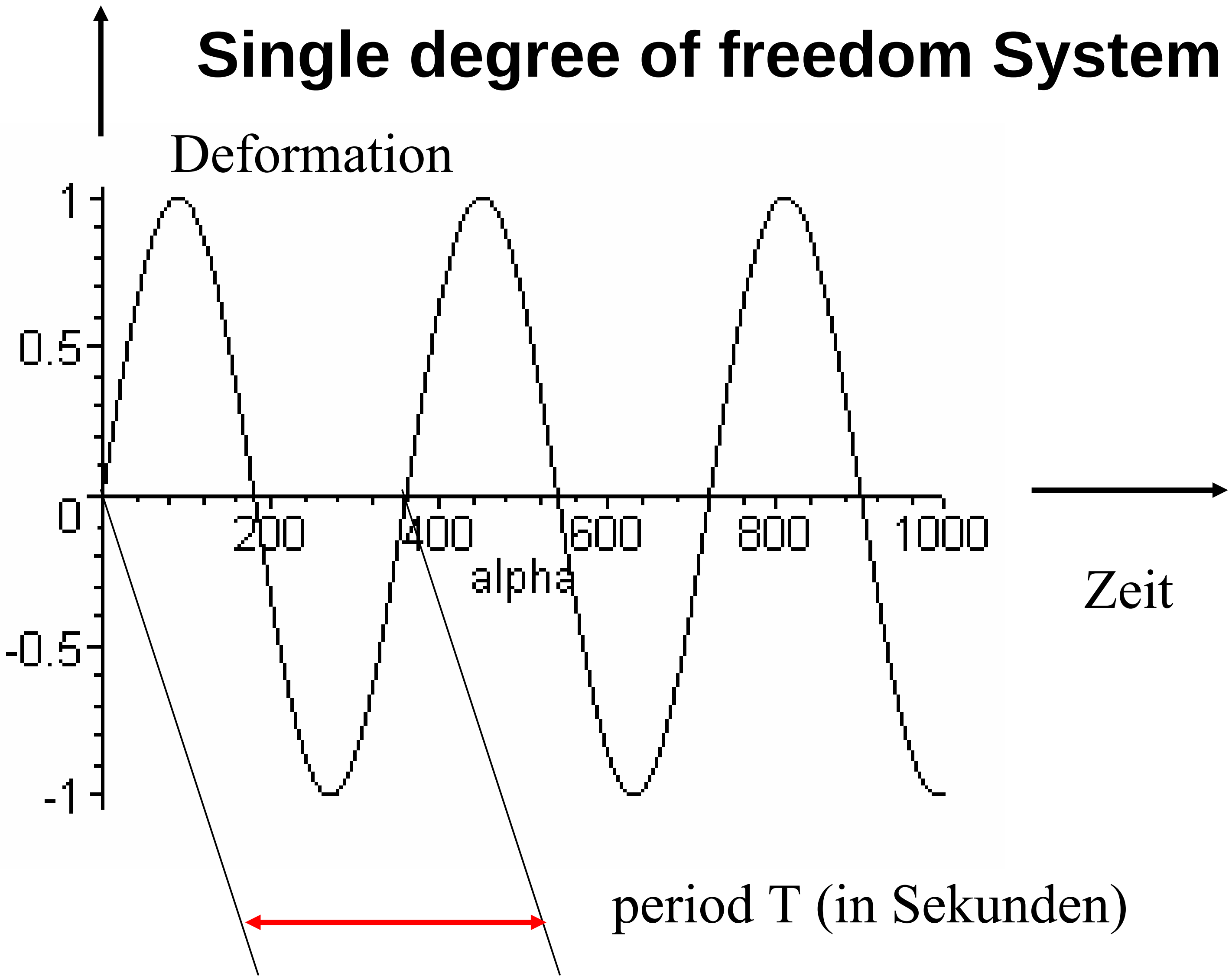


$$w_{g+0,3p} = 6\ mm = 0,6 cm$$

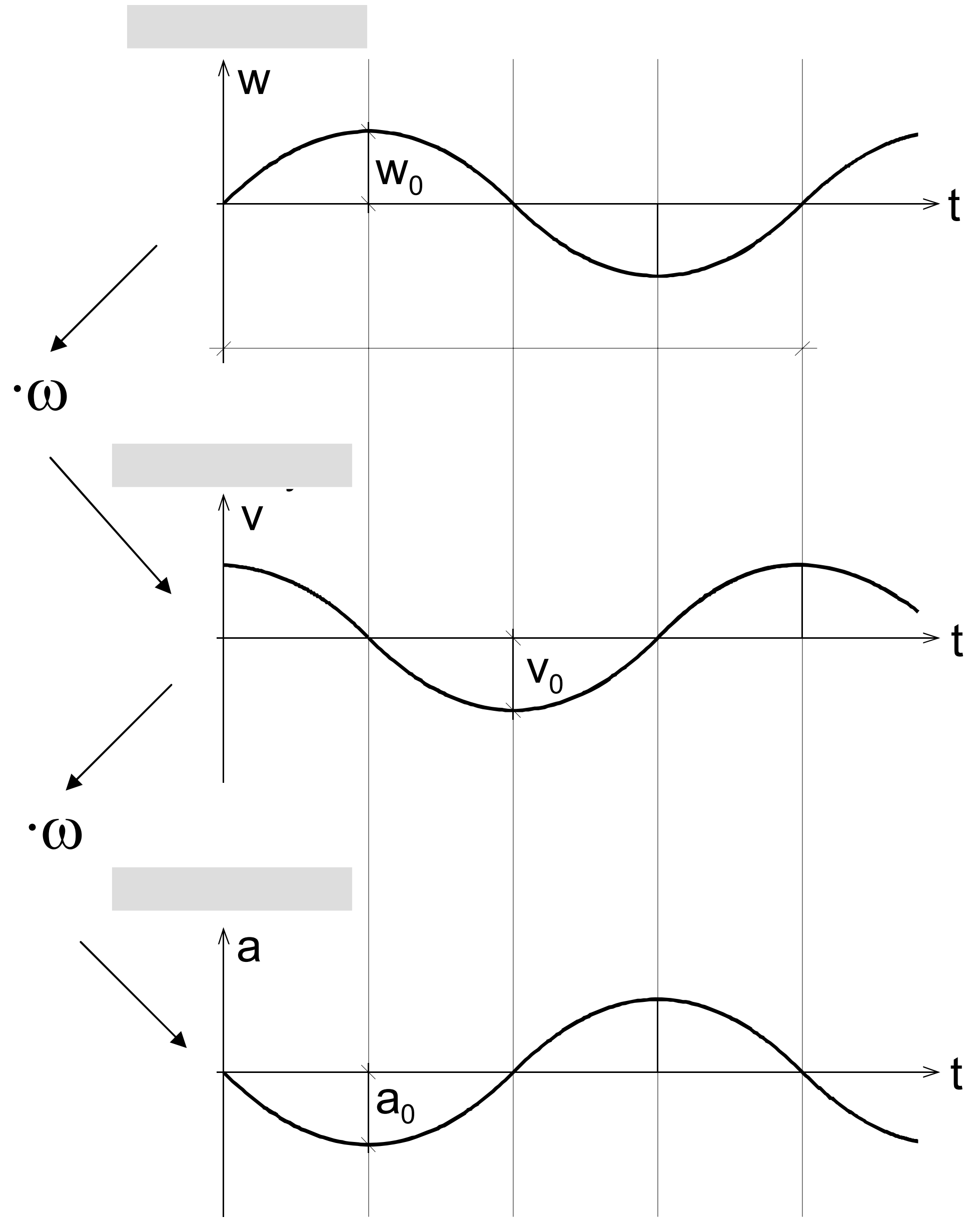
$$f = \frac{5}{\sqrt{0,8 \cdot 0,6}} = 7,2\ Hz$$

Factor for beam

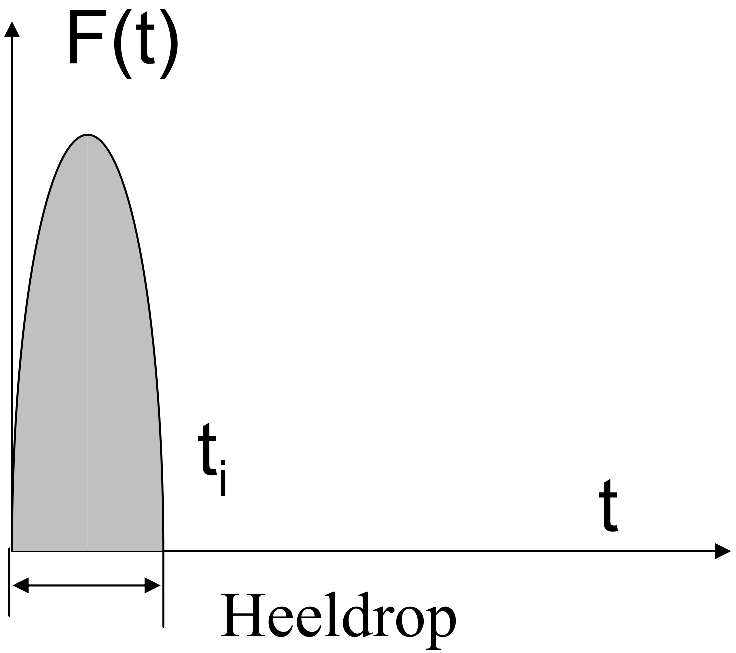
Single degree of freedom System



Deflection w
Velocity v
Acceleration a



Impuls



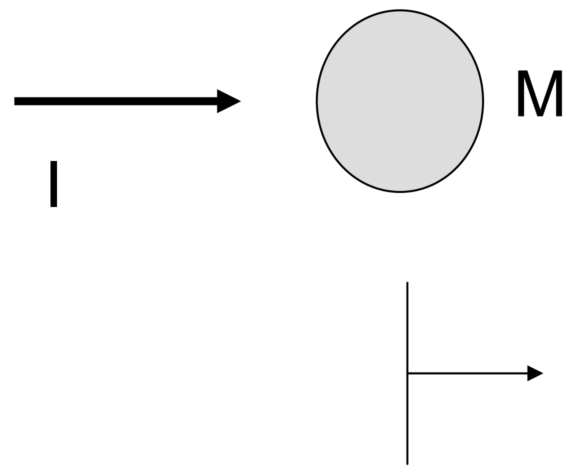
Impuls I
Ti=0
F=infinite

$$I = \int F(t) \cdot dt = M \cdot v = M \cdot \sqrt{2 \cdot g \cdot h} =$$

$$55 \cdot \sqrt{2 \cdot 9{,}81 \cdot 0{,}05} = 55\ kg \cdot m/s = 55\ N \cdot s$$

Impuls

Velocity v

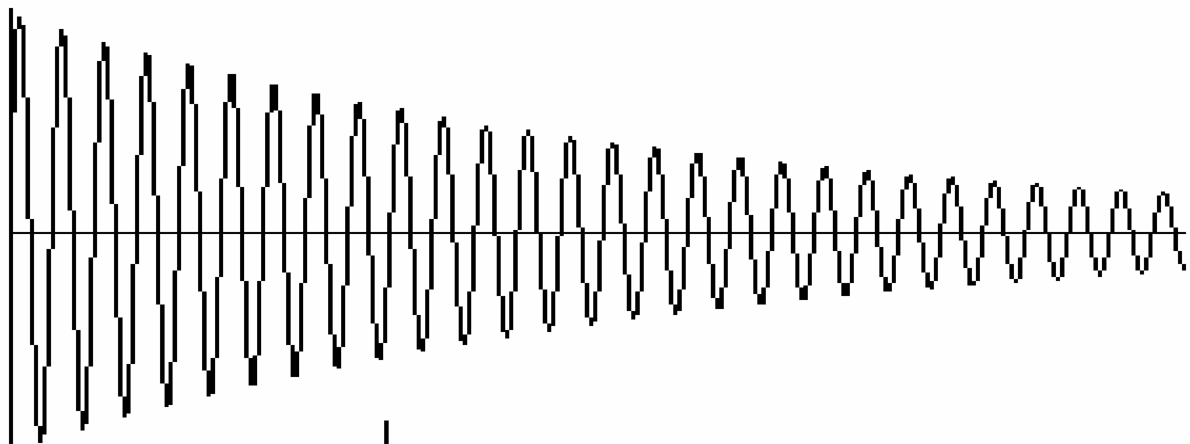


$$v = \frac{I}{M} \qquad \frac{N \cdot s}{kg} = \frac{m}{s}$$

$t = 0$

Impuls

$$w(t) = \frac{I}{M \cdot \omega \cdot \sqrt{1-\zeta^2}} \cdot e^{-\zeta \cdot \omega \cdot t} \cdot \sin\left(\sqrt{\left(1-\zeta^2\right)} \cdot \omega \cdot t\right)$$



$$\frac{I}{M \cdot \omega \cdot \sqrt{1-\zeta^2}} \cdot = 1 \quad \text{und} \quad \zeta = 0{,}01$$

Chair of Timber Structures
and Building Construction

Impuls – I = 1Ns, 7.3.3(5)

$$f_{1,n} = f_0 \cdot \sqrt{1 + \frac{EJ_b}{EJ_\ell} \frac{\ell^4}{b^4} \cdot n_4^4}$$

$f_{1,n}$ frequency of a plate

n number of waves in direction vertical to the main span

Number of frequencies $f_{1,n} \leq 40$ Hz

$$n_{40} = \left[\left(\left(\frac{40}{f_0} \right)^2 - 1 \right) \cdot \frac{b^4 EJ_\ell}{\ell^4 EJ_b} \right]^{0,25}$$

$$v = \frac{4 \cdot (0,4 + 0,6 \cdot n_{40})}{m \cdot b \cdot \ell + 200} \approx \frac{1}{m \cdot \frac{\ell}{2} \cdot b_{SI}} = \frac{1}{M}$$

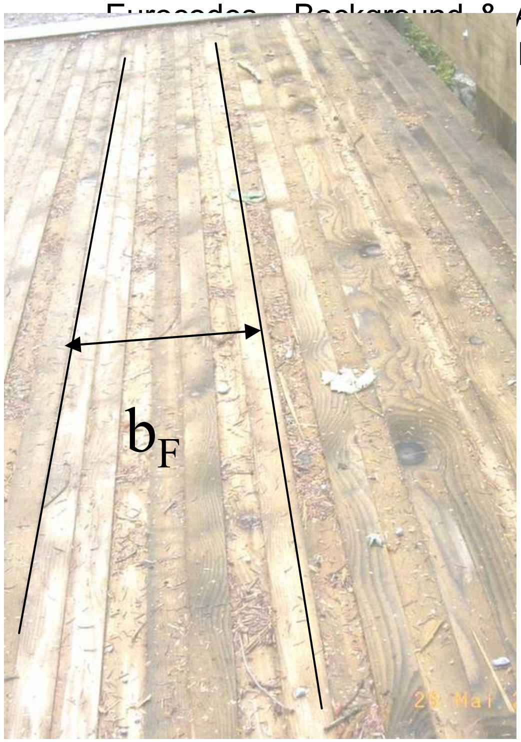
Impuls – Heeldrop I = 55 Ns

Experimental solution,
Found by testing

$$v = \frac{0{,}6}{m^{0,5} \cdot EI_{\ell}^{\,0,25} \cdot EI_{b}^{\,0,25}} \frac{m}{s}$$

7.3.3(2)

$$w = \frac{F_0}{K}$$



uniform load

$$w = \frac{5 \cdot q \cdot \ell^4}{384 \cdot EI}$$

Single load

$$w_F = \frac{F \cdot \ell^3}{48 \cdot EI_l \cdot b_F} \qquad b_F = \frac{\ell}{1,1} \cdot \sqrt[4]{\frac{EI_b}{EI_\ell}}$$

Serviceability limit states

	1	2	3	4
	Value	Limit	aim	EN 1995-1-1 2004 (E)
1	deflection w	w < l/X	Enough rigidity small deflection	7.2
4	frequency $w_{G,inst}$ quasi ständig, $g+\psi_2 p$	f >8 Hz (w < 0,5 cm) f < 8 Hz (w > 0,5 cm)	No resonance	7.3.3(2) 7.3.3(1)

3	Deflection Single load F=1kN	u <0,5 bis 4 mm	Small deformation Rigidity perpendicular to the main span	7.3.3(2) SIA 265
5	Impuls I=1 Ns (up to 40 Hz)	$v<b^{(f_1 \zeta - 1)}$ $50 < b < 150$ $\zeta = 0{,}01$	Velocity	7.3.3(2) SIA 265

Information

2	Durchbiegung $w_{G,inst}$ quasi ständig, $g+\psi_2 p$	$w < 6$ mm $w > 6$ mm	Frequenz keine Resonanz-untersuchung Frequenz Resonanz-untersuchung	DIN 1052, 2004 9.3

6	velocity heeldrop I=55Ns, t_i = 0,05s	$v<6\ b^{(f_1\zeta-1)}$	velocity	Mohr /bmh/

7.3.3(1) Special Investigation f < 8 Hz

7	Acceleration Resonance Walking	$a < 0{,}1\ m/s^2$	better performance	7.3.3.(1)
8	Acceleration Resonance Walking	a< 0,35 bis 0,7 m/s^2	poorer performance	7.3.3 (1)

Compare:

Kreuzinger, H.; Blaß, H.J.; Ehlbeck, J.; Steck, G.:
Erläuterungen zu DIN 1052:2004-08 –
Entwurf, Berechnung und Bemessung von Holzbauwerken.
Hrsg.: DGfH, Bruderverlag, Albert Bruder GmbH, Karlsruhe

Damping ratio

D, ζ R/(2 M ω)

δ,Λ logarithmic Damping ratio, 2 π D, 2 π ζ

$$\Lambda = \ln \frac{w_i}{w_{i+1}}$$

$$D \cong \frac{\Lambda}{2\pi}$$

Some values for D:

steel	0,005
concret	0,008
timber	0,010 bis 0,02

SIA

Wenn keine genaueren Informationen vorliegen, ist das Dämpfungsmass ζ (logarithmisches Dekrement geteilt durch 2π) ein Wert von 0,01 anzunehmen. Weitere Richtwerte sind:

-Holztragwerte ohne mech. Verbindungen	0,010
-Holztragwerte mit mech. Verbindungen	0,015
-Holzdecken ohne schwimmenden Estrich	0,010
-Decken aus Brettschichtholz mit schwimmenden Estrich	0,020
-Holzbalkendecken aus Brettstapeldecken mit einem schwimmenden Estrich	0,030

EN 1995-1-1:2004 (E)

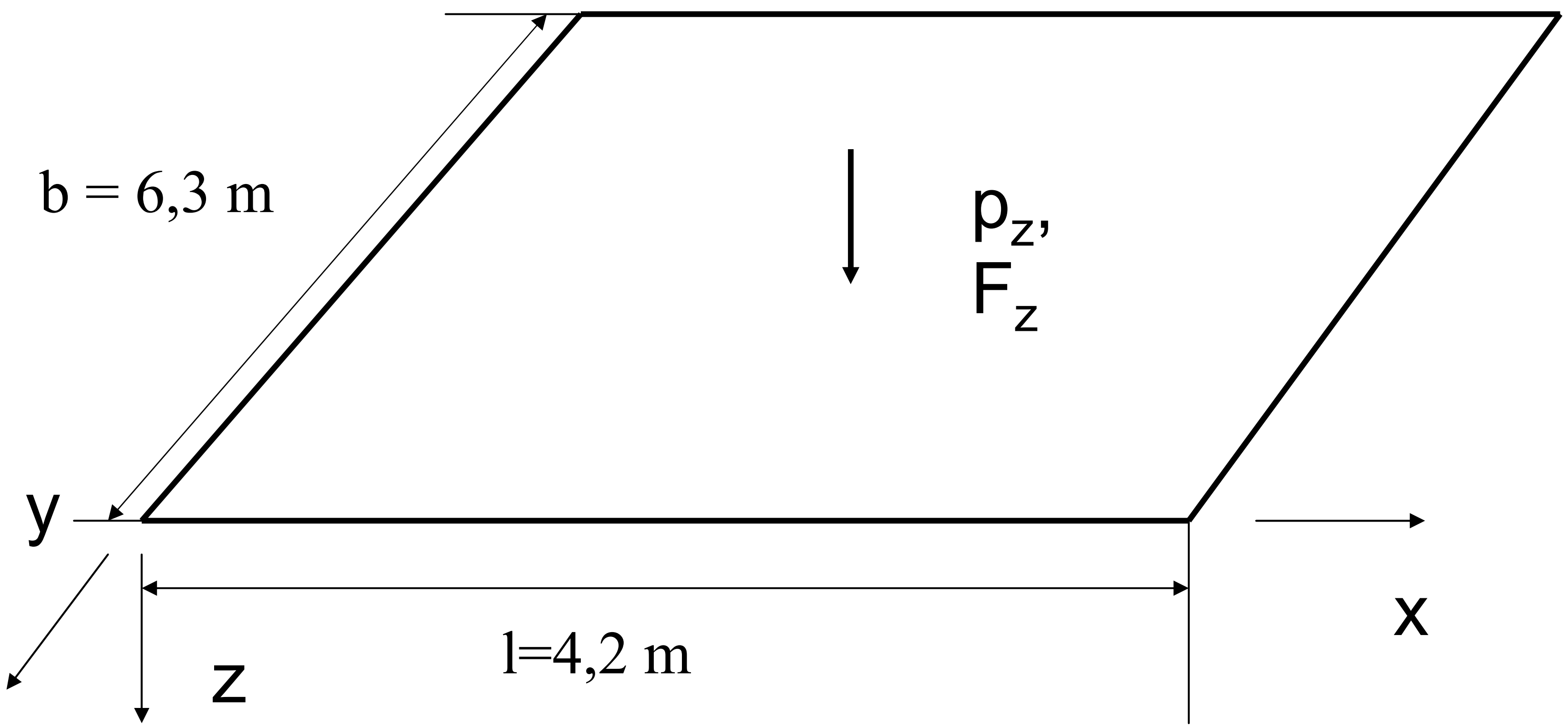
Eurocode 5

Design of timber structures

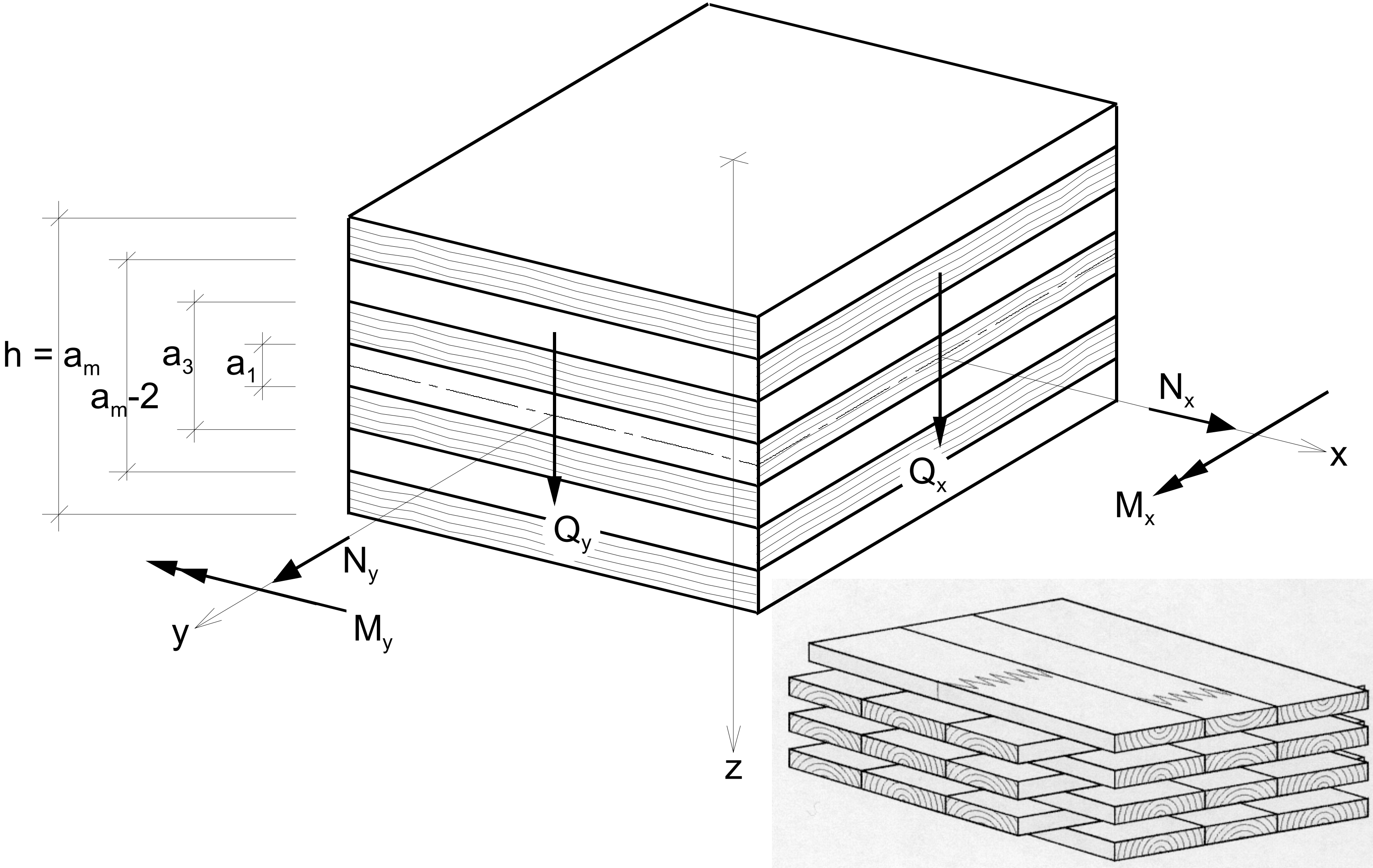
Part 1-1: General –
Common rules and rules for buildings

Schweizer Norm
Swiss code
SIA 265

Example - System



BSP

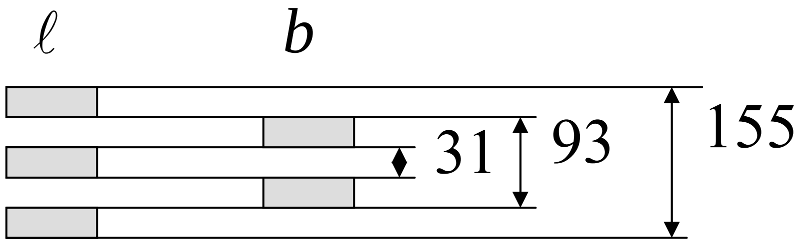


Self weight 3,23 kN/m^2
Trafic load 2,00 kN/m^2
Quasi ständige Kombination
nach DIN 1055-100, Gl 24

$$q_s = g + \Psi_2 \cdot p = 3{,}23 + 0{,}3 \cdot 2{,}0 = 3{,}83\ kN / m^2$$

Bending Stiffness

Decke: BSP 155 mm 5 Lagen je 31 mm



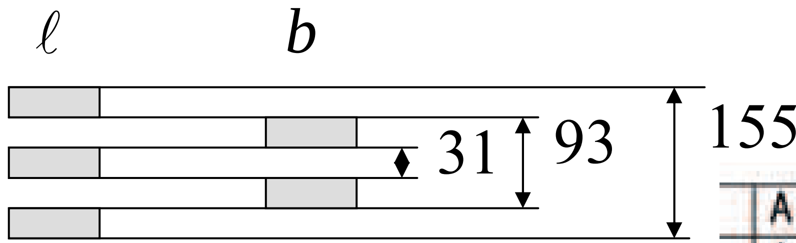
Biegesteifigkeiten:

$$EI_{\ell} = 11000 \cdot \frac{1}{12} \cdot \left(0{,}155^3 - 0{,}093^3 + 0{,}031^3\right) = 2{,}70\ MNm^2$$

$$EI_{b} = 11000 \cdot \frac{1}{12} \cdot \left(0{,}093^3 - 0{,}031^3\right) = 0{,}71\ MNm^2$$

$$EI_{\ell+b} = 11000 \cdot \frac{1}{12} \cdot 0{,}155^3 = 3{,}41\ MNm^2$$

Decke: BSP 155 mm 5 Lagen je 31 mm



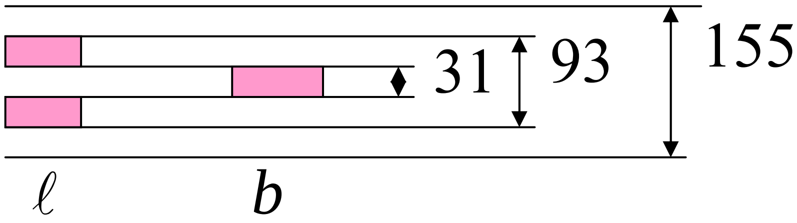
Biegesteifigkeiten:

A netto	I netto	W netto	i netto	Aq
A netto	I netto	W netto	i netto	Aq
A net	I net	W net	i net	Aq
A netto	I netto	W netto	i netto	Aq
[cm2]	[cm3]	[cm4]	[cm]	[cm2]
500	3.385	903	2,60	116
500	4.180	1.032	2,89	123
620	6.455	1.388	3,23	143
750	12.891	2.063	4,15	229
810	14.554	2.222	4,24	251
810	17.914	2.505	4,70	271
930	24.578	3.171	5,14	284

$$I_\ell = \frac{1}{12}\cdot\left(0{,}155^3 - 0{,}093^3 + 0{,}031^3\right) = 2{,}46\cdot 10^{-4}\, m^4 = 24600\ cm^4$$

Shear Stiffness

Decke: BSP 155 mm 5 Lagen je 31 mm



$$a_\ell = 155 - 31 = 124\ mm$$

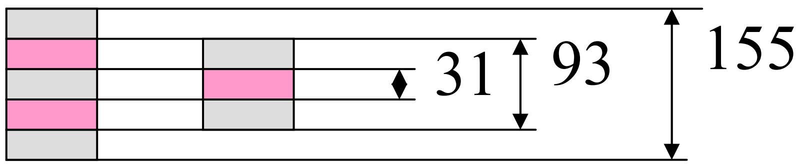
$$a_b = 93 - 31 = 62\ mm$$

$$S_\ell = \frac{a_\ell^{\,2} \cdot G_R \cdot 1}{n \cdot d_i} = \frac{0{,}124^2 \cdot 75 \cdot 1}{2 \cdot 0{,}031} = 18{,}6\ MN$$

$$S_b = \frac{a_b^{\,2} \cdot G_R \cdot 1}{n \cdot d_i} = \frac{0{,}062^2 \cdot 75 \cdot 1}{1 \cdot 0{,}031} = 9{,}3\ MN$$

Effective Stiffness

Decke: BSP 155 mm 5 Lagen je 31 mm



Näherung:

$$efEI = EI \cdot \frac{1}{1 + \frac{EI \cdot \pi^2}{S \cdot l^2}}$$

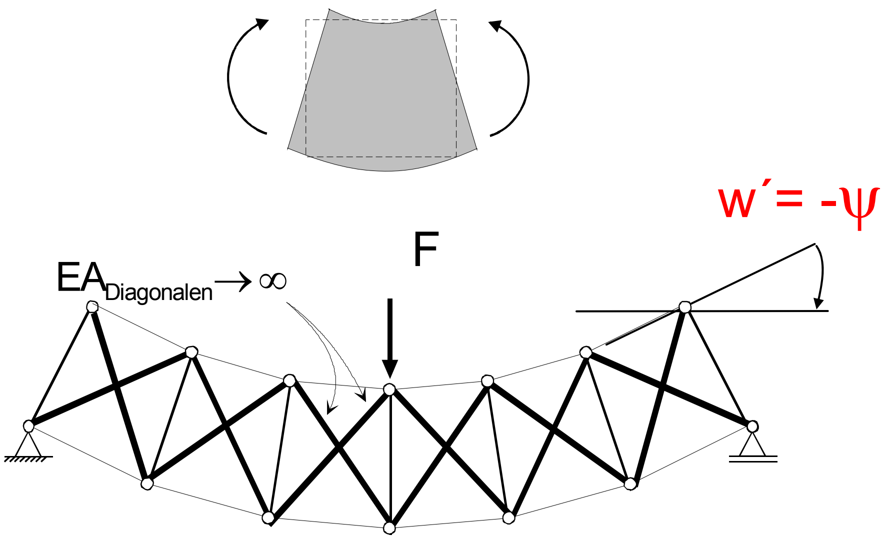
Wirksame Steifigkeiten:

$$efEI_\ell = 2{,}704 \cdot \frac{1}{1 + \frac{2{,}704 \cdot \pi^2}{18{,}6 \cdot 4{,}2^2}} = 2{,}50\ MNm^2$$

$$efEI_b = 0{,}71 \cdot \frac{1}{1 + \frac{0{,}71 \cdot \pi^2}{9{,}3 \cdot 6{,}3^2}} = 0{,}70\ MNm^2$$

Beispiel – Biege- Schubverformung

Biegung ohne Schub
(EI, GA→ ∞)

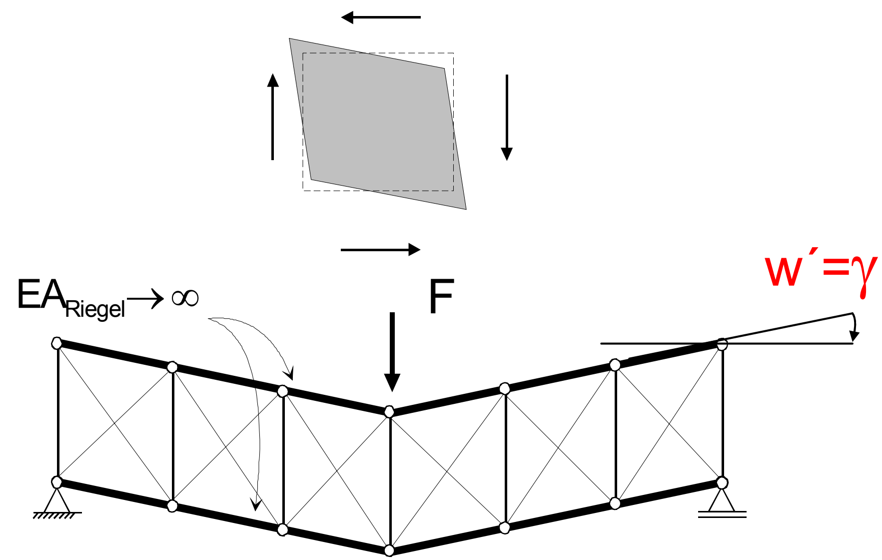


$$w = \frac{F \cdot \ell^3}{48 \cdot EI}$$

Im Fachwerkmodell:

- Elastische Gurte (EA),
- Dehnstarre Diagonalen (EA→ ∞)

Reine Schubverformung
(GA, EI→∞)



$$w = \frac{F \cdot \ell}{4 \cdot GA}$$

Im Fachwerkmodell:

- Dehnstarre Gurte (EA→ ∞),
- Elastische Diagonalen (EA)

Abschnitt 8.6, Anhang D

Deflection, Frequency

Deflection, single span

$$w = \frac{5 \cdot g \cdot \ell^4}{384 \cdot efEI_\ell} = \frac{5 \cdot 3{,}83 \cdot 10^{-3} \cdot 4{,}2^4}{384 \cdot 2{,}50}$$

$$= 6{,}21 \cdot 10^{-3}\ m = 6{,}21\, mm$$

Frequency

$$f = \frac{5}{\sqrt{0{,}8 \cdot w}} = \frac{5}{\sqrt{0{,}8 \cdot 0{,}621}} = 7{,}09\ Hz$$

Frequency plate

Anisotrope Platte, Trägerrost

m*: Masse (tausend t/m2)

k: Zahl der Wellen in x-Richtung

n: Zahl der Wellen in y-Richtung

n40: Zahl der Eigenfrequenzen unter 40 Hz

$$\psi_{k,n} = w_{k,n} \cdot \sin\frac{k\pi}{\ell} \cdot x \cdot \sin\frac{n\pi}{b} y$$

$$m\omega^2 = EJ_\ell \frac{k^4\pi^4}{\ell^4} + EJ_b \frac{n^4\pi^4}{b^4}; \qquad f_0 = \frac{\pi}{2\ell^2} \cdot \sqrt{\frac{EJ_\ell}{m}}$$

$$f_{m,n} = f_0 \sqrt{k^4 + \frac{EJ_b}{EJ_\ell} \cdot \frac{\ell^4}{b^4} \cdot n^4}$$

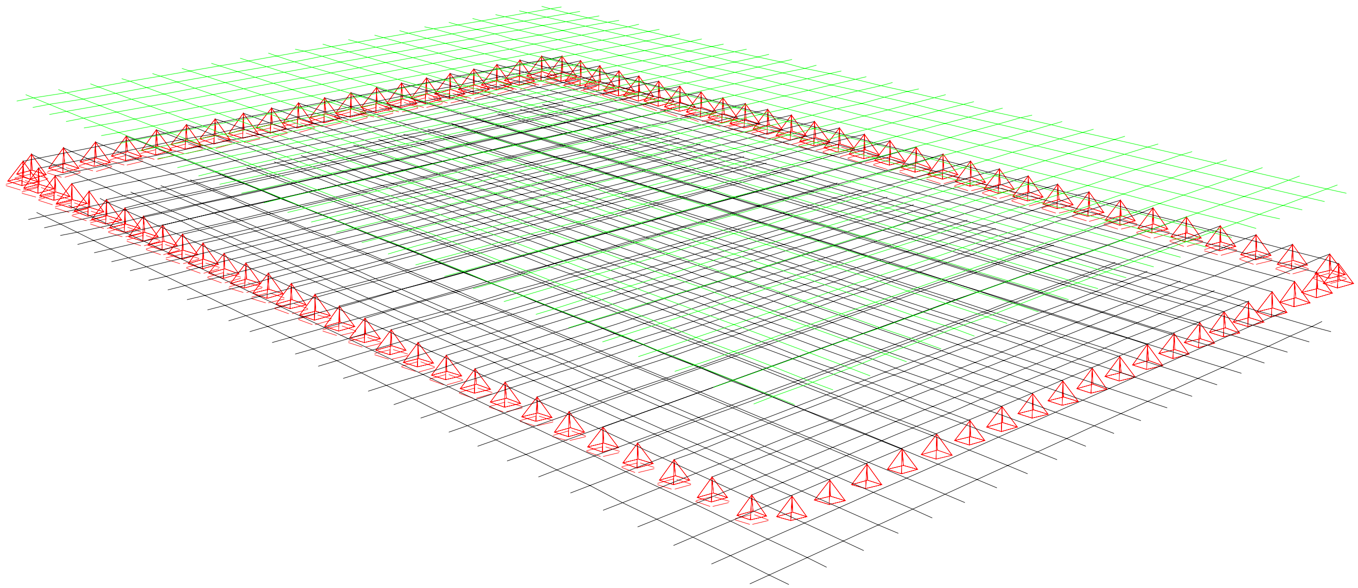
Frequency plate

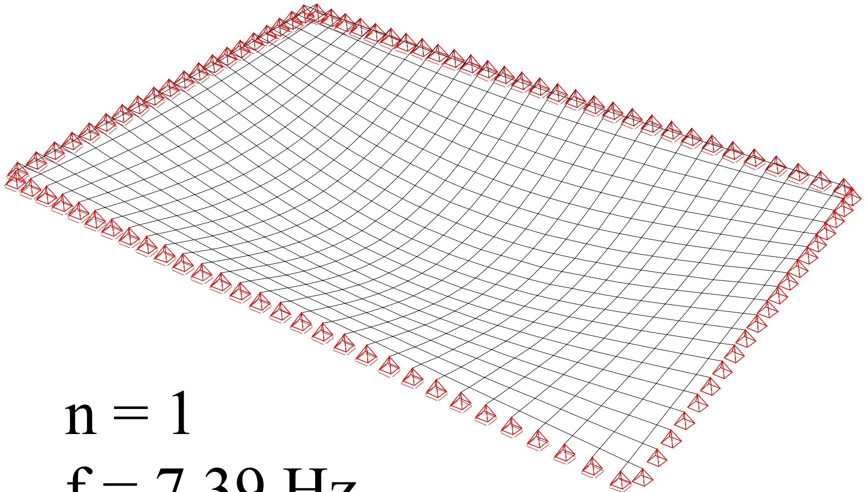
$$f_0 = \frac{\pi}{2 \cdot \ell^2} \cdot \sqrt{\frac{efEI}{m}} = \frac{\pi}{2 \cdot 4{,}2^2} \cdot \sqrt{\frac{2{,}50}{0{,}383 \cdot 10^{-3}}} = 7{,}19\ Hz$$

$$\sqrt{k^4 + \frac{EI_b \cdot \ell^4}{efEI \cdot b^4} \cdot n^4} = \sqrt{k^4 + \frac{0{,}7 \cdot 4{,}2^4}{2{,}50 \cdot 6{,}3^4} \cdot n^4} = \sqrt{k^4 + 0{,}055 \cdot n^4}$$

k Wellen in Tragrichtung	n Wellen senkrecht zur Tragrichtung								
	1	2	3	4	5				
1	7,39	8,88	16,8	27,9	42,8				
2	28,8	29,5	32,5	39,4					
3	66,5								

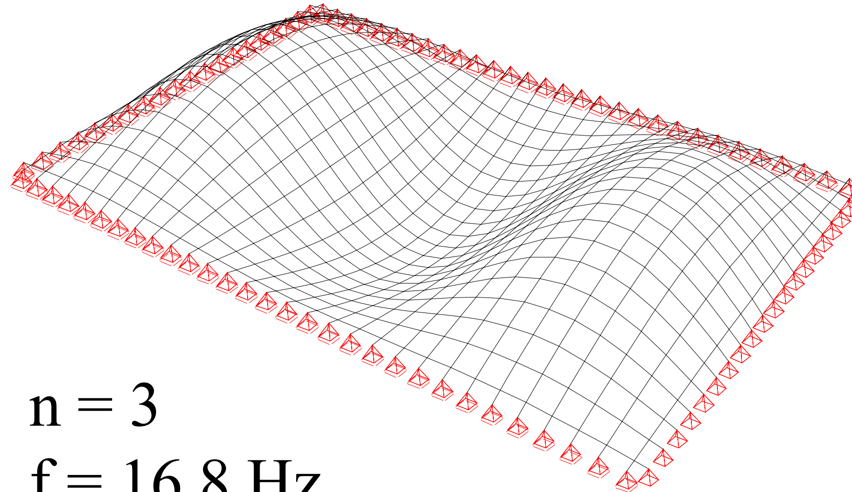
Frequencies





n = 1

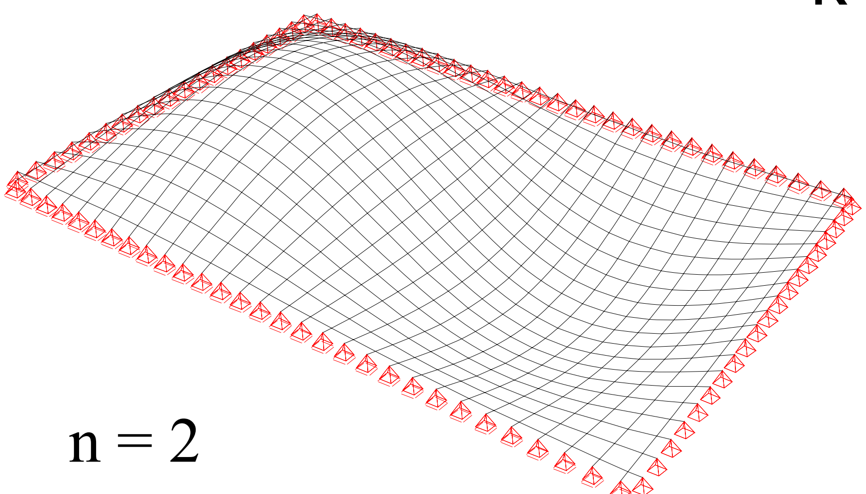
f = 7,39 Hz



n = 3

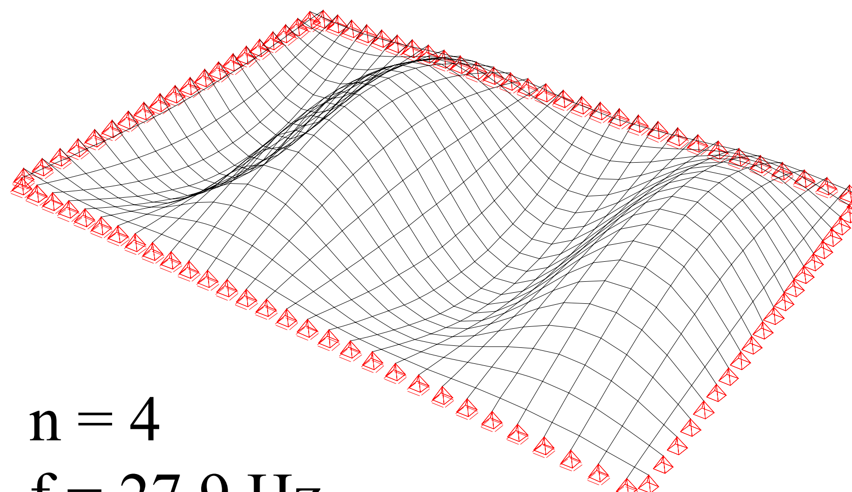
f = 16,8 Hz

k=1



n = 2

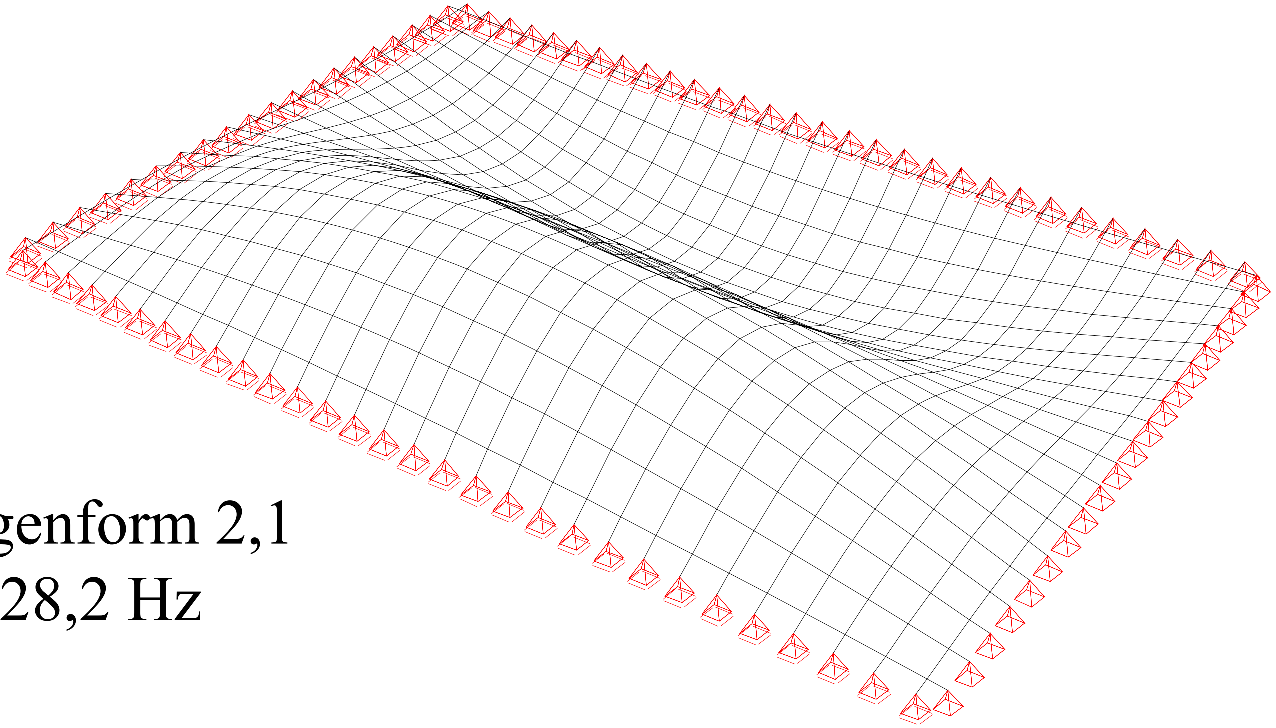
f = 8,88 Hz



n = 4

f = 27,9 Hz

k=2



Eigenform 2,1
f =28,2 Hz

deflection

k_{def} = 0,8, EDIN 1052, Tble 3.2

$$efEI(k_{def}) = \frac{efEI}{1 + k_{def}}$$

Quasi ständige Bemessungssituation nach Gleichung (42) DIN 1052				
1 W_{fin}	$(1 + k_{def})\cdot w_{G,inst}$ 1,8·5,25 = 9,4 mm	$\psi_2\cdot(1 + k_{def})\cdot w_{Q,inst}$ 0,3·(1+0,8)·3,25= 1,8 mm	11,2 mm	0,0112/ 4,2 = 1/375

1	deflection w	w < l/X	Enough rigidity Small deflection	7.2

Deflection single load

F = 1kN

$$w_F = \frac{F \cdot \ell^3}{48 \cdot EI_\ell \cdot b_F} = \frac{1 \cdot 10^{-3} \cdot 4{,}2^3}{48 \cdot 2{,}50 \cdot 2{,}78} = 0{,}22 \cdot 10^{-3}\ m = 0{,}22\ mm$$

$$b_F = \frac{\ell}{1{,}1} \cdot \sqrt[4]{\frac{EI_b}{EI_\ell}} = \frac{4{,}2}{1{,}1} \cdot \sqrt[4]{\frac{0{,}7}{2{,}50}} = 2{,}78\ m$$

0,22< 0,5 bis 4 mm

3	Durchbiegung Einzellast F=1kN	u <0,5 bis 4 mm	Querverteilung geringe Verformung	DIN ENV 1995-1-1 7.3.3 SIA 265

Frequency

$f_{1,1}$ = **7,19 Hz < 8 Hz !!**

4	frequency $w_{G,inst}$ quasi ständig, $g+\psi_2 p$	f >8 Hz (w < 0,5 cm) f < 8 Hz (w > 0,5 cm)	No resonance	7.3.3(2) 7.3.3(1)

Impuls I=1 Ns

$$v = \frac{4 \cdot (0{,}4 + 0{,}6 \cdot n_{40})}{m \cdot b \cdot \ell + 200} = \frac{4 \cdot (0{,}4 + 0{,}6 \cdot 5)}{383 \cdot 4{,}2 \cdot 6{,}3 + 200} = 0{,}00132 \; \frac{m}{s}$$

„better performance“

$$b^{(f \cdot \xi - 1)} = 100^{(7{,}19 \cdot 0{,}01 - 1)} = 0{,}014 \; \frac{m}{s}$$

5	Impuls I=1 Ns (up to 40 Hz)	$v<b^{(f_1 \zeta - 1)}$ 50 < b < 150 ζ = 0,01	Velocity	7.3.3(2) SIA 265

heeldrop I=55 Ns

$$v = \frac{0,6}{m^{0,5} \cdot EI_{\ell}^{\;0,25} \cdot EI_{b}^{\;0,25}} =$$

$$\frac{0,6}{383^{0,5} \cdot 2,50^{0,25} \cdot 0,70^{0,25}} = 0,027 \; \frac{m}{s}$$

6	velocity heeldrop I=55Ns, t_i = 0,05s	$v<6\ b^{(f_1\zeta-1)}$	velocity	Mohr /bmh/

Limit

6 . 0,014 = 0,084 m/s > 0,027

7.3.3 (1) f < 8 Hz: Special investigation

Decke: f = 7,19 Hz

Gehen: Resonanz

fs = 7,19 / 3 = 2,4 Hz

$$\frac{\alpha_3 \cdot F_0}{M} \cdot V(3 f_s / f_{1,1}) = \frac{0{,}1 \cdot 700}{2520} \cdot 33.3 = 0{,}925 \frac{m}{s^2}$$

$$0{,}4 \cdot 0{,}93 = 0{,}37\, m/s^2 > 0{,}1\, m/s^2$$

7	Beschleunigung Resonanz-untersuchung	a<0,1 m/s²	Beschleunigung Wohlbefinden	

Gehen: Resonanz

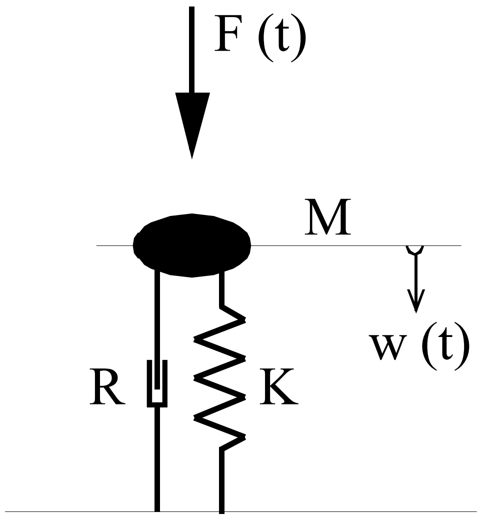
fs = 7,19 / 3 = 2,4 Hz

$$\frac{\alpha_3 \cdot F_0}{M} \cdot V(3 f_s / f_{1,1}) = \frac{0{,}1 \cdot 700}{2520} \cdot 33.3 = 0{,}925 \frac{m}{s^2}$$

$$0{,}4 \cdot 0{,}93 = 0{,}37\ m/s^2 < 0{,}7\ m/s^2$$

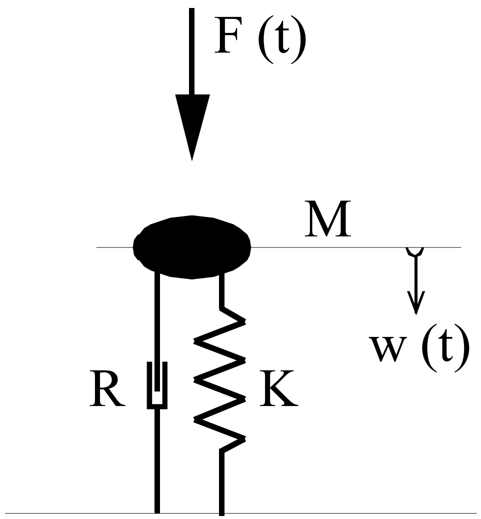
8	Beschleunigung Resonanz-untersuchung	a< 0,35 bis 0,7 m/s^2	Beschleunigung Spürbar, nicht störend	DIN1052, 9.3(3) besondere Unter-suchungen

Summary



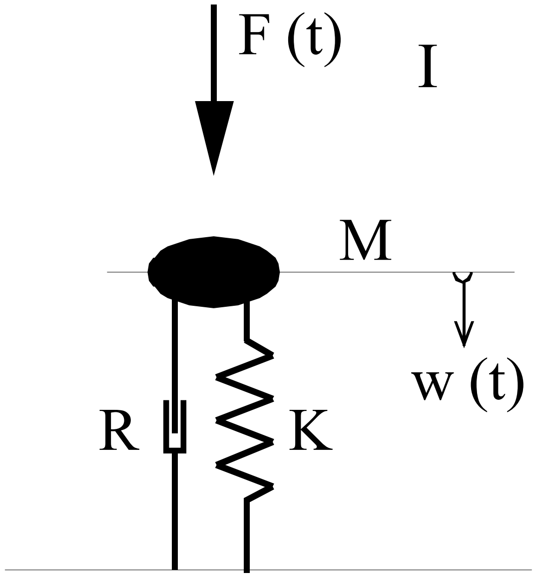
Deformation

$$w\left(\frac{1}{Rigidity}\right)$$



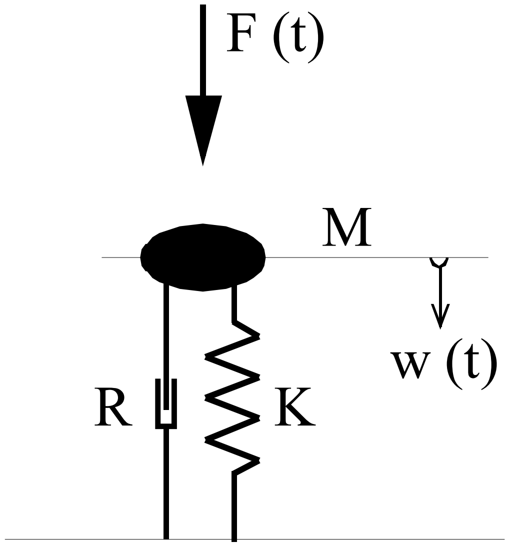
Frequency

$$f = f\left(\sqrt{\frac{rigidity}{mass}}\right)$$



Velocity

$$v\left(\frac{1}{mass}\right)$$



Acceleration

$$a\left(\frac{1}{mass}\frac{1}{damping}\right)$$

Wanted:	Timber floors
great stiffness	most
high mass	no
high frequency	not always
high damping	yes

Thank you very much for your attention!

Vielen Dank für's Zuhören !